

**NONDYNAMICAL AMPLITUDE RECONSTRUCTION IN KAON  
PHOTOPRODUCTION BY USING OF THE LATEST DATA  
AND COMPARING WITH PARTIAL WAVE ANALYSIS**

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Received 21 March 2005, in final form 9 May 2006, accepted 10 May 2006

The magnitudes of four independent complex amplitudes and two relative phases are determined in kaon photoproduction in three fixed incident photon energies. In order to reconstruct such amplitude determination in addition to differential cross section, we require six polarization parameters at different kaon scattering angles. Six available polarization parameters are used to calculate the complex amplitudes. This analysis indicates that the initial orientation of the spin polarization of the target nuclei is determinant for the probability of spin to flip after the interaction. A comparison between these results and the solution of the partial wave analysis has been presented.

PACS: 24.70.+s

## 1 Introduction

The nondynamical spin polarization analysis of the complex reaction amplitudes has been widely used in the last three decades by taking advantage of the measured polarization parameters [1]. The nondynamical spin polarization not only enables us to obtain dynamical information, it is also capable of checking the validity of conservation laws such as parity, time reversal and identical particles in hadronic interactions [2]. Such analysis can also provide experimentalists with the minimum number of experiments required for a complete determination of the magnitudes and phases of the amplitudes (called complete set) [3]. Once amplitudes are found, the dynamical models and validity of the assumptions made there could be investigated. The relations between the amplitudes of an elastic interaction and the lab observables are usually too complicated, but by introducing the optimal formalism [4] such reactions are considerably simplified and the reaction matrix is diagonalized as much as possible (block diagonalized). A number of important polarization experiments have been performed in the particle physics such as:

$$\begin{array}{ll} (1/2 + 1/2 \longrightarrow 1/2 + 1/2) & \text{(nucleon -nucleon scattering),} \\ (1 + 1/2 \longrightarrow 0 + 1/2) & \text{(kaon photoproduction),} \\ (0 + 1/2 \longrightarrow 0 + 1/2) & \text{(kaon-nucleon scattering).} \end{array}$$

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In this paper all efforts have been made to reconstruct the four independent hybrid reaction amplitudes and two relative phases for the interaction  $(1 + 1/2 \longrightarrow 0 + 1/2)$ . The results are then used to investigate the charged kaon photoproduction denoted as

$$\gamma + p \longrightarrow k^+ + \Lambda.$$

## 2 Theory

Choosing the proper formalism results in simpler relations between the observables and bilinear combination of the amplitudes. Let us consider the general interaction

$$S_A + S_B \longrightarrow S_C + S_D, \quad (1)$$

where  $S_A$ ,  $S_B$ ,  $S_C$ , and  $S_D$  are spins of the particles  $A$ ,  $B$ ,  $C$ , and  $D$  respectively. In order to obtain the reaction matrix in Eq. (1), the standard factorization procedure is used [5] to decompose this interaction into two constituent reactions each with one zero spin particle

$$\begin{aligned} S_A + 0 &\longrightarrow S_C + 0, \\ 0 + S_B &\longrightarrow 0 + S_D. \end{aligned} \quad (2)$$

The interaction matrix of the general interaction (1) is given in terms of the outer product of the two constituent reaction matrices which is given by [2]

$$M_1 = \sum_l \sum_\lambda D(\lambda, l) S^{\lambda l}, \quad (3)$$

where  $D(\lambda, l)$  are complex spin amplitudes and  $S^{\lambda l}$  are the spin-momentum tensors corresponding to the particles with spin  $S_A$  and  $S_C$  which will be a matrix of dimension  $(2S_A + 1) \times (2S_C + 1)$ ,  $l$  and  $\lambda$  are spin component along quantization axis for each particle. Considering a similar reaction matrix  $M_2$  for the second constituent reaction in (2), the general reaction matrix is given as

$$M = M_1 \otimes M_2 = \sum_{l, \lambda, L, \Lambda} D(\lambda, l; \Lambda, L) S^{\lambda l} \otimes S^{\Lambda L}. \quad (4)$$

All the dynamical information concerning the interaction mechanism contains in  $D$ 's. Since the spin momentum tensor in general is a  $(2S_C + 1) \times (2S_A + 1)$  matrix, there are  $(2S_C + 1) \times (2S_A + 1)$  independent complex amplitudes when only Lorentz invariance is considered.

Each of the constituent reaction in (2) is characterized by the initial density matrix  $\rho_i$  and a final density matrix  $\rho_f$ . The former is a  $(2S_A + 1) \times (2S_A + 1)$  matrix and the latter is a  $(2S_C + 1) \times (2S_C + 1)$  matrix. The final density matrix is given in terms of the initial density matrix [6]

$$\rho_f^{uv} = M \rho_i^{uv} M^\dagger. \quad (5)$$

The experimental observables are given by the expectation values of certain spin-momentum tensor  $Q$  in the final states as a  $(2S_C + 1) \times (2S_C + 1)$  matrix. Such observables are given by the quantum mechanics as a trace of the matrix obtained by multiplication of the final density

matrix and the final state spin-momentum tensor. Denoting these experimental observables by  $L(uv; \psi\omega)$  we have

$$L(uv; \psi\omega) = \langle Q^{\psi\omega} \rangle = \text{Tr}(Q^{\psi\omega} \rho_F^{uv}) = \text{Tr}(Q^{\psi\omega} M \rho_i^{uv} M^\dagger). \quad (6)$$

Now it is time to impose the optimal conditions to simplify the relations between the observables and the bilinear combination of the amplitudes. In this formalism the initial and final density matrices as well as the spin momentum tensors are defined in such a way that the elements of the general interaction matrix will be as diagonal as possible. The best way is to choose the density matrices and spin momentum tensors to be all hermitian [4]. In addition to Lorentz invariance, in the case of kaon photoproduction parity is also conserved, reducing the number of independent amplitudes to only six. Conservation laws are used in a variety of frames [2], here hybrid frame is used for more simplicity. In this frame the polarization direction of the photon is in the reaction plane and in the direction of its momentum, but the polarization of nucleon is in the perpendicular direction to the reaction plane. For charged kaon photoproduction, since photon has only two directions of polarization, the zero component of spin is omitted and only two directions of polarization amplitudes remain. There are two nonflip  $D(+, 11)$ ,  $D(+, 22)$  and two spin flip  $D(+, 12)$  and  $D(+, 21)$  amplitudes, where numbers 1 and 2 correspond to the positive and negative components of the nucleon spin respectively.

### 3 Amplitude Determination

The final procedure in amplitude determination [7] is the averaging and differentiating on the observables. The number of available experimental observables for charged kaon are  $d\sigma/d\Omega$  (the unpolarized differential cross-section),  $P(\theta)$  (the recoil nucleon polarization),  $T(\theta)$  (the conventional polarized target asymmetry),  $\Sigma(\theta)$  (the conventional polarized photon asymmetry) and the double polarization parameters namely  $H(\theta)$  and  $G(\theta)$  standing for, the values of these observables are collected from Refs. 7 and 8. These six experimental observables are not sufficient for complete determination of four independent complex amplitudes, but they are possible by introducing the following relations and using the optimal formalism [5]

$$\frac{1}{32}L(\tilde{A}, A; A) = |D(+, 11)|^2 + |D(+, 21)|^2 + |D(+, 12)|^2 + |D(+, 22)|^2, \quad (7)$$

$$\frac{1}{32}L(\tilde{A}, \Delta; A) = |D(+, 11)|^2 + |D(+, 21)|^2 - |D(+, 12)|^2 - |D(+, 22)|^2, \quad (8)$$

$$\frac{1}{32}L(\tilde{A}, \Delta; \Delta) = |D(+, 11)|^2 - |D(+, 21)|^2 - |D(+, 12)|^2 + |D(+, 22)|^2, \quad (9)$$

$$\frac{1}{32}L(\tilde{A}, A; \Delta) = |D(+, 11)|^2 - |D(+, 21)|^2 + |D(+, 12)|^2 - |D(+, 22)|^2, \quad (10)$$

where  $\tilde{A}$  and  $A$  represent the averaging on photon and nucleon states respectively and  $\Delta$  represent the differentiating. The first relation (7) corresponds to the unpolarized differential cross section  $d\sigma/d\Omega(\theta)$ . The factor  $\frac{1}{32}$  is the result of averaging and differentiating the observables with respect to different spin projections of target nucleon along quantization axis. The other three

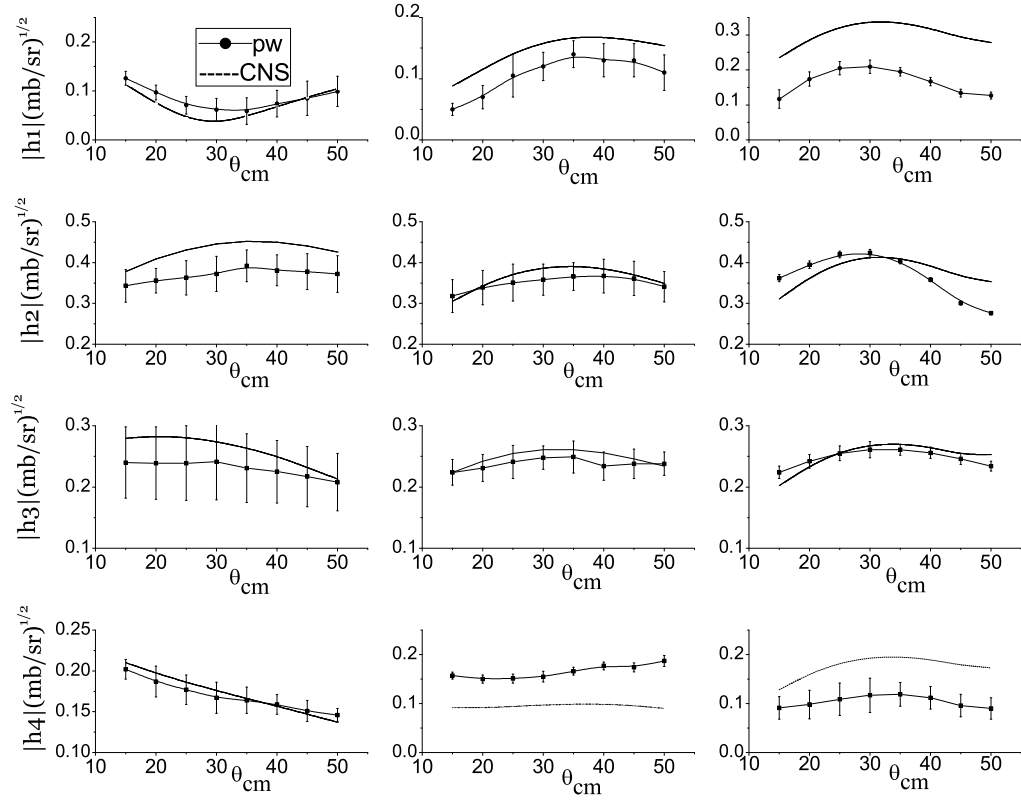


Fig. 1. The magnitudes of four independent amplitudes as a function of kaon center of mass scattering angles at fixed photon energies (1500 MeV) for charged kaon photoproduction.

observables are related as

$$L(\tilde{A}, \Delta; A) \sim \Sigma(\theta), \quad (11)$$

$$L(\tilde{A}, \Delta; \Delta) \sim T(\theta), \quad (12)$$

$$L(\tilde{A}, A, \Delta) \sim P(\theta). \quad (13)$$

Let us define the complex amplitudes in hybrid frame in terms of the two parameters  $\alpha_i$  and  $h_i$

$$D(+, 11) = -D(-, 11) = h_1 e^{i\alpha_1},$$

$$D(+, 21) = +D(-, 21) = h_2 e^{i\alpha_2},$$

$$D(+, 12) = +D(-, 12) = h_3 e^{i\alpha_3},$$

$$D(+, 22) = -D(-, 22) = h_4 e^{i\alpha_4},$$

then equations (7)–(10) give

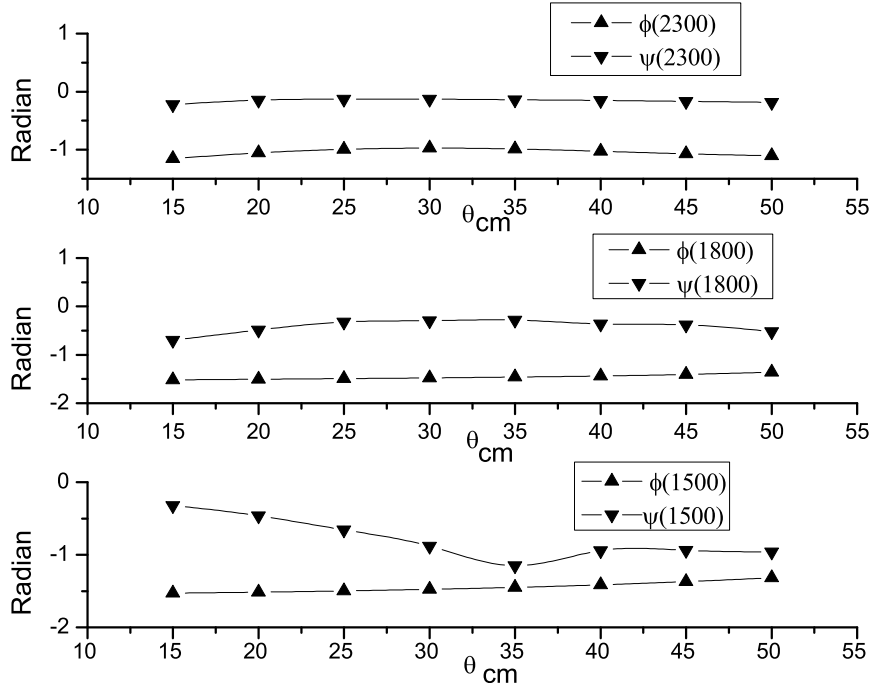


Fig. 2. The phase differences  $\Phi$  and  $\Psi$  are plotted in hybrid frame as a function of kaon center of mass scattering angles at fixed photon energies.

$$|h_1| = \frac{(d\sigma/d\Omega)^{1/2}}{2} [1 + \Sigma(\theta) + T(\theta) + P(\theta)]^{1/2}, \quad (14)$$

$$|h_2| = \frac{(d\sigma/d\Omega)^{1/2}}{2} [1 + \Sigma(\theta) - T(\theta) - P(\theta)]^{1/2}, \quad (15)$$

$$|h_3| = \frac{(d\sigma/d\Omega)^{1/2}}{2} [1 - \Sigma(\theta) - T(\theta) + P(\theta)]^{1/2}, \quad (16)$$

$$|h_4| = \frac{(d\sigma/d\Omega)^{1/2}}{2} [1 - \Sigma(\theta) + T(\theta) - P(\theta)]^{1/2}. \quad (17)$$

The magnitudes of four independent amplitudes are unambiguously constructed for the energy range of 1.5 to 2.3 GeV. For a complete determination of the magnitudes and phases, at least seven observables are needed. But only six observables are available; therefore, as mentioned before a subset of the relations are chosen so that the phase differences could be determined in terms of the two remaining observables namely  $H(\theta)$  and  $G(\theta)$  standing for double polarization parameters. The magnitudes of amplitudes and the phase differences are plotted versus the charged kaon center of mass scattering angles at different incident photon energies respectively in Fig. 1 and 2. In Fig. 1 we have also compared the results of the present work (p.w.) with the partial-wave analysis predictions using the CNS Data Analysis Center sp98 program [9].

#### 4 Conclusion

The kaon-photoproduction scattering amplitude analysis was performed using complete sets of observables at three incident photon energies. Symmetry constraints and optimal conventions have reduced the number of independent amplitudes to only four,  $D(+, 12)$ ,  $D(+, 21)$  are corresponded to spin flip and  $D(+, 11)$ ,  $D(+, 22)$  are corresponded to spin non-flip amplitudes. As indicated in Fig. 1 at all measured kaon scattering angles and three incident photon energies, the spin flip amplitudes  $D(+, 21)$  and  $D(+, 12)$  are considerably larger than non flip  $D(+, 11)$  and  $D(+, 22)$ . The magnitudes of amplitudes and also the phase differences  $\phi$  and  $\psi$  in the energy range of 1.5 to 2.3 GeV are smooth. In general we can say that the initial orientation of spin polarization of the target nuclei plays a significant role in the probability of spin to flip. We also have found that the results of the present work and the those of partial-wave analysis sp98 are in good agreements at all energies and kaon scattering angles.

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