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GLUONIC EFFECTS IN η- AND η'-NUCLEON AND NUCLEUS INTERACTIONS[1]

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Gluonic degrees of freedom play an important role in the masses of the η and η' mesons. We discuss η- and η'-nucleon and nucleus interactions where this glue may be manifest. Interesting processes being studied in experiments are η' production in proton-nucleon collisions close to threshold and possible $\eta-$nucleus bound-states.

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1 The axial U(1) problem

Gluonic degrees of freedom play an important role in the physics of the flavour-singlet $J^P = 1^+$ channel [1] through the QCD axial anomaly [2]. The most famous example is the axial U(1) problem: the masses of the η and η' mesons are much greater than the values they would have if these mesons were pure Goldstone bosons associated with spontaneously broken chiral symmetry [3]. This extra mass is induced by non-perturbative gluon dynamics [4–9].

Spontaneous chiral symmetry breaking is associated with a non-vanishing chiral condensate

$$\langle \text{ vac } | \bar{q}q | \text{ vac } \rangle < 0. \tag{1}$$

The non-vanishing chiral condensate also spontaneously breaks the axial U(1) symmetry so, naively, we expect a nonet of would-be pseudoscalar Goldstone bosons: the octet associated with chiral $SU(3)_L \otimes SU(3)_R$ plus a singlet boson associated with axial U(1) — each with mass squared $m^2_{\text{Goldstone}} \sim m_q$ where m_q denotes the light and strange quark masses. The pions and kaons are described well by this theory. The masses of the η and η' mesons are about 300-400 MeV too big to fit in this picture without additional physics. One needs extra mass in the singlet channel associated with non-perturbative gluon configurations and the QCD axial anomaly [2]. The strange quark mass induces considerable η-η' mixing. For free mesons the $\eta - \eta'$ mass matrix (at leading order in the chiral expansion) is

$$M^2 = \begin{pmatrix} \frac{4}{3}m^2_{\rm K} - \frac{1}{3}m^2_{\pi} & -\frac{2}{3}\sqrt{2}(m^2_{\rm K} - m^2_{\pi}) \\ -\frac{2}{3}\sqrt{2}(m^2_{\rm K} - m^2_{\pi}) & [\frac{2}{3}m^2_{\rm K} + \frac{1}{3}m^2_{\pi} + \tilde{m}^2_{\eta_0}] \end{pmatrix}. \tag{2}$$

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Here $\tilde{m}^2_{\eta_0}$ denotes the gluonic mass contribution in the singlet channel. It has a rigorous interpretation through the Witten-Veneziano mass formula [6, 7] and is associated with non-perturbative gluon topology, related perhaps to confinement [8] or instantons [9]. When we diagonalize this matrix

$$\begin{aligned} |\eta\rangle &= \cos\theta\,|\eta_8\rangle - \sin\theta\,|\eta_0\rangle \\ |\eta'\rangle &= \sin\theta\,|\eta_8\rangle + \cos\theta\,|\eta_0\rangle \end{aligned} \tag{3}$$

with

$$\eta_0 = \frac{1}{\sqrt{3}}\,(u\bar{u} + d\bar{d} + s\bar{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}\,(u\bar{u} + d\bar{d} - 2s\bar{s}) \tag{4}$$

we obtain values for the η and η' masses

$$m^2_{\eta',\eta} = (m^2_{\rm K} + \tilde{m}^2_{\eta_0}/2) \pm \frac{1}{2}\sqrt{(2m^2_{\rm K} - 2m^2_\pi - \frac{1}{3}\tilde{m}^2_{\eta_0})^2 + \frac{8}{9}\tilde{m}^4_{\eta_0}}. \tag{5}$$

The physical mass of the η is close to the octet mass $m_{\eta_8} = \sqrt{\frac{4}{3}m^2_{\rm K} - \frac{1}{3}m^2_\pi}$, within a few percent. However, to build a theory of the η treating it as a pure octet state risks losing essential physics associated with the singlet component and axial U(1) dynamics. In the absence of the gluonic term ($\tilde{m}^2_{\eta_0}$ set equal to zero), one finds $m_{\eta'} \sim \sqrt{2m^2_{\rm K} - m^2_\pi}$ and $m_\eta \sim m_\pi$. That is, without extra input from glue, in the OZI limit, the η would be approximately an isosinglet light-quark state ($\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$) degenerate with the pion and the η' would be a strange-quark state $|\bar{s}s\rangle$ — mirroring the isoscalar vector ω and ϕ mesons. The gluonic mass term is vital to understanding the physical η and η' mesons. [3]

2 Glue and η and η' nucleon interactions

Given that glue plays an important role in the masses of the η and η' mesons, it is worthwhile and interesting to look for possible manifestations of gluonic effects in dynamical processes involving these mesons. In the rest of this paper we consider η and η' production in proton-nucleon collisions close to threshold, and possible η–nucleus bound-states. These systems are being studied in experiments at COSY and GSI. We note that the η'–nucleon coupling constant is related, in part, to the flavour-singlet axial-charge extracted from polarized deep inelastic scattering experiments [14] – for a recent review see [15].

2.1 η and η' production in proton-nucleon collisions close to threshold

Since the singlet components of the η and η' couple to glue, it is natural to consider the process where glue is excited in the "short distance" (~ 0.2fm) interaction region of a proton-nucleon collision and then evolves to become an η' in the final state [16]. This gluonic induced production

[3] Taking the value $\tilde{m}^2_{\eta_0} = 0.73\text{GeV}^2$ [7] in the leading-order mass formula, Eq.(5), gives agreement with the physical masses at the 10% level. The corresponding $\eta - \eta'$ mixing angle $\theta \simeq -18°$ is within the range $-17°$ to $-20°$ obtained from a study of various decay processes in [10, 11]. Closer agreement with the physical masses can be obtained by introducing the singlet decay constant $F_0 \neq F_\pi$ and including higher-order mass terms in the chiral expansion [12, 13].

mechanism is extra to the contributions associated with meson exchange models [17–19]. Given the large gluonic effect in the mass, there is no reason, a priori, to expect it to be small. The contribution to the matrix elements for η' and η production is weighted by the singlet-component projection-factors $\cos\theta$ for the η' and $\sin\theta$ for the η where θ is the $\eta - \eta'$ mixing angle. The angle $\theta \sim -20$ degrees means that gluonic induced production should be considerably enhanced in η' production compared to η production.

What is the phenomenology of this gluonic interaction ?

Since glue is flavour-blind the gluonic production process has the same size in both the $pp \to pp\eta'$ and $pn \to pn\eta'$ reactions. CELSIUS [20] have measured the ratio $R_\eta = \sigma(pn \to pn\eta)/\sigma(pp \to pp\eta)$ for quasifree η production from a deuteron target up to 100 MeV above threshold. They observed that R_η is approximately energy-independent $\simeq 6.5$ over the whole energy range. The value of this ratio signifies a strong isovector exchange contribution to the η production mechanism [20]. This experiment is being repeated for η' production. The cross-section for $pp \to pp\eta'$ close to threshold has been measured by the COSY-11 Collaboration [21] who are now measuring the $pn \to pn\eta'$ process [22]. In the extreme scenario that the glue-induced production saturated the η' production cross-section, the ratio $R_{\eta'} = \sigma(pn \to pn\eta')/\sigma(pp \to pp\eta')$ would go to one after we correct for the final state interaction [19, 23] between the two outgoing nucleons. In practice, we should expect contributions from both gluonic and meson-exchange type mechanisms. It will be interesting to observe the ratio $R_{\eta'}$ and how it compares with R_η.

Gluonic induced production appears as a contact term in the axial U(1) extended chiral Lagrangian for low-energy QCD [16].

2.2 η-nucleus bound-states

New experiments at the GSI will employ the recoilless $(d,\ {}^3He)$ reaction to study the possible formation of η meson bound states inside the nucleus [24, 25], following on from the successful studies of pionic atoms in these reactions [26]. The idea is to measure the excitation-energy spectrum and then, if a clear bound state is observed, to extract the in-medium effective mass, m^*_η, of the η in nuclei through performing a fit to this spectrum with the η-nucleus optical potential.

Meson masses in nuclei are determined from the scalar induced contribution to the meson propagator evaluated at zero three-momentum, $\vec{k} = 0$, in the nuclear medium. Let $k = (E, \vec{k})$ and m denote the four-momentum and mass of the meson in free space. Then, one solves the equation

$$k^2 - m^2 = \mathrm{Re}\ \Pi(E, \vec{k}, \rho) \tag{6}$$

for $\vec{k} = 0$ where Π is the in-medium s-wave meson self-energy and ρ is the nuclear density. Contributions to the in medium mass come from coupling to the scalar σ field in the nucleus in mean-field approximation, nucleon-hole and resonance-hole excitations in the medium. The s-wave self-energy can be written as [27]

$$\Pi(E, \vec{k}, \rho)\Big|_{\{\vec{k}=0\}} = -4\pi\rho\left(\frac{b}{1 + b\langle\frac{1}{r}\rangle}\right). \tag{7}$$

Here $b = a(1 + \frac{m}{M})$ where a is the meson-nucleon scattering length, M is the nucleon mass and the mean inter-nucleon seperation is $\langle\frac{1}{r}\rangle$. Attraction corresponds to positive values of a. The denominator in Eq.(7) is the Ericson-Ericson double scattering correction.

The in-medium mass m_η^* is sensitive to the flavour-singlet component in the η, and hence to the non-perturbative glue associated with axial U(1) dynamics. An important source of the in-medium mass modification comes from light-quarks coupling to the scalar σ mean-field in the nucleus. Increasing the flavour-singlet component in the η at the expense of the octet component gives more attraction, more binding and a larger value of the η-nucleon scattering length, $a_{\eta N}$. Since the mass shift is approximately proportional to the η–nucleon scattering length, it follows that the physical value of $a_{\eta N}$ should be larger than if the η were a pure octet state.

This physics has been investigated by Bass and Thomas [28]. QCD arguments suggest that the gluonic mass term is suppressed at finite density due to coupling to the σ mean-field in the nucleus. [4] Phenomenology is used to estimate the size of the effect in the η using the Quark Meson Coupling model (QMC) of hadron properties in the nuclear medium [30]. Here one uses the large η mass (which in QCD is induced by mixing and the gluonic mass term) to motivate taking an MIT Bag description for the η wavefunction, and then coupling the light (up and down) quark and antiquark fields in the η to the scalar σ field in the nucleus working in mean-field approximation [30]. The strange-quark component of the wavefunction does not couple to the σ field. $\eta - \eta'$ mixing is readily built into the model.

The mass for the η in nuclear matter is self-consistently calculated by solving for the MIT Bag in the nuclear medium [30]:

$$m_\eta^*(\vec{r}) = \frac{2[a_P^2 \Omega_q^*(\vec{r}) + b_P^2 \Omega_s(\vec{r})] - z_\eta}{R_\eta^*} + \frac{4}{3}\pi R_\eta^{*3} B, \tag{8}$$

$$\left.\frac{\partial m_j^*(\vec{r})}{\partial R_j}\right|_{R_j = R_j^*} = 0, \qquad (j = \eta,\ \eta'). \tag{9}$$

Here Ω_q^* and Ω_s are light-quark and strange-quark Bag energy eigenvalues, R_η^* is the Bag radius in the medium and B is the Bag constant. The $\eta - \eta'$ mixing angle θ is included in the terms $a_P = \frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta$ and $b_P = \sqrt{\frac{2}{3}}\cos\theta + \frac{1}{\sqrt{3}}\sin\theta$ and can be varied in the model. One first solves the Bag for the free η with a given mixing angle, and then turns on QMC to obtain the mass-shift. In Eq. (8), z_η parameterizes the sum of the center-of-mass and gluon fluctuation effects, and is assumed to be independent of density [31].

The coupling constants in the model for the coupling of light-quarks to the σ mean-field in the nucleus are adjusted to fit the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. The Bag parameters used in these calculations are $\Omega_q = 2.05$ (for the light quarks) and $\Omega_s = 2.5$ (for the strange quark) with $B = (170\text{MeV})^4$. For nuclear matter density we find $\Omega_q^* = 1.81$ for the $1s$ state. This value depends on the coupling of light-quarks to the σ mean-field and is independent of the mixing angle θ.

Increasing the mixing angle increases the amount of singlet relative to octet components in the η. This produces greater attraction through increasing the amount of light-quark compared to strange-quark components in the η and a reduced effective mass. Through Eq.(7) increasing the mixing angle also increases the η-nucleon scattering length $a_{\eta N}$. We quantify this in Table 1 which presents results for the pure octet ($\eta = \eta_8$, $\theta = 0$) and the values $\theta = -10^\circ$ and -20°

[4] In the chiral limit the singlet analogy to the Weinberg-Tomozawa term does not vanish because of the anomalous glue terms. Starting from the simple Born term one finds anomalous gluonic contributions to the singlet-meson nucleon scattering length proportional to $\tilde{m}_{\eta_0}^2$ and $\tilde{m}_{\eta_0}^4$ [29].

	m (MeV)	m^* (MeV)	$\mathtt{Re}a$ (fm)
η_8	547.75	500.0	0.43
η (-10^o)	547.75	474.7	0.64
η (-20^o)	547.75	449.3	0.85
η_0	958	878.6	0.99
η' (-10^o)	958	899.2	0.74
η' (-20^o)	958	921.3	0.47

Tab. 1. Physical masses fitted in free space, the bag masses in medium at normal nuclear-matter density, $\rho_0 = 0.15$ fm^{-3}, and the corresponding meson-nucleon scattering lengths.

(the physical mixing angle). The values of $\mathtt{Re}a_\eta$ quoted in Table 1 are obtained from substituting the in-medium and free masses into Eq. (7) with the Ericson-Ericson denominator turned-off, and using the free mass in the expression for b. The effect of exchanging m for m^* in b is a 5% increase in the quoted scattering length. The QMC model makes no claim about the imaginary part of the scattering length. The key observation is that $\eta - \eta'$ mixing leads to a factor of two increase in the mass-shift of the η meson and in the scattering length obtained in the model. [5]

The density dependence of the mass-shifts in the QMC model is discussed in Ref. [30]. Neglecting the Ericson-Ericson term, the mass-shift is approximately linear. For densities ρ between 0.5 and 1 times ρ_0 (nuclear matter density) we find

$$m^*_\eta/m_\eta \simeq 1 - 0.17\rho/\rho_0 \qquad (10)$$

for the physical mixing angle $-20°$. The scattering lengths extracted from this analysis are density independent to within a few percent over the same range of densities.

3 Conclusions and Outlook

Glue plays an important role in the masses of the η and η' mesons. New experiments are measuring the interactions of these mesons with nucleons and nuclei. The glue which generates a large part of the η and η' masses can contribute to the cross-section for η' production in proton-nucleon collisions and to the possible binding energies of η and η' mesons in nuclei. It will be interesting to see the forthcoming data from COSY and GSI on these processes.

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[5]Because the QMC model has been explored mainly at the mean-field level, it is not clear that one should include the Ericson-Ericson term in extracting the corresponding η nucleon scattering length. Substituting the scattering lengths given in Table 1 into Eq. (7) (and neglecting the imaginary part) yields resummed values $a_{eff} = a/(1 + b\langle 1/r \rangle)$ equal to 0.44 fm for the η with the physical mixing angle $\theta = -20$ degrees, with corresponding reduction in the binding energy.

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