

**DISCUSSION OF THE  $\eta \rightarrow \pi^0 \gamma \gamma$  DECAY WITHIN A CHIRAL  
UNITARY APPROACH<sup>1</sup>**

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We improve the calculations of the  $\eta \rightarrow \pi^0 \gamma \gamma$  decay within the context of meson chiral lagrangians. We use a chiral unitary approach for the meson-meson interaction, thus generating the  $a_0(980)$  resonance and fixing the longstanding sign ambiguity on its contribution. This also allows us to calculate the loops with one vector meson exchange, thus removing a former source of uncertainty. In addition we ensure the consistency of the approach with other processes, first, by using vector meson dominance couplings normalized to agree with radiative vector meson decays, and, second, by checking the consistency of the calculations with the related  $\gamma \gamma \rightarrow \pi^0 \eta$  reaction. We find an  $\eta \rightarrow \pi^0 \gamma \gamma$  decay width of  $0.47 \pm 0.10$  eV, in clear disagreement with published data but in remarkable agreement with the most recent measurement.

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## 1 Introduction

The  $\eta \rightarrow \pi^0 \gamma \gamma$  decay has attracted much theoretical attention, since Chiral Perturbation Theory (ChPT) calculations have sizable uncertainties and produce systematically rates about a factor of two smaller than experiment. Within ChPT, the problem stems from the fact that the tree level amplitudes, both at  $O(p^2)$  and  $O(p^4)$ , vanish. The first non-vanishing contribution comes at  $O(p^4)$ , but either from loops involving kaons, kinematically suppressed due to the kaon masses, or from pion loops, again suppressed since they violate G parity and are thus proportional to  $m_u - m_d$ . The first sizable contribution comes at  $O(p^6)$  but the coefficients involved are not precisely determined. The use of tree level Vector Meson Dominance (VMD) to obtain the  $O(p^6)$  chiral coefficients by expanding the vector meson propagators, leads to results about a factor of two smaller than the "all order" VMD term, which means keeping the full vector meson propagator. All this said, it has become clear that the strict chiral counting has to be abandoned

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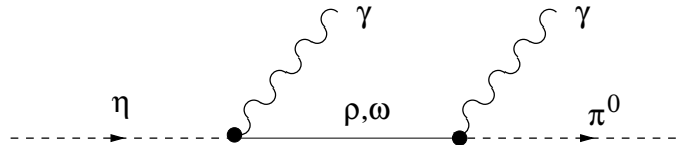


Fig. 1. Diagrams for the VMD mechanism.

since the  $O(p^6)$  and higher orders involved in the full (“all order”) VMD results are larger than those of  $O(p^4)$ .

Once the “all order” VMD results is accepted as the dominant mechanism, one cannot forget the tree level exchange of other resonances around the 1 GeV region. The  $a_0(980)$  exchange, which was taken into account approximately in [2], was one of the main sources of uncertainty, since even the sign of its contribution was unknown.

After the tree level light resonance exchange has been taken into account, we should consider loop diagrams, since meson-meson interaction or rescattering can be rather strong. First of all we find the already commented  $O(p^4)$  kaon loops from ChPT, but also the meson loops from the terms involving the exchange of one resonance. The uncertainty from the latter was roughly expected [2] to be about 30% of the full width.

Another relevant question is that no attempts have been made to check the consistency of  $\eta \rightarrow \pi^0 \gamma \gamma$  results with the related channel  $\gamma \gamma \rightarrow \pi^0 \eta$ . The reason is not surprising since there are no hopes within ChPT to reach the  $a_0(980)$  region where there are measurements of the  $\gamma \gamma \rightarrow \pi^0 \eta$  cross section. On the other hand, the explicit SU(3) breaking already present in the radiative vector meson decays has not been taken into account when calculating the VMD tree level contributions, and this effect changes the results obtained from VMD estimations by about a factor of two.

The former discussion has set the stage of the problem and the remaining uncertainties that allow for further improvement. In recent years, with the advent of unitarization methods, it has been possible to extend the results of ChPT to higher energies where the perturbative expansion breaks down and to generate resonances up to 1.2 GeV. In particular these ideas were used to describe the  $\gamma \gamma \rightarrow meson - meson$  reaction, with good results in all the channels up to energies of around 1.2 GeV [3]. With these techniques, and always within the context of meson chiral lagrangians, we will address three of the problems stated above: First, the  $a_0(980)$  contribution, second, the evaluation of meson loops from VMD diagrams and, third, the consistency with the crossed channel  $\gamma \gamma \rightarrow \pi^0 \eta$ . In particular, we will make use of the results in [3], where the  $\gamma \gamma \rightarrow \pi^0 \eta$  cross section around the  $a_0(980)$  resonance was well reproduced using the same input as in meson meson scattering, without introducing any extra parameters.

With these improvements we are then left with a model that includes the “all order” VMD and resummed chiral loops.

## 2 Mechanisms

Following [2] we consider the sequential VMD mechanism of Fig. 1 which can be easily derived from the VMD Lagrangians involving VVP and  $V\gamma$  couplings

$$\mathcal{L}_{VVP} = \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle, \quad \mathcal{L}_{V\gamma} = -4f^2 eg A_\mu \langle Q V^\mu \rangle, \quad (1)$$

where  $V_\mu$  and  $P$  are standard  $SU(3)$  matrices constructed with the nonet of vector mesons containing the  $\rho$ , and the nonet of pseudoscalar mesons containing the  $\pi$ , respectively. We also assume an ordinary mixing for the  $\phi$ , the  $\omega$ , the  $\eta$  and  $\eta'$ .

From Eq. (1) one can obtain the radiative widths for  $V \rightarrow P\gamma$  obtaining a fair agreement with the experimental data in the PDG but the results can be improved by incorporating  $SU(3)$  breaking mechanisms. For that purpose, *we normalize the couplings so that the branching ratios agree with experiment*. These will be called results with “normalized couplings”. In this way we are taking into account phenomenologically the corrections to the  $VP\gamma$  vertex from an underlying field theory.

The integrated width obtained using this sequential VMD contribution is  $\Gamma = 0.57$  eV (universal couplings);  $\Gamma = 0.30 \pm 0.06$  eV (normalized couplings), where the error has been calculated from a Monte Carlo Gaussian sampling of the normalization parameters within the errors of the experimental branching ratios.

Our VMD normalized result is within three standard deviations from the value presently given in the PDG:  $\Gamma = 0.84 \pm 0.18$  eV, but within one sigma of the more recent one presented in [4],  $\Gamma = 0.42 \pm 0.14$  eV. There are, however, other contributions that we consider next.

The contribution of pion loops to  $\eta \rightarrow \pi^0 \gamma \gamma$ , evaluated in [2], proceeds, to begin with, through the G-parity violating  $\eta \rightarrow \pi^0 \pi^+ \pi^-$  process but it is proportional to  $m_u - m_d$ , and we shall include it in the uncertainties together with other isospin violating contributions.

The main meson loop contribution comes from the charged kaon loops at  $O(p^4)$  and proceeds via  $\eta \rightarrow \pi^0 K^+ K^- \rightarrow \pi^0 \gamma \gamma$ . Note that these loops are also suppressed due to the large kaon masses. That is why the  $\eta \rightarrow \pi^0 a_0(980) \rightarrow \pi^0 \gamma \gamma$  mechanism was included explicitly in [2], with uncertainties in the size and sign of the  $a_0(980)$  couplings. As commented in the introduction, the chiral unitary approach solves this problem by generating dynamically the  $a_0(980)$  in the  $K^+ K^- \rightarrow \pi^0 \eta$  amplitude. Note that this is not the case for the  $\rho$  or  $\omega$  that appear in the sequential VMD mechanism, since the  $\rho$  and  $\omega$  are genuine states in our chiral approach (they are not dynamically generated) and hence they are treated as ordinary particles.

We can illustrate this approach by revisiting the work done in [3] on the related process  $\gamma\gamma \rightarrow \pi^0 \eta$  where the chiral unitary approach was successfully applied around the  $a_0(980)$  region. Since for the  $\eta$  decay the low energy region of  $\gamma\gamma \rightarrow \pi^0 \eta$  is also of interest, we will include next the VMD mechanisms also in this reaction. Once we check that we describe correctly  $\gamma\gamma \rightarrow \pi^0 \eta$ , the results can be easily translated to the eta decay. We will finally add other anomalous meson loops that are numerically relevant for eta decay but not for  $\gamma\gamma \rightarrow \pi^0 \eta$ .

In [3] it was shown that, within the unitary chiral approach, the  $\gamma\gamma \rightarrow \pi^0 \eta$  amplitude around the  $a_0(980)$  region, diagrammatically represented at one loop in Fig. 2, factorizes as

$$-it = (\tilde{t}_{\chi K} + \tilde{t}_{AK^+K^-}) t_{K^+K^-, \pi^0 \eta} \quad (2)$$

with  $t_{K^+K^-, \pi^0 \eta}$  the full  $K^+ K^- \rightarrow \pi^0 \eta$  transition amplitude.

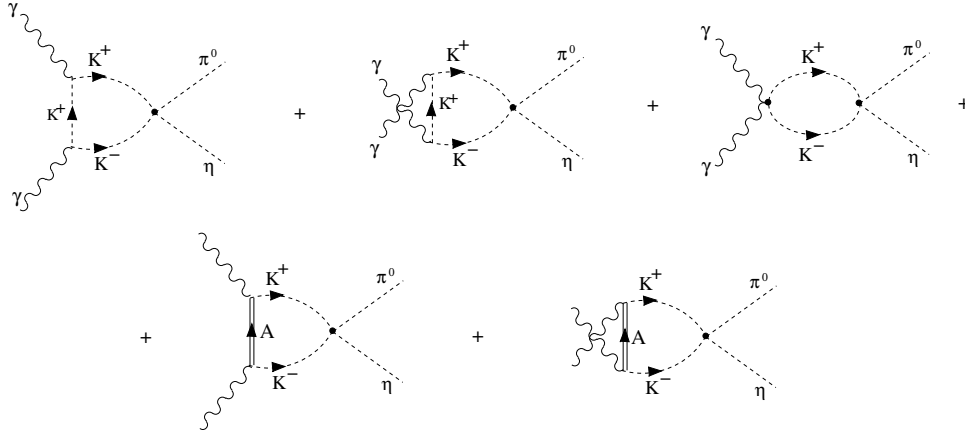


Fig. 2. Diagrams for the chiral loop contribution.

The first three diagrams correspond to  $\tilde{t}_{\chi K} t_{K^+K^-, \pi^0\eta}$  of Eq. (2). The meson-meson scattering amplitude was evaluated in [6] by summing the Bethe Salpeter (BS) equation with a kernel formed from the lowest order meson chiral Lagrangian amplitude and regularizing the loop function with a three momentum cut off (equivalently we could have also used dimensional regularization. The equivalence of both formulations is shown in the appendix of [7]). Other approaches like the inverse amplitude method or the  $N/D$  method all give similar results in the meson scalar sector. The BS equation with coupled channels can be solved algebraically, leading to the following solution in matrix form

$$t(s) = [1 - t_2(s)G(s)]^{-1}t_2(s), \quad (3)$$

with  $\sqrt{s}$  the invariant mass of the two mesons,  $t_2$  the lowest order chiral amplitude and  $G(s)$  a diagonal matrix,  $\text{diag}(G_{\overline{K}K}, G_{\eta\pi})$ , accounting for the loop functions of two mesons. These  $G$  functions were regularized in [6] by means of a cut off. The basic equivalence between the cut off regularization and dimensional regularization has been set in [7] ( see appendix). A thorough discussion on why the cut off regularization respects chiral symmetry and gauge invariance is given in [9].

In Eq. (2) there is another term,  $\tilde{t}_{AK^+K^-} t_{K^+K^-, \pi^0\eta}$ , which corresponds to the last two diagrams of Fig. 2 where the axial vector meson  $K_1(1270)$  is exchanged.

In addition to the axial vector meson exchange in loops considered, we have to include the loops with vector meson exchange for completeness. In fact, some of the uncertainties estimated in [2] were linked to these loops. For consistency, once again we have to sum the series obtained by iterating the loops in the four meson vertex.

Of course, when introducing loops with vector meson exchange we have to consider loops involving a  $K^{*+}$  or a  $K^{*0}$  exchanged between the photons, which were not present at tree level.

In the  $\eta \rightarrow \pi^0\gamma\gamma$  case, the meson loop diagrams correspond to those of  $\pi^0\eta \rightarrow \gamma\gamma$  but considering the  $\pi^0$  as an outgoing particle.

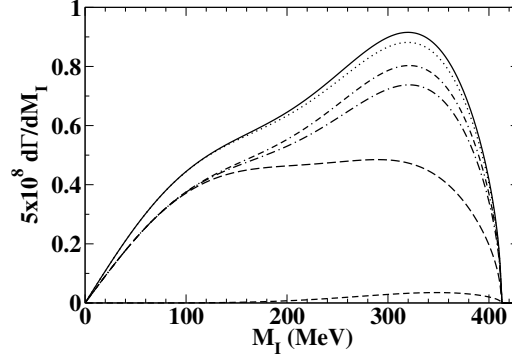


Fig. 3. Contributions to the two photon invariant mass distribution. From bottom to top, short dashed line: chiral loops from Eq.(2); long dashed line: only tree level VMD; dashed-dotted line: coherent sum of the previous mechanisms; double dashed-dotted line: idem but adding the resummed VMD loops; continuous line: idem but adding the anomalous terms, which is the full model presented in this work (we are also showing as a dotted line the full model but substituting the full  $t_{K^+K^-, \eta\pi^0}$  amplitude by its lowest order).

Since we are considering all the VMD diagrams and the chiral loops, we still have to take into account another kind of loop diagrams [2] which involve two anomalous  $\gamma \rightarrow 3M$  vertices. Despite being  $O(p^8)$  it has been found [2] that they can have a non-negligible effect on the  $\eta$  decay.

### 3 Results

Using the model described in the previous section, we plot in Fig. 3 the different contributions to  $d\Gamma/dM_I$ . We can see that the largest contribution is that of the tree level VMD (long dashed line). Let us recall that this is a new result as long as we are using the VMD couplings normalized to agree with the vector radiative decays. The resummation of the loops in Fig. 2 using Eq.(2), (short dashed line) gives a small contribution (0.011 eV in the total width), but when added coherently to the tree level VMD, leads to an increase of 30% in the  $\eta$  decay rate (dashed-dotted line). More interestingly, the shape of the  $\gamma\gamma$  invariant mass distribution is appreciably changed with respect to the tree level VMD, developing a peak at high invariant masses. The resummed VMD loops leads, through interference, to a moderate increase of the  $\eta$  decay rate (double dashed dotted line), smaller than that of the chiral loops considered before. The last ingredient is the contribution of the anomalous mechanisms (continuous line), leading again to a moderate increase of the  $\eta$  decay rate, also smaller than the chiral loops from Eq.(2). These anomalous mechanisms have a very similar shape to the tree level VMD and interfere with it in the whole range of invariant masses. Altogether the final result is

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = 0.47 \pm 0.10 \text{ eV} \quad (4)$$

where the theoretical error have been obtained considering the uncertainties from the vector meson radiative decays, the contribution of the  $1^{+-}$  axial-vector mesons and the isospin violating terms.

Note that although we have considered a new error source from the uncertainties in the vector radiative decays, which turns out to be the largest one, we still have reduced the uncertainty from previous calculations.

The result of Eq. (4) is in remarkable agreement with the latest experimental number  $\Gamma = 0.42 \pm 0.09 \pm 0.08$  eV [4], based on 1600 events and also with those of [5] of  $\Gamma = 0.35 \pm 0.20$  eV based on a smaller statistics of about 120 events. and lie within two sigmas from the earlier one in the PDG  $\Gamma = 0.84 \pm 0.18$  eV. Confirmation of those preliminary results would therefore be important to test the consistency of this new approach. Furthermore, precise measurements of the  $\gamma\gamma$  invariant mass distributions would be of much help given the differences found with and without loop contributions.

Finally, we would like to make some comments concerning the comparison of recent experimental data with former calculations of chiral perturbation theory plus VMD estimates. The chiral models prior to our work have to be looked at in perspective. They rely upon data on radiative decay of vector mesons which have changed considerably in recent years. We use updated data and find that if the earlier calculations would have used the present data they would get eta decay widths about one half what they got. Thus, comparing our result and the old ones is somewhat misleading. And it is also misleading to compare the experiments with these calculations without this warning. We find it also inappropriate to compare experimental results with what chiral perturbation theory would give at order  $O(p^6)$  since, as we have mentioned here, these results are obtained from a VDM model projecting over  $O(p^6)$  the results of the full model, which provides a width about a factor two larger than its  $O(p^6)$  projection. These considerations are appropriate concerning the recent preliminary results of [8] and the claims of agreement with old ChPT results and disagreement with the most recent data [4].

In other words, a strict chiral perturbative calculation in terms of powers of momentum is not practical for this problem, and the full model predictions, which accounts also for higher powers of momentum, have to be taken for reference.

In addition, previous works in the literature have large uncertainties arising from ignorance of the  $a_0(980)$  contribution. We have improved all this and other small things and evaluated theoretical errors from different sources. In summary the most relevant things of the present work are:

- 1) Use of updated data for radiative decay of vector mesons. If previous chiral calculations are updated making use of these new data the result comes about a factor of two smaller.
- 2) Unitarized chiral perturbation theory allows one to determine precisely the contribution of the  $a_0(980)$  resonance, which was a major source of uncertainty in the past.
- 3) A careful determination of theoretical errors has been done, which was also lacking before.

Our work has made the most accurate calculation, so far, within the framework of chiral dynamics and can be taken as reference of what chiral theory predicts for this reaction. Previous chiral works have to be seen in their value as making the first predictions for this ratio, but should not be used as a reference for a quantitative prediction since they contain intrinsic uncertainties of more than a factor two.

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