acta physica slovaca vol. 56 No. 4, 463 – 472 August 2006

CALCULATION OF THE EXPONENT $\lambda_g(x,t)$ BASED ON THE BEHAVIOR OF STEEPLY RISING GLUON DISTRIBUTION FUNCTION AT LOW-x

G.R. Boroun[1], **B. Rezaie**

Physics Department, Razi University, Kermanshah 67149, Iran

Received 12 December 2005, in final form 13 April 2006, accepted 25 April 2006

The gluon distribution function exponent, λ_g, is calculated by using power law behavior of gluon distribution function at small x. Also, at low x, the derivative of structure function with respect to t and the gluon distribution function are determined. In the calculations, the leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equations are usedi. The calculated values are compared with other methods. The obtained results are very close to the calculated values by the leading order Glück, Reya and Vogt and in agreement with experimental data.

PACS: 13.60.Hb, 14.70.Dj

1 Introduction

Deep inelastic Lepton-Nucleon scattering experiments have traditionally shed light on the nature of the partons within the proton and the strong QCD interactions between them [1]. The standard perturbative QCD framework predicts that at a regime of low values of the Bjorken variable x and large values of Q^2 (Q^2 is the four momenta transfer in deep inelastic scattering process), a nucleon consists predominantly of the sea quarks and gluons. The gluons coupled only through the strong interaction, consequently the gluons are not directly probed in the DIS. But at low-x, structure function $F_2(x,Q^2)$ is dominated by gluons and Dokshitzer- Gribov- Lipatov- Altarelli-Parisi (DGLAP) equations [2] can be approximately solved that led to several approximate phenomenological schemes [3–5]. These methods of approximate determination of the gluon density are based on the simplification of the convolution $P_{qg}\otimes g$ by the expansion of the gluon density. The result is the gluon distribution function $G(k.x)$ proportional to the derivative of $F_2(x,Q^2)$, with respect to $\ln Q^2$; i.e. $\partial F_2(x,Q^2)/\partial \ln Q^2$, where k is associated with the choice of a point of expansion. The methods of approximate determination of gluon density are discussed in the following section.

Prytz reported a method to obtain an approximate relation between the unintegrated gluon density and the scaling violations of the quark structure function at low x at leading order [3]. He expanded $G(\frac{x}{1-z})$ using the Taylor expansion formula at $z=\frac{1}{2}$. Hence one gets:

$$dF_2/dt = 10\alpha_s/27\pi G(2x)$$

[1]E-mail address: boroun@razi.ac.ir

for fixed t (the variable t is defined as $t = \ln(\frac{Q^2}{\Lambda^2})$ and Λ is the QCD cut- off parameter and α_s is the running coupling constant), which is the main result for the leading order analysis [3].

Bora and Choudhury also presented a method [5] to find the gluon distribution from the F_2 proton structure function and its scaling violation at low x using the expansion of gluon distribution around $z = 0$. In the limit $x \rightarrow 0$, the last result becomes [5]

$$dF_2/dt = 15\alpha_s/36\pi G\left(\frac{4}{3}x\right).$$

Also, Gay Ducati and Goncalves presented a complete method [6] to find the gluon distribution by using the expansion of the $G(\frac{x}{1-z})$ at an arbitrary $z = a$ that the gluon distribution for a point of expansion $a<1$ (the better choice is at $a = 0.75$) can be expressed by

$$dF_2/dt = 10\alpha_s/27\pi G\left[\frac{x}{1-a}\left(\frac{3}{2} - a\right)\right].$$

We know that in the DGLAP formalism at leading order the gluon splitting function is singular as $x \rightarrow 0$. Thus the gluon distribution will become large as $x \rightarrow 0$, this will generate a steeply rising gluon distribution at small x. That is, the perturbative QCD predicts a very strong power law rise of the gluon density in the limit $x \rightarrow 0$, where as usual, x denotes the momentum fraction carried by the gluon and Q^2 is the scale at which the distribution is probed. Over the x, Q^2 range of HERA data this solution mimics a power law behavior as follows

$$xg(x, Q^2) \sim x^{-\lambda_g}.$$

In this work, we concentrate on the computation of λ_g into x and $t(t = \ln\frac{Q^2}{\Lambda^2})$ variables with the use of the leading order Dokshitzer- Gribov- Lipatov- Altarelli- Parisi (LO-DGLAP) evolution equations. Then with the determined $\lambda_g(x,t)$, the gluon distribution function and the derivative of the structure function are calculated.

The contents of our paper are as follows. In section 2 we give a formalism for the exponent gluon density into the solution of the DGLAP equations with the starting distribution provided along the QCD parton distribution function. Finally, a numerical analysis of our solutions is presented and the obtained results are compared with other methods which are followed by conclusions and results.

2 Theoretical calculations

The precision measurements of the ep scattering cross-section show that $G(x, Q^2)$ strongly rises towards low-x for values of $Q^2 \geq 4$ GeV^2 [7]. This leading contributions can be resumed using the Balitsky- Fadin- Kuraev- Lipatov (BFKL) [8], and eventually give rise to the characteristic behavior at very small x

$$G(x,t) = f(t)x^{-\lambda_g(x,t)}. \tag{1}$$

In the DGLAP evolution equations, the quark densities and the nonsinglet contribution F_2^{NS} can be ignored safely at low-x. Thus, the Altarelli- Parisi (AP) evolution equation for F_2 becomes(for four flavours)

$$\frac{dF_2}{dt} = \frac{10\alpha_s(t)}{9\pi}\int_x^1 dz P_{qg}(z) G\left(\frac{x}{z}, t\right) \tag{2}$$

where $G(x,t) = xg(x,t)$ and $g(x,t)$ are the gluon momentum density and the gluon number of proton, respectively. Also, the leading order splitting function is defined as

$$P_{qg}(z) = \frac{1}{2}[z^2 + (1-z)^2] \tag{3}$$

and $\alpha_s(t) = \frac{12\pi}{(33-2N_f)t}$ is the strong coupling constant (N_f = number of flavours). Substituting the steep power law behavior (Eq. (1)) into Eq. (2), we get

$$\frac{dF_2}{dt} = \frac{5\alpha_s(t)}{9\pi}\int_x^1 dz[z^2 + (1-z)^2]f(t)(\frac{x}{z})^{-\lambda_g(x,t)}. \tag{4}$$

This equation can be rearranged as

$$\frac{dF_2}{dt} = \frac{5\alpha_s(t)}{9\pi}G(x,t)\int_x^1 dz[z^2 + (1-z)^2]z^{\lambda_g(x,t)} \tag{5}$$

or

$$\begin{aligned}\frac{dF_2}{dt} &= \frac{5\alpha_s(t)}{9\pi}G(x,t)\left[\frac{2}{3+\lambda_g(x,t)}(1-x^{3+\lambda_g(x,t)}) + \frac{1}{1+\lambda_g(x,t)}(1-x^{1+\lambda_g(x,t)})\right.\\ &- \left.\frac{2}{2+\lambda_g(x,t)}(1-x^{2+\lambda_g(x,t)})\right].\end{aligned} \tag{6}$$

Now, Let us assume the derivative of the structure function F_2 is a constant in terms of t [9, 10]. Therefore, the derivative of the Eq. (6) with respect to t, gives

$$0 = \frac{5}{9\pi}A_{\lambda_g(x,t)}G(x,t)\frac{d\alpha_s}{dt} + \frac{5\alpha_s}{9\pi}B_{\lambda_g(x,t)}G(x,t)\frac{d\lambda_g(x,t)}{dt} + \frac{5\alpha_s}{9\pi}A_{\lambda_g(x,t)}\frac{dG(x,t)}{dt}, \tag{7}$$

where

$$\begin{aligned}A_{\lambda_g(x,t)} &= \frac{2}{3+\lambda_g(x,t)}(1-x^{3+\lambda_g(x,t)}) + \frac{1}{1+\lambda_g(x,t)}(1-x^{1+\lambda_g(x,t)})\\ &- \frac{2}{2+\lambda_g(x,t)}(1-x^{2+\lambda_g(x,t)})\end{aligned} \tag{8}$$

and

$$\begin{aligned}B_{\lambda_g(x,t)} &= \frac{2}{(2+\lambda_g(x,t))^2}(1-x^{2+\lambda_g(x,t)}) - \frac{2}{(3+\lambda_g(x,t))^2}(1-x^{3+\lambda_g(x,t)})\\ &- \frac{1}{(1+\lambda_g(x,t))^2}(1-x^{1+\lambda_g(x,t)}) + \left(\frac{2}{2+\lambda_g(x,t)}x^{2+\lambda_g(x,t)}\right.\\ &- \left.\frac{2}{3+\lambda_g(x,t)}x^{3+\lambda_g(x,t)} - \frac{1}{1+\lambda_g(x,t)}x^{1+\lambda_g(x,t)}\right)\ln x.\end{aligned} \tag{9}$$

In Eq. (7), $dG(x,t)/dt$ is obtained from the DGLAP evolution equation [2]. At low x, the quarks can be neglected in the AP evolution equation [11]. Hence, we have

$$\begin{aligned}\frac{dG(x,t)}{dt} &= \frac{3\alpha_s(t)}{\pi}\left[\left\{\left(\frac{11}{12}-\frac{N_f}{18}\right)+\ln(1-x)\right\}G(x,t)\right.\\ &+ \left.\int_x^1 dz\frac{zG(\frac{x}{z},t)}{1-z}-\frac{G(x,t)}{1-z}+\left(z(1-z)+\frac{1-z}{z}\right)G\left(\frac{x}{z},t\right)\right]. \qquad (10)\end{aligned}$$

Putting Eq. (1) in Eq. (10), we obtain:

$$\begin{aligned}\frac{dG(x,t)}{dt} &= \frac{3\alpha_s(t)}{\pi}\left[\left\{\left(\frac{11}{12}-\frac{N_f}{18}\right)+\ln(1-x)\right\}G(x,t)\right.\\ &+ G(x,t)\left\{\int_x^1 dz\left[\frac{z^{1+\lambda_g(x,t)}}{1-z}-\frac{1}{1-z}\right.\right.\\ &+ \left.\left.\left.z(1-z)z^{\lambda_g(x,t)}+\frac{1-z}{z}z^{\lambda_g(x,t)}\right]\right\}\right] \qquad (11)\end{aligned}$$

or

$$\frac{dG(x,t)}{dt}=\frac{12}{\beta_0 t}G(x,t)I_{\lambda_g(x,t)}, \qquad (12)$$

where

$$\begin{aligned}I_{\lambda_g(x,t)} &= \left(\frac{11}{12}-\frac{N_f}{18}\right)+\ln(1-x)\\ &\quad +\int_x^1 dz\left[\frac{z^{\lambda_g(x,t)+1}-1}{1-z}+(1-z)(z^{\lambda_g(x,t)+1}+z^{\lambda_g(x,t)-1})\right] \qquad (13)\end{aligned}$$

and

$$\begin{aligned}\int_x^1 \quad dz \quad &\left[\frac{z^{\lambda_g(x,t)+1}-1}{1-z}+(1-z)(z^{\lambda_g(x,t)+1}+z^{\lambda_g(x,t)-1})\right]=\frac{2}{2+\lambda_g(x,t)}\\ \times \quad &(1-x^{2+\lambda_g(x,t)})-(1-x)-\frac{1}{2}(1-x^2)\\ + \quad &\frac{1}{\lambda_g(x,t)}(1-x^{\lambda_g(x,t)})-\frac{1}{1+\lambda_g(x,t)}(1-x^{1+\lambda_g(x,t)})\\ + \quad &\sum_{N=4}^{\infty}\left[\frac{1}{N+\lambda_g(x,t)}(1-x^{N+\lambda_g(x,t)})-\frac{1}{N-1}(1-x^{N-1})\right]. \qquad (14)\end{aligned}$$

Substituting the derivative of gluon density ($dG(x,t)/dt$) according to Eq. (12) into Eq. (7), gives

$$\frac{d\lambda_g(x,t)}{dt}B_{\lambda_g(x,t)}=\frac{1}{t}A_{\lambda_g(x,t)}\left[1-\frac{36}{25}I_{\lambda_g(x,t)}\right], \qquad (15)$$

that

$$\int_{\lambda_{0g}}^{\lambda_g} d\lambda_g(x,t)\frac{B_{\lambda_g(x,t)}}{A_{\lambda_g(x,t)}[1-\frac{36}{25}I_{\lambda_g(x,t)}]}=\int_{t_0}^{t}\frac{dt}{t}. \qquad (16)$$

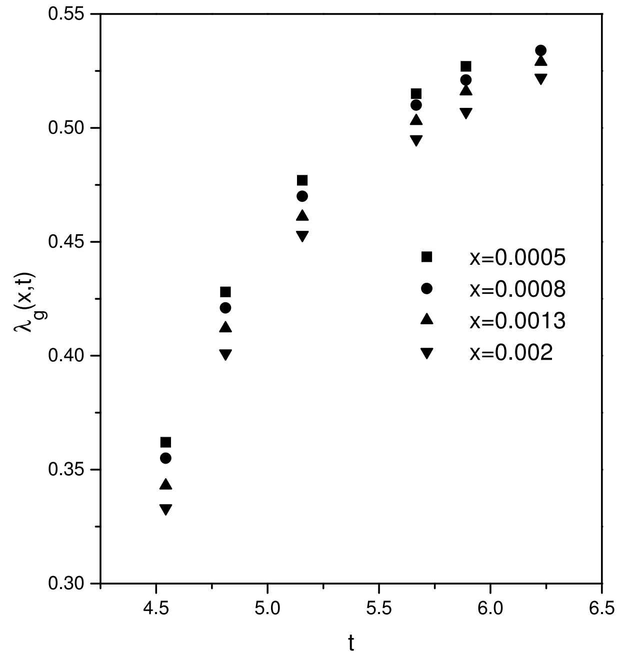

Fig. 1. Exponent $\lambda_g(x,t)$ plotted against t at several x values.

The quantity λ_g is calculated by solving this integral as a function of t at fixed x. The $\lambda_g(x,t)$ values are calculated for any x and t variables (λ_{0g} is the exponent at the starting scale $t_0(=\ln(\frac{Q_0^2}{\Lambda^2}))$). With respect to the calculated values for $\lambda_g(x,t)$ and Eq. (12), we can calculate the gluon distribution function. Therefore integration Eq. (12) becomes:

$$G(x,t) = G(x,t_0)\ \exp\left[\frac{12}{\beta_0}\int_{t_0}^{t} I_{\lambda_g(x,t)}\frac{dt}{t}\right]. \tag{17}$$

In this equation $G(x,t_0)$ is the input gluon distribution function and $I_{\lambda_g(x,t)}$ is defined by Eq. (13).

3 Conclusions and Results

It is known that perturbative QCD predicts a universal growth of the gluon distribution function at large t and small x. Because, in the DGLAP formalism at leading order the gluon splitting functions are singular as $x\rightarrow 0$. Over the x and t range of HERA data this solution mimics a power behavior [12], thus the gluon distribution function will become large as $x\rightarrow 0$. Hence, we used this point and obtained a description of the exponent $\lambda_g(x,t)$ and gluon distribution function given by Eqs. (16) and (17) into x and t variables.

We have therefore attempted to see how the predictions with this approach are compared with those of the QCD gluon distribution, like leading order Glück, Reya and Vogt (LO-GRV)

Tab. 1. The values of $\lambda_g(x,t)$, the gluon distribution function $G(x,t)$ and the derivative of structure function at t constant ($Q^2 = 20$ GeV^2) for any x values. $G(x,t)_{LO-GRV}$ obtained from Ref.[13].

x	λ_g	$G(x)$	$G(x)_{LO-GRV}$	$\frac{dF_2}{dt}$
0.000268	0.521	31.929	40.263	0.649
0.000328	0.519	29.629	36.852	0.603
0.000500	0.515	25.247	30.518	0.516
0.000800	0.510	20.994	24.566	0.430
0.001300	0.503	17.240	19.469	0.355
0.002000	0.495	14.380	15.711	0.297
0.003200	0.485	11.685	12.306	0.243
0.005000	0.472	9.5080	9.6450	0.200
0.010000	0.444	6.7400	6.4250	0.144
0.032000	0.355	3.4640	2.9190	0.078

[13]. Our calculations are presented in the full kinematic range available for any x values and representative Q^2 values $6.5, 8.5, 12, 20, 25, 35\,\mathrm{GeV}^2$ [14]. The results obtained are presented in Table 1 only at $Q^2 = 20$ GeV^2. In this Table, We see the results for $\lambda_g(x,t)$; as can be seen, λ_g rise with t at x constant (see Fig. 1) and decreases almost smoothly with x at t constant (see Fig. 2). Of course, as x increases above 10^{-2}, λ_g falls sharply with increasing x.

Based upon the calculated values for $\lambda_g(x,t)$, the gluon distribution functions by Eq. (17) are calculated. In Fig. 3, we compare our results with the already existing description (LO-GRV [13]). The difference between the two increases as x decreases, such difference exists as Q^2 increases. As can be seen, the values $G(x,Q^2)$ increase as x decreases. These results are in very good agreement with the QCD results.

To illustrate better our results, we can calculate the derivative of structure function by Eq. (6) that can be seen in Table 1. In doing so, the $\lambda_g(x,t)$ and $G(x,t)$ data given in Table 1 are used. In Fig. 4, we see that the derivative of F_2 with respect to t, increases as x decreases. This is capable of with perturbative quantum chromodynamics (PQCD) information for gluon density at this x range.

Tab. 2. The calculated values of the derivative of structure function, compared with the experimental data [9] at $Q^2 = 20$ GeV^2. The total error is the squared sum of the statistical and systematic uncertainties, given as absolute values in Ref. [9]. Also the percentage of difference between $\frac{dF_2}{dt}$ theory and experimental are given in this table.

x	$\frac{dF_2}{dt}$ (Our results)	$\frac{dF_2}{dt}$ (Exper. results)	$\Delta\frac{dF_2}{dt}/\frac{dF_2}{dt}(\%)$
0.00085	0.42	0.45 ± 0.08	6.6
0.00155	0.33	0.30 ± 0.09	10.0
0.00268	0.26	0.25 ± 0.05	4.0
0.00465	0.21	0.23 ± 0.04	8.7

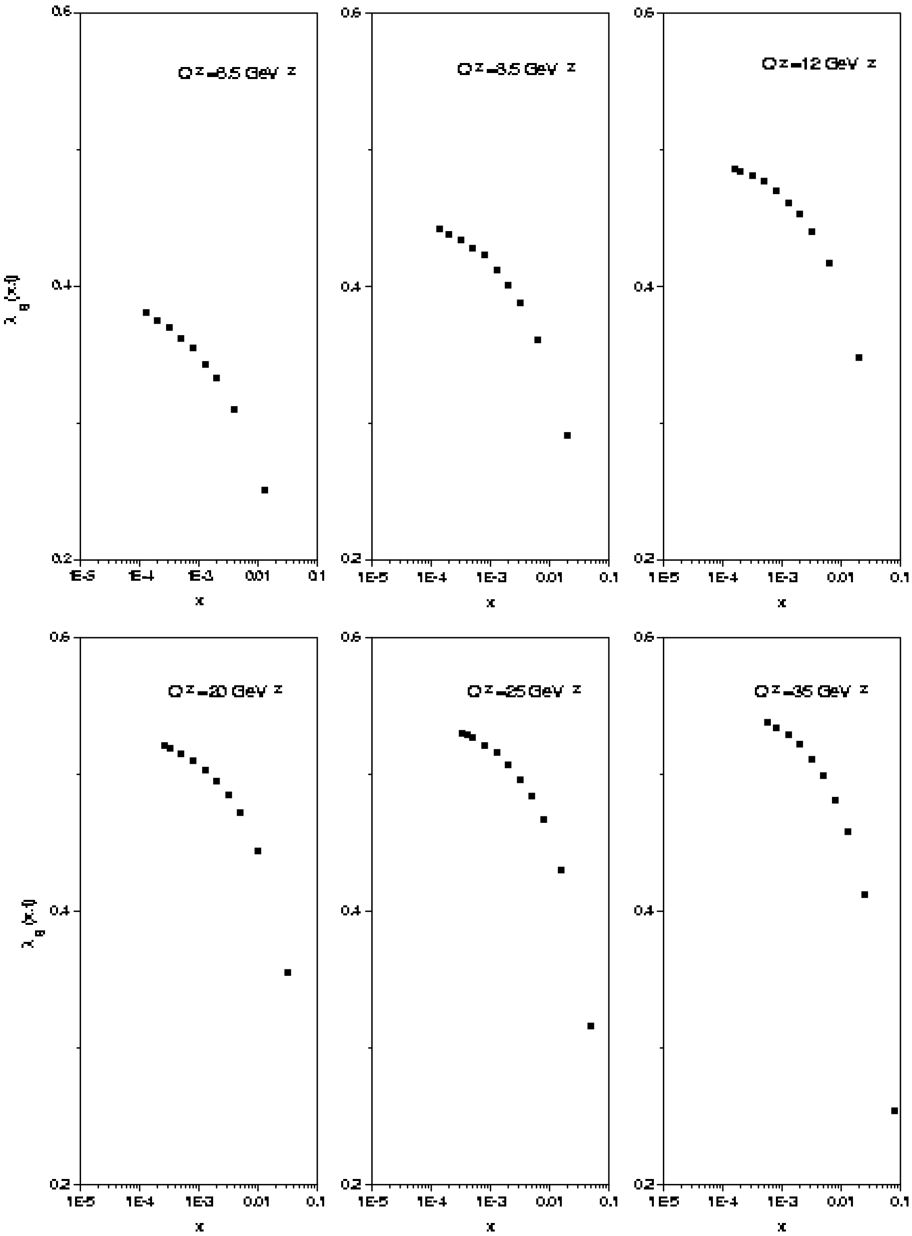

Fig. 2. Exponent $\lambda_g(x,t)$ plotted against x at several Q^2 values.

In order to test the validity and correctness of our calculations for $\lambda_g(x,t)$, $G(x,t)$ and $(\partial F_2/\partial t)$ at Eqs. (16), (17) and (6), we compare results with the experimental data [9] shown in

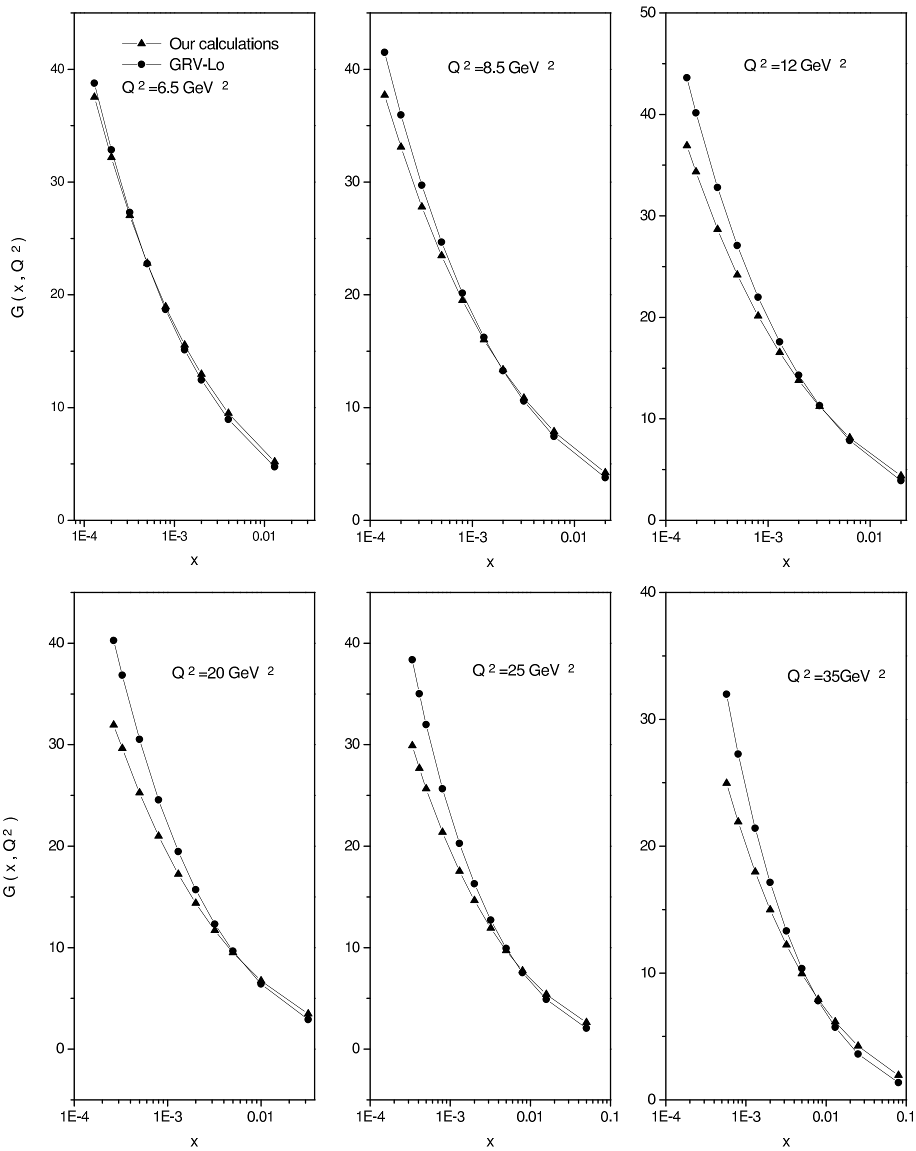

Fig. 3. $G(x, t)$ as function of x at Q^2 (in GeV^2) values 6.5; 8.5; 12; 15; 20; 25; 35 that compared with $G(x, t)$ obtained from LO-GRV [13].

Table 2 for $Q^2 = 20$ GeV^2. The percentage of the difference between our calculations is compared with experimental data as given in Table 2. Based on these calculations, we have tried to estimate whether the violation is consistent with the analytical prediction of formula (6); Hence we compared our results for the gluon distribution function with some methods $[3, 5, 6]$ proposed

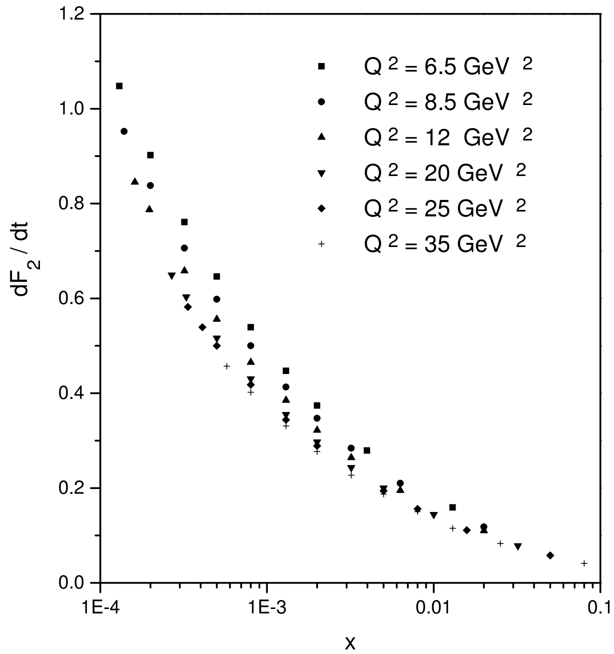

Fig. 4. The derivative of structure functions $(\partial F_2/\partial t)$ plotted as functions of x for fixed Q^2.

in the literature to isolate the gluon density by expansion, and tested validity results with the standard QCD parton distribution [13] (see Table 3).

To summarize, we have obtained an approximation method based upon growth of the gluon distribution function at small x. In this method, the exponent $\lambda_g(x,t)$ is computed from the LO-DGLAP evolution equations into x and t components. Hence, we can propose a form of gluon distribution at low x which is based on $\lambda_g(x,t)$ exponent. Then we have tested its validity by comparing with a global fit due to LO-GRV. As can be seen, the gluon distribution will increase as usual when x decreases. These results are in good agreement with the LO-GRV94. Then, the derivative of structure functions $(\partial F_2/\partial t)$ are calculated into gluon distribution functions and $\lambda_g(x,t)$ at any x and t value, and are compared with the experimental results. It is concluded that the proposed method in addition to being very simple, provides relatively accurate values

Tab. 3. The percentage of difference between the our method and the other results for the gluon distribution, compared with LO-GRV.

x	Our results	Prytz. results [3]	Bora. results [5]	D.&G. results [6]
0.00085	14.1%	37.1%	44.1%	37.1%
0.00155	10.2%	43.9%	50.2%	43.9%
0.00268	6.5%	38.3%	45.1%	38.3%
0.00465	2.0%	23.6%	32.1%	23.6%

for the $\lambda_g(x,t)$, gluon distribution function and derivative of structure function. Moreover, we solve only leading order evolution equations. We expect that next- to- leading order equations are more correct and their solutions will give better fit to global data and parameterizations.

References

[1] R.G. Roberts: *The structure of the proton*, Cambridge University Press, Cambridge 1990

[2] Yu.L. Dokshitzer: *Sov. Phys. JETPG.* **46** (1977) 641; G. Altarelli, G. Parisi: *Nucl. Phys. B* **126** (1997) 298; V.N. Gribov, L.N. Lipatov: *Sov. J. Nucl. Phys.* **28** (1978) 822

[3] K. Pretz: *Phys. Lett. B* **311** (1993) 286

[4] A.M. Cooper-Sarkar et al.: *Z. Phys. C* **39** (1988) 281

[5] K. Bora, D.K. Choudhury: *Phys. Lett. B* **354** (1995) 151

[6] M.B. Gay Ducati, V.P.B. Goncalves: *Phys. Lett. B* **390** (1997) 401

[7] A.M. Cooper-Sarkar, R.C.E. Devenish: *Acta. Phys. Polon. B* **34** (2003) 2911

[8] E.A. Kuraev, L.N. Lipatov, V.S. Fadin:*Phys. Lett. B* **60** (1975) 50; *Sov. Phys. JETP.* **44** (1976) 433; *ibid.* **45** (1977) 199; Ya.Ya. Balitsky, L.N. Lipatov: *Sov. Nucl. Phys.* **28** (1978) 822

[9] M. Derrick et al. *ZEUS Collab: Phys. Lett. B* **345** (1995) 576

[10] S. Aid et al.: $H1$ *Collab: Phys. Lett. B* **354** (1995) 494

[11] L.F. Abbott et al.: *Phys. Rev. D* **22** (1980) 582

[12] R.D. Ball, F. Fote: *Phys. Lett. B* **335** (1994) 77

[13] M. Glück, E. Reya, A. Vogt: *Z.Phys.C* **67** (1995) 433

[14] C. Adloff et al.: $H1$ *Collab: Eur. Phys. J. C* **21** (2001) 33