

**STUDYING THE DOUBLE HIGGS PRODUCTION  
VIA  $e^+e^- \rightarrow b\bar{b}HH$  AT FUTURE  $e^+e^-$  COLLIDERS**

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We examine the double Higgs production process with the reaction  $e^+e^- \rightarrow b\bar{b}HH$ . We evaluate the total cross-section of  $b\bar{b}HH$  and calculate the total number of events considering the complete set of Feynman diagrams at tree-level. The numerical computation is done for the Higgs mass range 100 – 200 GeV and for the energy which is expected to be available at a possible Next Linear  $e^+e^-$  Collider with a center-of-mass energy 500, 1000 GeV and luminosity 1000 fb<sup>-1</sup>.

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## 1 Introduction

In the Standard Model (SM) [1] of particle physics, there are three types of interactions of fundamental particles: gauge interactions, Yukawa interactions and the Higgs boson self-interaction. The Higgs boson [2] plays an important role in the SM; it is responsible for generating the masses of all the elementary particles (leptons, quarks, and gauge bosons). In the future, after the Higgs boson is discovered, one of the most important problems will be studying its self-interaction. It is necessary to clarify the nature of the spontaneous breaking of the gauge symmetry which provides nonzero masses of intermediate bosons and fermions. However, the Higgs-boson sector is the least tested in the SM, in particular the Higgs boson self-interaction. In the SM, the profile of the Higgs particle is uniquely determined once its mass  $M_H$  is fixed [3]; the decay width, the branching ratios, and the production cross-sections are given by the strength of the Yukawa couplings to fermions and gauge bosons, the scale of which is set by the masses of these particles. Unfortunately, the mass of the Higgs boson is a free parameter.

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The only available information on  $M_H$  is the lower limit  $M_H \geq 114.1$  GeV established at the CERN  $e^+e^-$  collider LEP2 [4]. The collaborations have also reported a  $2.1\sigma$  excess of events beyond the expected SM backgrounds consistent with a SM like Higgs boson with a mass of  $M_H = 115_{-0.9}^{+1.3}$  GeV [4]. Furthermore, the accuracy of the electroweak data measured at LEP, SLAC Large Detector (SLC), and the Fermilab Tevatron provides sensitivity to  $M_H$ : the Higgs boson contributes logarithmically,  $\propto \log(\frac{M_H}{M_W})$ , to the radiative corrections to the  $W/Z$  boson propagators. A recent analysis yields the value  $M_H = 81_{-33}^{+42}$  GeV corresponding to a 95% C.L. upper limit of  $M_H < 193$  GeV [5].

The trilinear Higgs self-coupling can be measured directly in pair-production of Higgs particles at hadron and high-energy  $e^+e^-$  linear colliders. Higgs pairs can be produced through double Higgs-strahlung of  $W$  or  $Z$  bosons [6–9],  $WW$  or  $ZZ$  fusion [7, 10–13]; moreover, through gluon-gluon fusion in  $pp$  collisions [14–16] and high-energy  $\gamma\gamma$  fusion [7, 10, 17] at photon colliders. The two main processes at  $e^+e^-$  colliders are double Higgs-strahlung and  $WW$  fusion:

$$\begin{aligned} \text{double Higgs-strahlung} & : e^+e^- \rightarrow ZHH \\ WW \text{ double-Higgs fusion} & : e^+e^- \rightarrow \bar{\nu}_e\nu_e HH. \end{aligned} \quad (1)$$

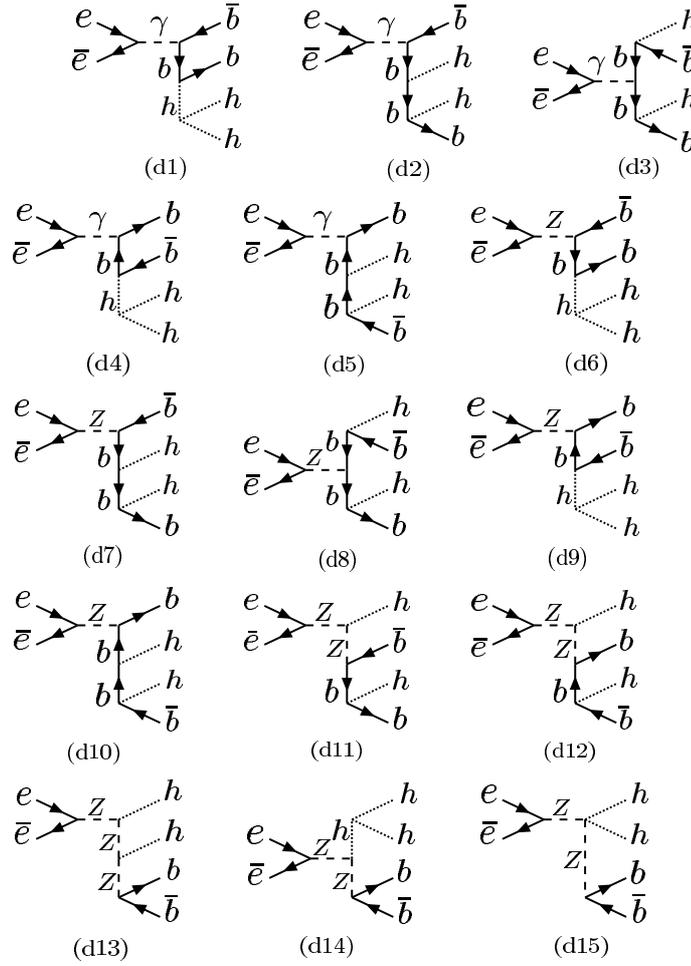
Since the electron- $Z$  coupling is small, the  $ZZ$  fusion process of Higgs pairs is suppressed by an order of magnitude. However, the process  $e^+e^- \rightarrow ZHH$  has been extensively studied [6–9]. This three-body process is important because it is sensitive to Yukawa couplings. The inclusion of four-body processes with heavy fermions  $b$ ,  $e^+e^- \rightarrow b\bar{b}HH$ , in which the SM Higgs boson is radiated by a  $b(\bar{b})$  quark at future  $e^+e^-$  colliders [18–20] with a c.m. energy in the range of 500 to 1000 GeV, as in the case of DESY TeV Energy Superconducting Linear Accelerator (TESLA) machine [21], is necessary in order to know its impact on the three-body mode processes and also to search for new relations that could have a clear signature of the Higgs boson production.

The Higgs coupling with bottom quarks, one of the largest couplings in the SM, is directly accessible in the process where the Higgs boson is radiated off bottom quarks  $e^+e^- \rightarrow b\bar{b}HH$ . This process depends on the Higgs boson triple self-coupling, which could lead us to obtain the first non-trivial information on the Higgs potential. We are interested in finding regions that could allow the observation of the process  $b\bar{b}HH$  at the next generation of high energy  $e^+e^-$  linear colliders. We consider the complete set of Feynman diagrams at tree-level (Fig.1) and use the CALCHEP [22] packages for the evaluation of the amplitudes and of the cross-section.

This paper is organized as follows: In Sec. II we present the total cross-section for the process  $e^+e^- \rightarrow b\bar{b}HH$  at next generation linear  $e^+e^-$  colliders. In Sec. III, we give our conclusions.

## 2 Cross section of the Higgs pairs production in the SM at next generation linear positron-electron colliders

In this section we present numerical results for  $e^+e^- \rightarrow b\bar{b}HH$  with double Higgs production. We carry out the calculations using the framework of the Standard Model at next generation linear  $e^+e^-$  colliders. We use CALCHEP [22] packages for calculations of the matrix elements and cross-sections. These packages provide automatic computation of the cross-sections and distributions in the SM as well as their extensions at the tree level. The process  $e^+e^- \rightarrow b\bar{b}HH$  is estimated and a complete set of Feynman diagrams at tree-level is included. We consider the


 Fig. 1. Feynman Diagrams at tree-level for  $e^+e^- \rightarrow b\bar{b}HH$ .

high energy stage of a possible Next Linear  $e^+e^-$  Collider with  $\sqrt{s} = 500, 1000$  GeV and design luminosity  $1000 \text{ fb}^{-1}$ .

For the SM parameters, we have adopted the following: the Weinberg angle  $\sin^2 \theta_W = 0.232$ , the mass ( $m_b = 4.5$  GeV) of the bottom quark and the mass ( $m_{Z^0} = 91.2$  GeV) of the  $Z^0$ , with the mass  $M_H$  of the Higgs boson as input [23].

In order to illustrate our results of the production of Higgs pairs in the SM, we present a plot for the total cross-section as a function of Higgs boson mass  $M_H$  for the process  $e^+e^- \rightarrow b\bar{b}HH$  in Fig. 2. We observe in this figure that the total cross-section for the double Higgs production of  $b\bar{b}HH$  is of the order of  $0.03 \text{ fb}$   $e^+e^-$  machine works with very high luminosity. The cross-section is shown for unpolarized electrons and positrons beams. The cross-section is at the

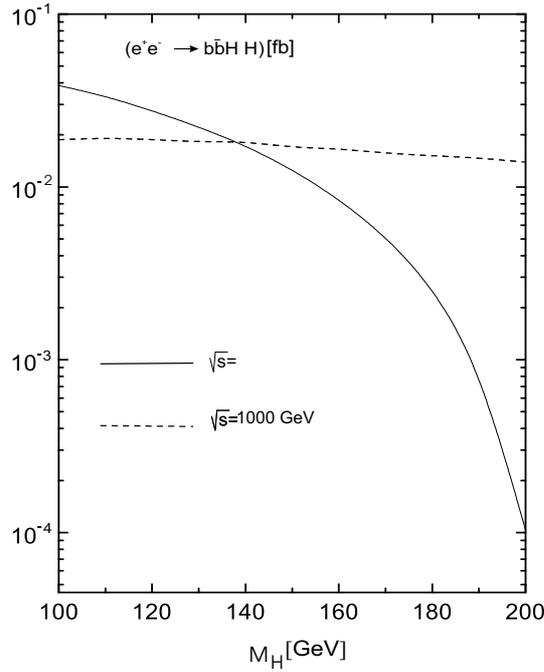


Fig. 2. Total cross-section of the Higgs pairs production  $e^+e^- \rightarrow b\bar{b}HH$  as a function of the Higgs mass  $M_H$  for  $\sqrt{s} = 500, 1000 \text{ GeV}$  with  $m_b = 4.5 \text{ GeV}$ .

level of a fraction of femtobarn and decreases with rising energy beyond the threshold region. However, the cross-section increases with rising self-coupling in the vicinity of the SM value. The sensitivity to the  $HHH$  self-coupling is analyzed in Ref. [24] for  $\sqrt{s} = 800 \text{ GeV}$  and  $M_H = 130 \text{ GeV}$  by varying the trilinear coupling  $\kappa\lambda_{HHH}$  within the range  $\kappa = -1$  and  $+2$ .

Figure 3 shows the total cross-section as a function of the center-of-mass energy  $\sqrt{s}$  for two representative values of the Higgs mass  $M_H = 110, 130 \text{ GeV}$ . We observe that the cross-section is very sensitive to the Higgs boson mass and decreases when  $M_H$  increases.

As shown in Table 1, for center-of-mass energies of 500-1000 GeV and high luminosity, the possibility of observing the process  $b\bar{b}HH$  is promising. Thus, a high luminosity  $e^+e^-$  linear collider is a very high precision machine in the context of Higgs physics.

We include a contours plot for the number of events of the studied process as a function of  $M_H$  and  $\sqrt{s}$  in Fig. 4. These contours are obtained from Table 1.

Finally, measurement of the trilinear Higgs self-coupling, which is the first non-trivial test of the Higgs potential, could be accessible in the double Higgs production processes  $e^+e^- \rightarrow b\bar{b}HH$  at high energies.

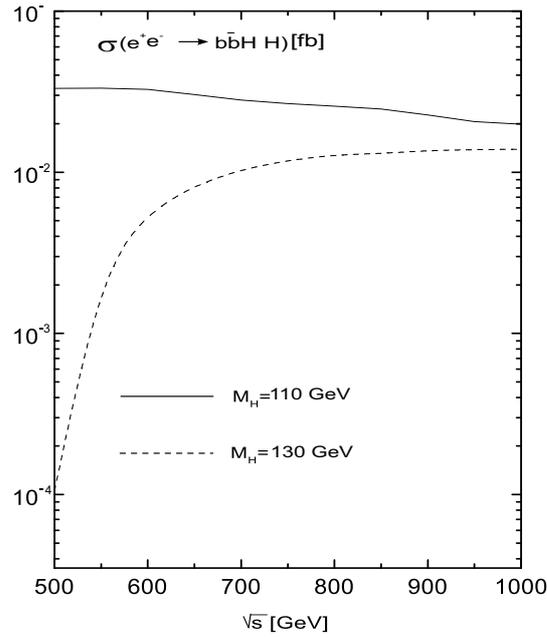


Fig. 3. Total cross-section of the Higgs pairs production  $e^+e^- \rightarrow b\bar{b}HH$  as a function of the center-of-mass energy  $\sqrt{s}$  for two representative values of the Higgs mass  $M_H = 110, 130$  GeV with  $m_b = 4.5$  GeV.

Tab. 1. Total production of Higgs pairs in the SM for  $\mathcal{L} = 1000$  fb $^{-1}$  and  $m_b = 4.5$  GeV.

Total Production of Higgs Pairs $M_H$ (GeV)	$e^+e^- \rightarrow b\bar{b}HH$	
	$\sqrt{s} = 500$ GeV	$\sqrt{s} = 1000$ GeV
100	38	18
120	27	18
140	17	18
160	8	16
180	2	15
200		13

### 3 Conclusions

In conclusion, we have analyzed the double Higgs production in association with  $b(\bar{b})$  quarks ( $e^+e^- \rightarrow b\bar{b}HH$ ) at the Next Generation Linear  $e^+e^-$  Colliders. The study of this process is important in order to know its impact on the three-body process and it could be useful to probe anomalous  $HHH$  coupling given the following conditions: very high luminosity, excellent  $b$  tag-

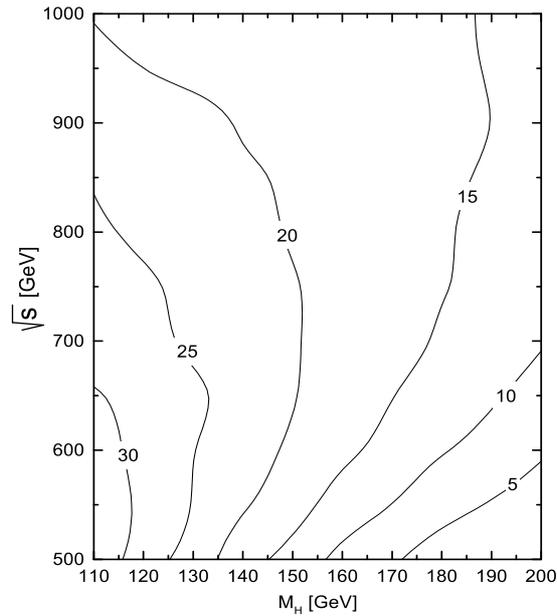


Fig. 4. Contours plot for the number of events as a function of  $M_H$  and  $\sqrt{s}$ .

ging performances, center-of-mass large energy and intermediate range Higgs boson mass. The results appear promising but judgment regarding the usefulness of the process must be reserved until further study has been carried out which includes background analysis and perhaps precise detector efficiency.

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