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DECAYS OF η AND η' AND WHAT CAN WE LEARN FROM THEM[1]?

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A short overview of η and η' decays is given with an emphasis on what can be learned for the strong interaction from them. The talk consists of a short introduction to Chiral Perturbation Theory, a discussion of $\eta \to 3\pi$ beyond p^4 and some of the physics involved in $\eta' \to \eta\pi\pi, 3\pi$ as well as an overview of anomalous processes.

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1 Introduction

This talk gives an introduction to some of the basic strong interaction issues we face in η and η' decays. It will not cover weak decays. Simple dimensional analysis leads to branching ratios below 10^{-11} for weak η-decays and below 10^{-12} for weak η' decays. These will obviously not be observed in the near future. It is possible to construct models that enhance η and η' decays due to physics beyond the standard model to observable rates but these models tend to be ugly in order to avoid the very stringent constraints from kaon decays and other sources, some examples can be found in [1].

Let me remind you of the η-handbook [1] where you can find a series of lectures on the basis of η and η' physics. The theory lectures relevant for this talk in there are those by Kroll [2], Bijnens and Gasser [3], Ametller [4], Holstein [5], Shore [6] and Bass [7]. There are also a set of related theory talks in this conference, those by Bass [8], Borasoy [9], Oset [10] and Martemyanov [11], while most of the other talks today are the experimental talks related to η and η' decays at KLOE and WASA.

2 Pseudoscalars are special

The Lagrangian of Quantum Chromodynamics (QCD) is obviously invariant under the interchange of the three light quarks, u, d and s, if they have equal mass. This leads to the vector symmetry $U(3)_V$. But

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right] . \tag{1}$$

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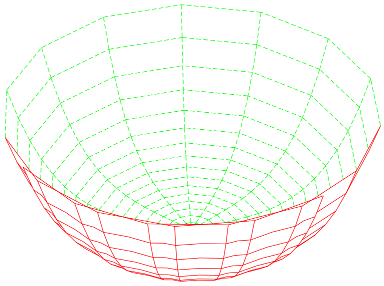

Fig. 1. The potential $V(\phi)$ for an unbroken symmetry.

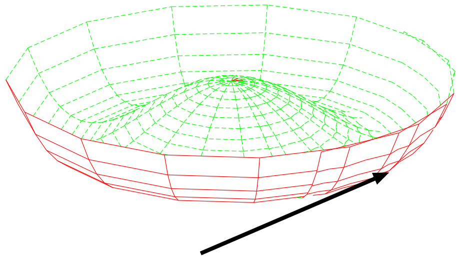

Fig. 2. The potential $V(\phi)$ for a spontaneously broken symmetry. The arrow indicates the choice of vacuum.

Here q_L and q_R are respectively the left and right handed quark spinors, $\not{D} = \gamma^\mu D_\mu$ is the contraction of the covariant derivative including the gluon field with the Dirac gamma matrices and m_q is the (current) quark mass. So, if $m_q = 0$, we have an enlarged (chiral) symmetry: $U(3)_L \times U(3)_R$, where the left and right handed particles can be rotated into each other independently.

But hadrons do not come in parity doublets: the chiral symmetry must be broken in the real world. There exists also a few (very) light hadrons: $\pi^0\pi^+\pi^-$ and K, η. The existence of the latter as well as the nonexistence of the degenerate parity doublets can be understood from spontaneous Chiral Symmetry Breaking.

Let us first discuss spontaneous symmetry breaking for a simple $U(1)$ symmetry for a complex scalar field $\phi(x) \to e^{i\alpha}\phi(x)$. This means that when plotting the potential $V(\phi)$ along the z-axis as a function of the real and imaginary part of ϕ along the x and y axis, it should be symmetric for rotations around the z-axis. A potential $V(\phi)$ for the unbroken case is shown in Fig. 1. Note that the vacuum, the lowest point is unique. All excitations around the vacuum require climbing up the sides of the potential and are thus massive. On the contrary, in Fig. 2 we show a potential corresponding to the $U(1)$ symmetry being spontaneously broken. The lowest energy state is now not unique anymore. This means that due to the continuous symmetry there is a continuum of vacua or ground states with the same energy. There exists thus a massless mode which is the moving along the valley at the bottom of the potential. This mode is the Goldstone Boson and can in this case, be parameterized by the angle around the z-axis. The symmetry is spontaneously broken because we have to choose one vacuum, indicated by the arrow. We can see that the vacuum is not invariant under the symmetry group since the vacuum expectation value $\langle\phi\rangle \neq 0$. There are also predictions for the interactions of this Goldstone mode. The symmetry is still present, so the angle around the z-axis should not matter. This means that only changes in the angle can contribute and a direct consequence is that the Goldstone Bosons do not interact at low energies. This leads to the low energy theorems. A nice review about Goldstone Bosons and their physics is [12].

Note that the degeneracy of the set of vacuum states, sometimes referred to as the vacuum

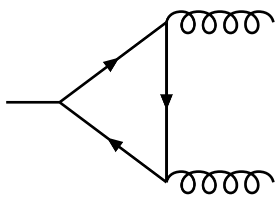

Fig. 3. The triangle diagram which leads to a nonzero divergence for the singlet axial current.

manifold, is determined by the group structure of the symmetry group and its broken part, and it is on this manifold that the Goldstone Bosons in some sense live.

For QCD, we do not have a simple field ϕ that has a vacuum expectation value but it is instead $\langle \overline{q}q \rangle \neq 0$. The chiral symmetry group $U(3)_L \times U(3)_R$ is spontaneously broken to the diagonal or vector subgroup $U(3)_V$. The resulting Goldstone bosons we identify with the pseudoscalars π, K, η.

This raises another problem, why is the η' not light? This is referred to as the $U(1)_A$-problem. The symmetry group can be decomposed into simple groups as

$$U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \,. \tag{2}$$

The spontaneous breaking of $SU(3)_L \times SU(3)_R$ to $SU(3)_V$ leads to 8 Goldstone Bosons and here we have π, K, η as light particles so this is fine. The $U(1)_V$ part is baryon number and is not broken. The axial $U(1)_A$ should not be a good symmetry of the theory in order to explain why the η' is heavy, even if the Lagrangian of QCD has this symmetry. The reason is that in quantum field theory, the Lagrangian alone does not fully specify the theory. One also needs to introduce a regularization/renormalization procedure. The latter is not compatible with all global symmetries and in particular the current corresponding to $U(1)_A$ is not conserved, this is known as the Adler-Bell-Jackiw anomaly [13]. The triangle diagram of Fig. 3 leads to

$$\partial_\mu A^{0\mu} = 2\sqrt{N_f}\,\omega\,, \qquad \omega = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,G_{\mu\nu}G_{\alpha\beta}\,. \tag{3}$$

The operator ω consists of gluons so the nonzero divergence is strongly interacting and it means that $U(1)_A$ is not a good symmetry for QCD. The η' is thus allowed to be heavy.

Unfortunately, when we look at mechanisms how ω could produce the η' mass we see that ω is a total derivative. These can normally be neglected in the Lagrangian so how can it have an effect? The answer was found by 't Hooft [14]. Due to special configurations with nonzero winding number $\nu = \int d^4x\,\omega \neq 0$, called instantons, there can be an effect and it leads to a large η' mass.

This solution lead to a new problem, the strong CP-problem. One can add a term to QCD,

$$\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{QCD} - \theta\omega, \tag{4}$$

that breaks CP symmetry strongly. Experimental limits are $|\theta| \leq 10^{-10}$ and we need to understand that small value.

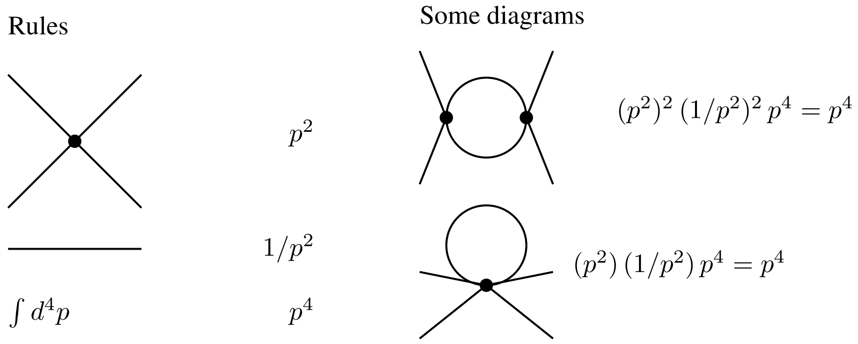

Fig. 4. On the left hand side the power counting rules for ChPT for a lowest order vertex, a propagator and a loop integration. The right side shows how this leads to the same order for two different one-loop diagrams.

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

Tab. 1. The number of free parameters in ChPT at various orders for two and three flavours and the partially quenched case.

The η' has thus large and very interesting nonperturbative effects and has a strong coupling to the gluon like no other hadron. Due to the fact that $m_s \neq \hat{m} = (m_u + m_d)/2$, this also affects η physics. This is one of the major reasons why studying η and η' is a very interesting subject.

3 Standard Chiral Perturbation Theory

A major tool in η decay studies is Chiral Perturbation Theory (ChPT) [15–17]. Introductions can be found in [18]. It is an effective field theory based on the Goldstone Bosons from the spontaneous breaking of chiral symmetry as degrees of freedom. It is an expansion in momenta and quark masses and the power counting is really dimensional counting, called p-counting for a generic momentum. The expected breakdown scale is the scale at which physics which is not included becomes relevant. This is resonances, so the breakdown scale is of order of the rho meson mass, m_ρ, somewhat dependent on the channel one looks at.

The fact that a power counting in momenta leads to a well defined perturbative expansion follows from the fact that the interactions of Goldstone Bosons vanish at zero momentum. The absence of the latter for other strongly interacting states is why it is so difficult to build an effective theory including resonances. Power Counting is shown for the example of $\pi\pi$ scattering in Fig. 4.

ChPT is a nonrenormalizable field theory. That means that, while in principle predictive, the number of parameters increases strongly order by order. At lowest order there are only two parameters [19], at p^4 ten [17] and at p^6 there are 90 [20]. Other cases are shown in Table 1.

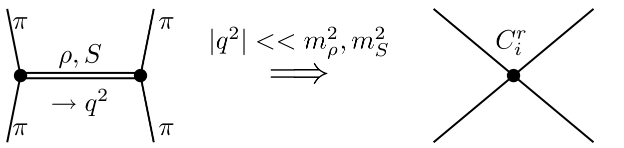

Fig. 5. A schematic view of resonance saturation of the C_i^r.

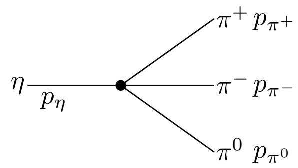

Fig. 6. The decay $\eta \rightarrow 3\pi$ and the momenta associated with it.

In practice the p^6 coefficients C_i^r are often determined from resonance saturation depicted schematically in Fig. 5. This is at present a major restriction on high order ChPT predictions. It works well for the terms not involving quark masses, OK for those with quark masses but dominated by vectors and it is not known how well the scalar dominated ones are predicted using this approximation. Many calculations have been performed to order p^6. A recent review is [21].

A major improvement happened recently for $\pi\pi$ and πK scattering where the combination of p^6 calculations and dispersion relations lead to a much better understanding of these systems.

4 The decay $\eta \rightarrow 3\pi$ beyond p^4

One calculation which is not completed yet at p^6 order in ChPT is in fact $\eta \rightarrow 3\pi$. This does not mean that nothing is known beyond order p^4. This section reviews that and is basically identical to the discussion in [3].

The kinematics are given in terms of the via s, t, u defined by

$$\begin{aligned} s &= (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \qquad t = (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \qquad s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0 \,. \end{aligned} \tag{5}$$

The two different amplitudes are

$$\begin{aligned} \langle \pi^0\pi^+\pi^- \text{out}|\eta\rangle &= i\,(2\pi)^4\;\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\,A(s,t,u)\,, \\ \langle \pi^0\pi^0\pi^0 \text{out}|\eta\rangle &= i\,(2\pi)^4\;\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\,\overline{A}(s_1,s_2,s_3)\,. \end{aligned} \tag{6}$$

The pions are in an $I = 1$ state which means that the amplitude is proportional to $m_u - m_d$ or α_{em}. The $\mathcal{O}(\alpha_{em})$ effect is small, but large via the kinematical effects of $m_{\pi^+} - m_{\pi^0}$. The photonic decay $\eta \rightarrow \pi^+\pi^-\pi^0\gamma$ needs to be included directly. Isospin leads to

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,. \tag{7}$$

The lowest order amplitude is [22]

$$A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2}\right\}\,, \tag{8}$$

or, with $Q^2 \equiv (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2)$ and $\hat{m} = (m_u + m_d)/2$, it becomes

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\,\frac{1}{3\sqrt{3}F_\pi^2}\,M(s,t,u)\,, \tag{9}$$

with at lowest order,

$$M(s,t,u) = \left(3s - 4m_\pi^2\right)\left(m_\eta^2 - m_\pi^2\right) \,. \tag{10}$$

That the decay rate $\Gamma\left(\eta \to 3\pi\right)$ is thus proportional to Q^{-4}, allows a PRECISE measurement of Q. To illustrate this we take Q from the baryon mass difference, $Q \approx 24$, and obtain at lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV . An alternative determination from $m_{K^+}^2 - m_{K^0}^2 \sim Q^{-2}$ gives $Q = 20.0 \pm 1.5$ and leads to a lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

The p^4 calculation [23] gives a very large enhancement

$$\left(\int dLIPS|A_2 + A_4|^2\right) \Big/ \left(\int dLIPS|A_2|^2\right) = 2.4\,, \tag{11}$$

with $LIPS$ meaning Lorentz invariant phase-space. A major source of the large effect is the large S-wave final state rescattering. The p^6 calculation is partially done but has been stalled since two years.

The higher orders have been estimated via dispersion relations that mainly include the effects of the final state rescattering. There have been two calculations, [24] and [25]. They used different methods but similar approximations. I will present a simplified version of the analysis of [24] here as performed in [3]. A more extensive description of [24] is [26].

Up to $\mathcal{O}(p^8)$ there are no absorptive parts from $\ell \geq 2$ which allows to write [24, 27]

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)\,. \tag{12}$$

The M_I "roughly" correspond to contributions with isospin 0,1,2. The M_I satisfy dispersion relation with 2 or 3 subtractions in terms of their discontinuities

$$\begin{aligned} M_{0,2}(s) &= a_{0,2} + b_{0,2}s + c_{0,2}s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\frac{\mathrm{disc}M_{0,2}(s')}{s'-s-i\varepsilon}\,, \\ M_1(s) &= a_1 + b_1 s + \frac{s^2}{\pi}\int \frac{ds'}{s'^2}\frac{\mathrm{disc}M_1(s')}{s'-s-i\varepsilon}\,, \end{aligned} \tag{13}$$

The constraint on s, t, u of (5) implies that there are only 4 free constants, not 8 in (13) via

$$M(s,t,u) = a + bs + cs^2 - d(s^2 + tu) + \text{dispersive}\,. \tag{14}$$

The quantities c and d can be determined from more convergent dispersion relations in terms of known parameters via

$$\begin{aligned} c &= c_0 + \frac{4}{3}c_2 = \frac{1}{\pi}\int \frac{ds'}{s'^3}\left\{\mathrm{disc}M_0(s') + \frac{4}{3}\mathrm{disc}M_2(s')\right\}\,, \\ d &= -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)} + \frac{1}{\pi}\int \frac{ds'}{s'^3}\left\{s'\mathrm{disc}M_1(s') + \mathrm{disc}M_2(s')\right\} \end{aligned} \tag{15}$$

We now restrict the discontinuities to their respective two-body cuts and further split them into the part coming from the forward channel, the $M_I(s)$ part on the rhs of (16), and that via the t, u channels, the $\hat{M}_I(s)$ part on the rhs of (16), explicitly:

$$M_I(s) = \frac{1}{\pi}\int \frac{ds'}{s'-s-i\varepsilon}\sin\delta_I(s)e^{-i\delta_I(s)}\left\{M_I(s) + \hat{M}_I(s)\right\}\,. \tag{16}$$

The $\delta_I(s)$ are the S-wave isospin I scattering phases.

The ambiguities inherent in the possible solutions can be solved by going over to a new set of functions that automatically include the forward cuts via

$$\Omega_I(s) = \exp\left\{(s/\pi)\int ds'/s'\ \delta_I(s')/(s'-s-i\varepsilon)\right\}. \tag{17}$$

The new dispersion relations are:

$$\begin{aligned}
\frac{M_0(s)}{\Omega_0(s)} &= \alpha_0 + \beta_0 s + \gamma_0 s^2 + \frac{s^2}{\pi}\int ds' \frac{\sin\delta_0(s')\,\hat{M}_0(s')}{|\Omega_0(s')|s'^2(s'-s-i\varepsilon)}\,,\\
\frac{M_1(s)}{\Omega_1(s)} &= \beta_1 s + \frac{s}{\pi}\int ds' \frac{\sin\delta_1(s')\,\hat{M}_1(s')}{|\Omega_1(s')|s'(s'-s-i\varepsilon)}\,,\\
\frac{M_2(s)}{\Omega_2(s)} &= \frac{s^2}{\pi}\int ds' \frac{\sin\delta_2(s')\,\hat{M}_2(s')}{|\Omega_2(s')|s'^2(s'-s-i\varepsilon)}\,.
\end{aligned} \tag{18}$$

So, we now need to find a set of $\pi\pi$ phases, $\delta_{0,1,2}(s)$, and solve Eq. (18) for M_1, M_2, M_3. We have to fix the four constants $\alpha_0, \beta_0, \gamma_0, \beta_1$. Two of them, γ_0 and β_1, can be determined from the more convergent dispersion relations (15) with the result at p^4:

$$\gamma_0 \approx 0\,, \quad \beta_1 \approx -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)} \tag{19}$$

The values of α_0, β_0 depend on where in the s,t,u plane matching is done. Ref. [24] uses that the lowest order, Eq. (10), has an Adler zero at $s_A = 4/3\, m_\pi^2$. This zero can move but must remain in the neighbourhood also at higher orders. They match the position of s_A and the slope of the amplitude there to the $\mathcal{O}(p^4)$ expressions. Ref. [25] matches the amplitude at several places in the s,t,u plane to the $\mathcal{O}(p^4)$ expressions. A very simplified analysis to understand the results can be done by neglecting the $\hat{M}_I$ [3]. The total corrections found in [24, 25] are very similar. But the distributions differ significantly as can be seen from the results along the $s = u$ line shown in Fig. 7 for the three approaches. The Dalitz plot distributions thus provide a check on the various input assumptions. There has also been a refitting of the [25] method of solving to the newer data by [11, 28].

The Dalitz plot distributions are parameterized by

$$1 + ay + by^2 + cx^2 \quad \text{charged decay}\,, \quad 1 + g(x^2+y^2) \quad \text{neutral decay}\,, \tag{20}$$

normalized at $x = y = 0$. The kinematical variables are

$$\begin{aligned}
x &= \sqrt{3}\,\frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2M_\eta Q_\eta}(u-t)\,,\\
y &= \frac{3T_0}{Q_\eta} - 1 = \frac{3}{2m_\eta Q_\eta}\left\{(m_\eta - m_{\pi^0})^2 - s\right\} - 1\,, \quad Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}
\end{aligned} \tag{21}$$

Also of interest is the ratio $r \equiv \Gamma(\eta\to\pi^0\pi^0\pi^0)/\Gamma(\eta\to\pi^+\pi^-\pi^0) = 1.44 \pm 0.04$ [29]. Results from the theory at p^4 and dispersive improvements [3, 25] are given in Table 2 and the experimental results for the neutral and charged decays in Tables 3 and 4. You can judge the agreement yourselves.

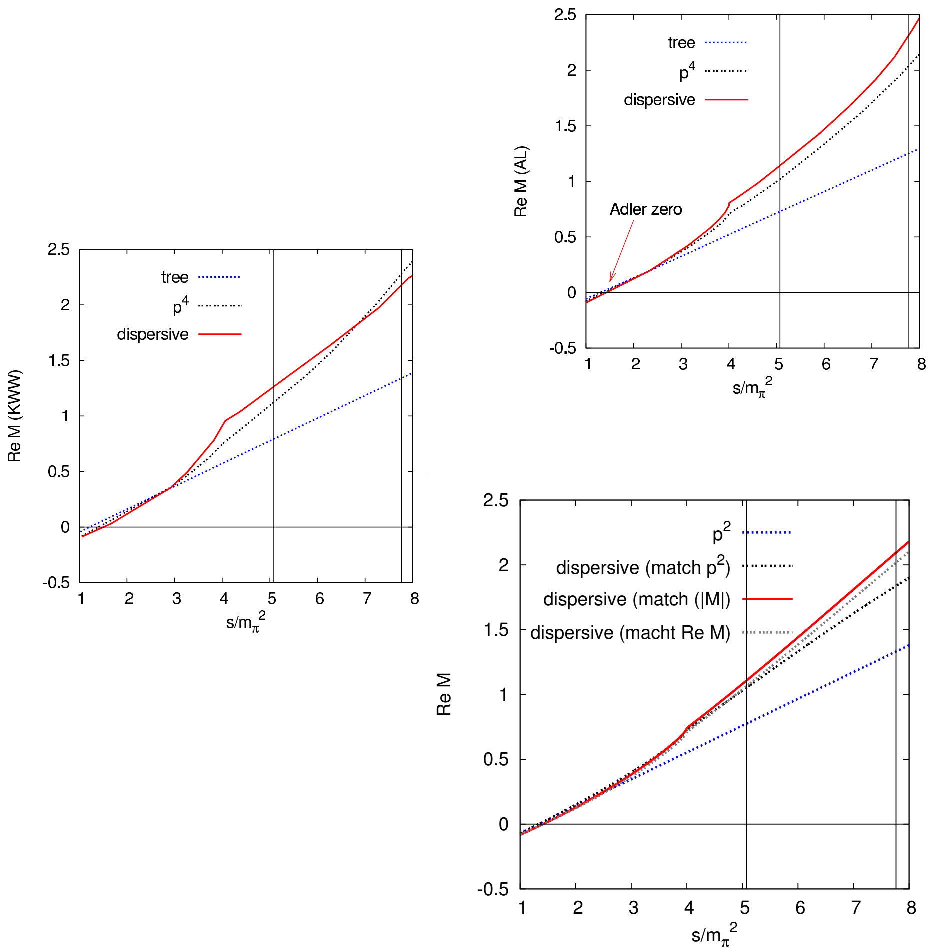

Fig. 7. The real part of the $\eta \to \pi^+\pi^-\pi^0$ amplitude plotted along the line $s = u$ with the physical region for the decay indicated. The plots are adapted from the results in: left [25], top right [24] and bottom right the simplified version of [3]. The vertical lines note the edges of the physical region.

	a	b	c	g
tree	-1.00	0.25	0.00	0.000
one-loop	-1.33	0.42	0.08	0.03
dispersive (KWW)	-1.16	0.26	0.10	$-0.021(7)$
tree dispersive	-1.10	0.31	0.001	-0.013
absolute dispersive	-1.21	0.33	0.04	-0.014

Tab. 2. Theory results for the distributions

	g
Alde	-0.044 ± 0.046
Crystal Barrel	-0.104 ± 0.039
Crystal Ball	-0.062 ± 0.008
SND	-0.020 ± 0.023
KLOE	-0.026 ± 0.014

Tab. 3. Experiment: neutral decay

	a	b	c
Layter	-1.08 ± 0.14	0.034 ± 0.027	0.046 ± 0.031
Gormley	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04
Crystal Barrel	-0.94 ± 0.15	0.11 ± 0.27	
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 fixed
KLOE	-1.072 ± 0.009	0.117 ± 0.008	0.047 ± 0.008

Tab. 4. Experimental results for the charged decay

5 Chiral Lagrangians and η'

To treat the η' we have to go back and look at the mechanism that gave the η' its mass. This is the $U(1)_A$ anomaly as discussed earlier. There exists a limit in QCD where the anomaly is not there, namely when the number of colours, N_c, is not kept at three but sent to infinity keeping $N_c\alpha_S$ constant [30]. In this limit many aspects of QCD simplify, see e.g. [31] for an overview and introduction. The η' becomes a Goldstone Boson and it can be introduced in the Lagrangian. This was done by Veneziano, DiVecchia, Witten, Schechter and others [32–36]. One treats the parameter θ as an external field and adds the singlet degree of freedom ϕ_0 to the Goldstone boson matrix U as follows:

$$\tilde{\phi} = \theta + \frac{\sqrt{2}\phi_0}{F}, \quad U = e^{i\sqrt{2}\phi_0/F_0} e^{i\sqrt{2}M/F}. \tag{22}$$

Under a general $U(3)_L \times U(3)_R$ transformation $\tilde{\phi}$ is invariant and $U \to g_R U g_L^\dagger$. The Lagrangian must then be constructed being invariant under the full $U(3)_L \times U(3)_R$. The transformation of θ as an external field assures that the effect of the anomaly is correctly accounted for. This approach has two problems. It is not clear whether it is a convergent procedure with a derivative expansion for the η' and the number of free parameters is very large. In fact, since $\tilde{\phi}$ is invariant one can add free functions $F_i(\tilde{\phi})$ instead of all the usual parameters of ChPT as well as a series of extra terms. To lowest order there are 5 such functions [17] but at next order there are 57 [37].

Luckily, if the large N_c counting itself is included into the power counting things become simpler. By looking at

$$\partial_\mu A^{0\mu}[\mathcal{O}(N_c)] \quad = \quad m_q P[\mathcal{O}(N_c)] \; + \; \omega[\mathcal{O}(1)]\,, \tag{23}$$

we see that we can take ω as a perturbation and treat $\tilde{\phi}$ as a quantity of order $1/\sqrt{N_c}$. The leading part is then the usual chiral Lagrangian but with the nonet U and a mass term for the singlet added as discussed in [32–35]. Treating ω as a perturbation is the basis of *all* the large N_c chiral Lagrangian predictions for the η'.

6 The decays $\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$

We can now use the methods discussed in the previous section but some problems remain. There are large $\pi\pi$ rescatterings possible in the S-wave channel, which are $1/N_c$ suppressed but sizable. In other words, how do we deal with the "σ"? The other problem is that ρ and ω are present in

	$\eta' \to \eta\pi^0\pi^0$	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \pi^0\pi^0\pi^0$	$\eta' \to \pi^0\pi^+\pi^-$
	1.0 keV	1.9 keV	455 eV	405 eV
Exp	42 ± 6 keV	89 ± 10 keV	311 ± 77 eV	≤ 1005 eV

Tab. 5. The prediction from the lowest order chiral Lagrangian for η' decays and their experimental values.

the final states. One thus obviously needs to go beyond ChPT. This also means that experiment in these decays will provide us with needed clues on how we can go beyond ChPT. Several attempts at resummation exist. One example is the work by Borasoy and collaborators [9, 38] and the unitary extension of chiral perturbation theory. The latter can be traced back from [10].

The lowest order Lagrangian has three terms

$$\mathcal{L} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle\ [(a)]\ + \frac{F^2}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle\ [(b)]\ - \frac{1}{2}m_0^2\phi_0^2\,. \tag{24}$$

The two decays are very different in their origin. The amplitude for $\eta' \to \eta\pi\pi$ comes from the term (a) in Eq. (24) with

$$A(\eta' \to \eta\pi\pi) = \frac{m_\pi^2}{6F}\left(s\sqrt{2}\cos(2\theta) - \sin(2\theta)\right) \tag{25}$$

while $\eta' \to \pi\pi\pi$ is produced from term (b) in Eq. (24) with an amplitude

$$A(\eta' \to \pi\pi\pi) \sim \frac{m_u - m_d}{F_\pi}\,. \tag{26}$$

The predictions from these are shown in Table 5 and compared with the experimental values. The two types of decays are quite different. The decays $\eta' \to \eta\pi\pi$ are very much off the predictions while the $\eta' \to 3\pi$ agree reasonably well. In the former decay, no isospin breaking is needed. The factor of m_π^2 which suppresses the decay in fact disappears at higher orders. The next order in $1/N_c$ allows for terms like $\eta'\eta\partial_\mu\pi\partial^\mu\pi$ which remove this suppression. That alone allows one to fit the rate but not the slope. We therefore need to study the distributions in phase space to uncover the type of effects that are important. The precision of the branching ratios also needs improvement to check the factor of 2 prediction from isospin. The 3π decay is isospin violating and always needs an overall factor of $m_u - m_d$. We thus do not expect as dramatic effects compared to lowest order but to check good quantitative agreement needs better experimental precision.

7 Anomalies

A major topic in η and η' decays is the study of the nonabelian anomaly [39] and its effective Lagrangian, the Witten-Wess-Zumino term [40, 41]. An obviously incomplete list of processes and experiments is given in Table 6.

The ones labeled $g-2$ are interesting also for the muon anomalous magnetic moment. They show up as subparts of the light-by-light hadronic contribution there, as shown in Fig. 9. The order p^6 Lagrangian is known [42]. An earlier review for anomalous η process is [43].

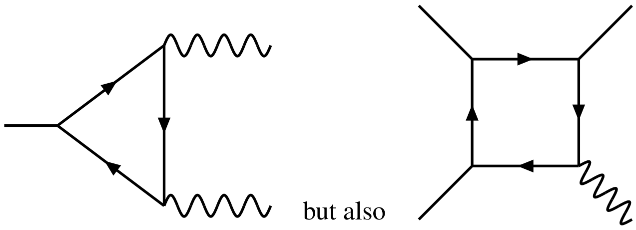

Fig. 8. Some of the graphs for the anomaly. Now with external photons or weak bosons rather than gluons.

$\pi^0 \to \gamma\gamma$	Primex, e^+e^-	$\eta' \to \pi^+\pi^-\gamma^{(*)}$	WASA	
$\eta \to \gamma\gamma$	Primex, e^+e^-	$\eta \to \gamma^{(*)}\gamma^{(*)}$	WASA,KLOE, CLEOc	g-2
$\eta' \to \gamma\gamma$	e^+e^-	$\eta' \to \gamma^{(*)}\gamma^{(*)}$	WASA,KLOE, CLEOc	g-2
$\gamma\pi^0\pi^+\pi^-$	Primakoff	$\eta' \to \rho^0\gamma$	WASA	
$\eta \to \pi^+\pi^-\gamma^{(*)}$	WASA,KLOE			

Tab. 6. Some anomalous processes and the experiments where we expect improvements. The symbol $\gamma^{(*)}$ stands for (if allowed) γ, e^+e^-, $\mu^+\mu^-$ or an off-shell photon in tagged $\gamma\gamma$ collisions.

8 Conclusions

There is a rich field of physics to be explored in η and η' decays. Some of these have been discussed in this talk. We look forward to find out more from WASA, KLOE and the other experiments presented at this meeting. Let me conclude by giving simply a list of topics/questions.

- Precision physics: Q from $\eta \to 3\pi$.
- Understanding physics: π^0 versus η versus η'.
- All channels: do the flavour singlet degrees of freedom differ significantly from the nonsinglet?
- Glue is important for η' in its mass. Can we detect it also in other places or is the rest *merely* a problem of final state interactions?
- requires getting at the mechanisms behind η, η' decays.
- High quality distributions are a must.

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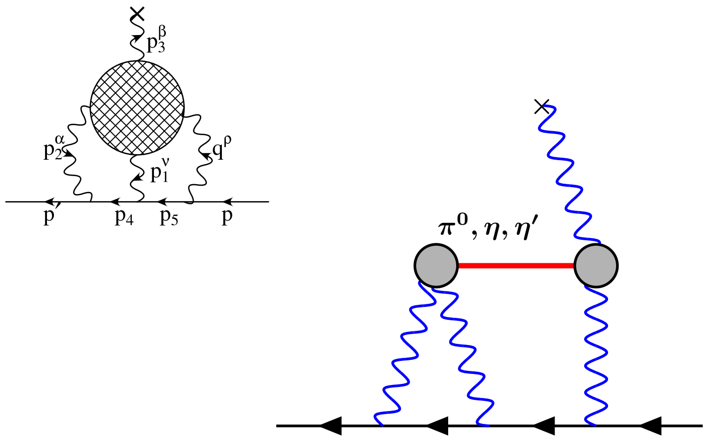

Fig. 9. The light-by-light hadronic contribution to the muon $g-2$ and a subprocess with anomalous vertices.

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