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ON THE LOWER SCALAR MESON STATES[1]

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The processes $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$ at the channel with $I^G J^{PC} = 0^+0^{++}$ are analysed using the analyticity properties of scattering matrix elements. The investigation is focussed on the properties of the $f_0(665)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ states. The analysis supports the $f_0(665)$ as the very broad resonance. It suggests further to see the $f_0(980)$ state as predominantly the $\eta\eta$ bound state with dominant $(\overline{q}q)(\overline{q}q)$ components. The quark content of other states is inferred and $f_0(1500)$ appears as a mixed state with dominant glueball component.

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The scalar meson states of vacuum quantum numbers are playing an important role in elementary particle physics. One of the reasons is that they provide interesting laboratory for the Quantum Chromodynamics (QCD) predictions so successful in explaining the known spectra of elementary particles in terms of quarks and gluons. Moreover it is expected that the theoretical QCD predictions of existence of glueballs, the elementary particles composed solely from gluons without quarks, would be first verified experimentally just in this sector of light mesons. This expectation is based on the fact that their theoretically calculated masses should be within interval of the present day experimentally known resonant states. The identification of these resonant states from the scattering experiments is therefore very important. The problem is, however, that the observed resonant states appear at energies at which it is not possible to exclude their interactions with coupled scattering channels which makes the otherwise well functioning Breit-Wigner one pole resonant formula unreliable and instead the many coupled channels formalism has to be applied. On the other hand, it is expected that the glueball states interact with various channels differently from the ordinary particles made of quarks, which makes the hope for their identification.

Because the existence of coupled channels appears as branch points in scattering amplitudes and the presence of resonant states as their pole singularities it is very important to know the topology of the coupled scattering amplitudes and the whole scattering matrix corresponding

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to interactions of mutually coupled channels has to be considered. The general outline of this method of experimental data analysis has been proposed and successfully applied [1]. It is based on the first principles - analyticity, causality, unitarity and Lorentz-invariance and allows to be immediately applied to experimental data analysis.

Spectrum of scalar mesons with quantum numbers $I^G J^{PC} = 0^+0^{++}$ is observed in the s-wave of $\pi\pi$ mesons scattering. The coupled channel processes besides the elastic scattering are mainly the $K\overline{K}$ and $\eta\eta$ production channels. These three reactions are simultaneously described by 3×3 S-matrix determined on the eight-sheeted Riemann surfaces in the s-variable, which is the invariant total energy squared. The Riemann surfaces are there due to the right-hand cuts along the real axis of the s complex plane starting at $4\mathrm{m}_\pi^2$, $4\mathrm{m}_K^2$, and $4\mathrm{m}_\eta^2$. They are caused by the S-matrix unitarity where $\mathrm{S}^\dagger$ is hermitian conjugate to S. In one channel case only the $\pi\pi \to \pi\pi$ elastic process is opened and sign (Imk_1) = + and - for the two Riemann sheets. In two channel case, i.e. for processes $\pi\pi \to \pi\pi$ and the $K\overline{K}$ production channel $\pi\pi \to K\overline{K}$ the sign (Imk_1, Imk_2) = + +, - +, - -, + - for four Riemann sheets. If all three channels $\pi\pi \to \pi\pi$, $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ are opened the 8 Riemann sheets I, II, ... VIII correspond to Sign (Imk_1, Imk_2, Imk_3) = + + +, - + +, - - +, + - +, + - -, - - -, - + -, + + -.

The S-matrix unitarity can be used to find out the analytic continuation of all S-matrix elements, which are real analytic complex functions, from the 1st sheet to all the connected Riemann sheets [1]. The S_{11} matrix element corresponds to the $\pi\pi \to \pi\pi$ scattering process, the S_{12} to the $\pi\pi \to K\overline{K}$ and S_{13} to the $\pi\pi \to \eta\eta$ and similarly the other cross channels. The result of this procedure is expressed in Tab. 1.

For the two coupled channels, i.e. for the coupled processes $\pi\pi \to \pi\pi$, $\pi\pi \to K\overline{K}$ and $K\overline{K} \to K\overline{K}$ the both cuts on the real s axis at s = $4\mathrm{m}_\pi^2$ and s = $4\mathrm{m}_K^2$ play the role and the 2×2 S-matrix elements analytical continuation to Riemann sheets II, III and IV is expressed by the first three rows and four columns of Tab. 1. One can see that the zero due to the resonance in S_{11} will appear as the 2^{nd} sheet pole at the other two coupled processes. As $\mathrm{D}_{33} = \mathrm{S}_{11}\mathrm{S}_{22} - \mathrm{S}_{12}^2$ it will appear also on the 3^{rd} sheet at coupled processes but at the shifted position. The magnitude of the shift depends on the value of S_{12} and in the absence of the coupling (S_{12} = 0) there is no shift of poles relative to the zero on the 1^{st} sheet.

From the above relations one can see that by starting from resonance zeros on the 1^{st} Riemann

Tab. 1. The 3×3 S-matrix elements analytically continued from the 1^{st} Riemann sheet to all eight adjacent Riemann sheets. There D_{kl} denotes the minor of the element S_{kl}, so $\mathrm{D}_{kk} = \mathrm{S}_{ll}\mathrm{S}_{mm} - \mathrm{S}_{lm}^2$, and $\mathrm{D}_{kl} = \mathrm{S}_{kl}\mathrm{S}_{mm} - \mathrm{S}_{km}\mathrm{S}_{lm}$ for k, l, m = 1, 2, 3 in cycle. The S matrix is symmetric due to the time reversal.

I	II	III	IV	V	VI	VII	VIII
S_{11},	$1/\mathrm{S}_{11}$,	$\mathrm{S}_{22}/\mathrm{D}_{33}$,	$\mathrm{D}_{33}/\mathrm{S}_{22}$,	$\mathrm{detS}/\mathrm{D}_{11}$,	$\mathrm{D}_{11}/\mathrm{detS}$,	$\mathrm{S}_{33}/\mathrm{D}_{22}$,	$\mathrm{D}_{22}/\mathrm{S}_{33}$
S_{12},	$\mathrm{iS}_{12}/\mathrm{S}_{11}$,	$-\mathrm{S}_{12}/\mathrm{D}_{33}$,	$\mathrm{iS}_{12}/\mathrm{S}_{22}$,	$\mathrm{iD}_{12}/\mathrm{D}_{11}$,	$-\mathrm{D}_{12}/\mathrm{detS}$,	$\mathrm{iD}_{12}/\mathrm{D}_{22}$,	$\mathrm{D}_{12}/\mathrm{S}_{33}$
S_{22},	$\mathrm{D}_{33}/\mathrm{S}_{11}$,	$\mathrm{S}_{11}/\mathrm{D}_{33}$,	$1/\mathrm{S}_{22}$,	$\mathrm{S}_{33}/\mathrm{D}_{11}$,	$\mathrm{D}_{22}/\mathrm{detS}$,	$\mathrm{detS}/\mathrm{D}_{22}$,	$\mathrm{D}_{11}/\mathrm{S}_{33}$
S_{13},	$\mathrm{iS}_{13}/\mathrm{S}_{11}$,	$-\mathrm{iD}_{13}/\mathrm{D}_{33}$,	$-\mathrm{D}_{13}/\mathrm{S}_{22}$,	$-\mathrm{iD}_{13}/\mathrm{D}_{11}$,	$\mathrm{D}_{13}/\mathrm{detS}$,	$-\mathrm{S}_{13}/\mathrm{D}_{22}$,	$\mathrm{iS}_{13}/\mathrm{S}_{33}$
S_{23},	$\mathrm{D}_{23}/\mathrm{S}_{11}$,	$\mathrm{iD}_{23}/\mathrm{D}_{33}$,	$\mathrm{iS}_{23}/\mathrm{S}_{22}$,	$-\mathrm{S}_{23}/\mathrm{D}_{11}$,	$-\mathrm{D}_{23}/\mathrm{detS}$,	$\mathrm{iD}_{23}/\mathrm{D}_{22}$,	$\mathrm{iS}_{23}/\mathrm{S}_{33}$
S_{33},	$\mathrm{D}_{22}/\mathrm{S}_{11}$,	$\mathrm{detS}/\mathrm{D}_{33}$,	$\mathrm{D}_{11}/\mathrm{S}_{22}$,	$\mathrm{S}_{22}/\mathrm{D}_{11}$,	$\mathrm{D}_{33}/\mathrm{detS}$,	$\mathrm{S}_{11}/\mathrm{D}_{22}$,	$1/\mathrm{S}_{33}$

sheet the resonance representations in terms of poles and zeros on the full Riemann surface is obtained. In the two coupled channel case there the three types of resonances generated by the 1^{st} sheet zeros can be distinguished depending whether they are caused by zeros in S_{11}, S_{22} or in both of them. In the three channel case such classification includes seven different causes of resonances.

In two channel case it is convenient to use transformation $z = (k_1 + k_2)/(m_K^2 - m_\pi^2)^{1/2}$ and to map the four Riemann sheets into one z-plane. This mapping effectively removes the kinematical threshold branch points of both channels and apart of the dynamical left hand cuts leaves us just with poles and zeros. The left hand cut is far from the physical region and can be simply taken into account. Since the kinematical cuts of coupled channels have been removed but the other analytical properties of scattering amplitudes bave been preserved the corresponding S matrix elements have only zeros and poles in the new complex variable z. At this point it is convenient to use the fact that the S matrix elements can be expressed in terms of one function and to write them as it was derived in [2]:

$$S_{11} = d(-z^{-1})/d(z); S_{22} = d(z^{-1})/d(z); \text{and}\det S = d(-z)/d(z). \tag{1}$$

These are expressions analogous to Le Couteur-Newton relations [3, 4]. The d(z) does not have the branch points and can be factorized to the resonance and background parts d = $d_B d_{res}$. It turns out that since the $\pi\pi$ background contribution is practically equal to 1, we have

$$d_B = z^{-4}(1 - z_0 z)^4(1 + z_0^* z)^4 \qquad d_{res}(z) = z^{-M}\prod_{n=1}^{M}(1 - z_n * z)(1 + z_n z), \tag{2}$$

where d_B comes from the $K\overline{K}$ background contribution and do not contribute to the $\pi\pi$ scattering amplitude and M is the number of pairs of the conjugate zeros. The z_0 and z_n positions are free parameters in our scattering data analyses to reproduce the scattering matrix elements. Due to the unitarity equation they can be written as: S_{11} = η expδ_1, $S_{12} = (1 - \eta^2)^{1/2}$exp($\delta_{12}$), and $S_{22} = \eta$expδ_2, where inelasticity η and $\pi\pi \to \pi\pi$ phase shift δ_1 and $\pi\pi \to K\overline{K}$ phase shift δ_{12} are known from the scattering experiments and $\delta_1 + \delta_2 = \delta_{12}$.

After describing in details the two coupled channel formalism let us skip for the reason of clarity its mathematical extension to the three channels. The three channel mathematical formalism is only more complex but in essence it follows the same ideas. It is clear that in that case the coupling of $\pi\pi \to \pi\pi$, $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ processes have to be taken into account and also all the other combined processes as described in Tab. 1. Unfortunately, the experimental measurements are providing us mainly with the data from the above three $\pi\pi$ scattering processes. In the analysis we are taking into account the following resonances coupled to these processes: f_0(600), f_0(980), f_0(1370), f_0(1500) and f_0(1710). The experimental data from $\pi\pi$ threshold energy up to 1.9 GeV as far as they are available for the coupled processes are used as input for the analysis [5].

By fitting the experimental data the zero and pole clusters describing resonances in these processes are showing remarkable features. First, they confirm that their best positions correspond to the theoretical expectation given by Tab. 1, i.e. they confirm the fact that certain clusters most adequately represent the multichannel states. From their locations on various Riemann sheets their properties can be traced back giving us information of their dynamical origin. The second feature is their remarkable stability when moving from the $\pi\pi \to \pi\pi$, $\pi\pi \to K\overline{K}$ coupled

Tab. 2. Comparison of resonance parameters from the two coupled channel analysis, the three coupled channel analysis and of the averaged data presented in [5, 6].

	MeV	2 Chnls	3 Chnls	PDG
$f_0(600)$	E_r	600 ± 16	661 ± 14	$400 \div 1200$
	Γ_r	605 ± 28	595 ± 22	$600 \div 1000$
$f_0(980)$	E_r	985 ± 5	962 ± 18	980 ± 10
	Γ_r	27 ± 8	74 ± 20	$40 \div 100$
$f_0(1370)$	E_r	1310 ± 22	1384 ± 20	$1200 \div 1500$
	Γ_r	410 ± 29	154 ± 25	$200 \div 500$
$f_0(1500)$	E_r	1490 ± 30	1513 ± 28	1507 ± 5
	Γ_r	220 ± 24	318 ± 27	109 ± 7
$f_0(1710)$	E_r	1700 ± 25	1699 ± 22	1714 ± 5
	Γ_r	86 ± 16	168 ± 23	140 ± 10

channels to the $\pi\pi \to \pi\pi$, $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ coupled channels. This can be seen by calculating the resonance energy E_r and the half-width Γ_r of each of the resonance from their positions in the complex s plane $s_r^{1/2} = E_r - i\Gamma_r$, Tab. 2.

Some results have been included to Particle Data Group [6] but in short we can say that research of scalar meson's spectrum is of great significance in our understanding of their quark and gluon structure [7], however more precise experimental data would be of great help. Starting from the available data suitable for our analysis we can formulate in short these observations:

The pole positions do not shift much from two to three channel case; the existence of broad $f_0(600)$ resonance is confirmed; absence of poles on Riemann sheets VI and VII indicates for $f_0(980)$ to be an $\eta\eta$ bound state; $f_0(1300)$ is coupled more strongly to $K\overline{K}$ than to $\pi\pi$ and $\eta\eta$; $f_0(1500)$ seems to be mixed state with a dominant glueball component, and the $f_0(1710)$ is more strongly coupled to $\eta\eta$ than to other two channels and has a dominant s quark component. This can give an insight in resolving the assignment of scalar mesons below 1.9 GeV to lower nonets and to investigate the possible mixing of these states.

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