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SUPERFLUID ^{3}He: A LABORATORY FOR STUDYING TURBULENCE[1]

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Superfluid ^{3}He is an ideal system for testing ideas in other areas of physics. The superfluid at low temperatures is almost 100 % condensate and thus described by a single set of simple equations. In this paper we discuss the application of the superfluid to understanding turbulence since turbulence in the superfluid takes the very simple form of a tangle of identical quantized vortices. Various observations and implications are discussed.

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1 Introduction

Turbulence is ubiquitous. It occurs on all scales from the subnuclear to the cosmological, has a great impact on human activity yet is not at all well understood. As Feynman described it,“turbulence is the last great unsolved problem of classical physic”.

One way for us to find an entry to the problem of turbulence is to find an easier system on which to apply our preliminary efforts. Such simpler systems are provided by the superfluids [1]. Superfluid ^{3}He especially provides the ideal model substance for studying turbulence as the normal fluid component of the liquid is too viscous to sustain turbulence which only occurs in the condensate component. Superfluid ^{3}He forms in the same way as the superconducting electron gas in a metal by the creation of Cooper pairs. The interactions between the atoms in liquid helium are very weak and the Cooper pairs thus only form at temperatures around 1 mK (depending on pressure). If we consider a superfluid at very low temperatures where there is essentially no normal fluid, then all the particles contribute to the Cooper pair condensate which are all in the same quantum mechanical wave function, ψ. Since the momentum/velocity is given by $\bigtriangledown\psi$ then the curl of the velocity must be zero (since $\bigtriangledown \times \bigtriangledown\psi = 0$ by definition) and the flow must be irrotational. Rotation can only be introduced into the liquid by the generation of vortices. Turbulence in the superfluid is thus made up of a tangle of vortices. While vortices can occur on all scales in a normal fluid system, in a superfluid condensate with all the constituent particles in the same quantum mechanical state, the vortices must be quantized, since as we take a particle around the vortex core to its starting point the wave function must return to its initial value thus fixing the circulation around the vortex, $\int \nu \mathrm{d}r$, in units of the quantum of circulation which

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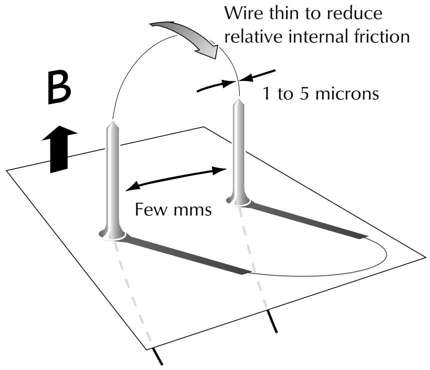

Fig. 1. A vibrating wire resonator.

in superfluid ^{3}He is $h/2m_3$, the factor of 2 coming from the fact that the constituent particles are the Cooper pairs with mass $2m_3$. This provides the great simplification. In a normal fluid the constituent eddies of turbulence can be on any scale. In the superfluids all the vortices are identical.

At the lowest temperatures we can reach ($\sim$ 80 μK), the properties of superfluid ^{3}He are unique. The only impurity capable of dissolving in superfluid ^{3}He is ^{4}He and we know the form of the solubility (from measurements at higher temperatures) to go as $T^{-3/2}\exp(-\varepsilon/k_BT)$, where ε is the phase separation energy of $\sim$ 0.8 K). At 85 μK this gives a ^{4}He solubility of 1 in 10^{4000}. If the whole visible Universe were filled with liquid ^{3}He that would only amount to about 10^{200} atoms, that is, there would be *no* chance of finding a *single* dissolved ^{4}He atom in a whole visible Universe filled with liquid ^{3}He at these temperatures. In other words, the liquid is *absolutely* pure. At these temperatures the number of unpaired atoms left in the superfluid is only about 1 in 10^8 so that the superfluid is essentially a completely pure condensate. It is an interesting side issue that the wave function of the superfluid mimics closely the structure of space-time such that vortices in ^{3}He share many of the properties of cosmic strings.

The Cooper pairs in the superfluid are formed with angular momentum and spin both unity, $L = 1, S = 1$. This means that there are several possible phases. The A phase has only $\Uparrow\Uparrow$ and $\Downarrow\Downarrow$ pairs all with the same angular momentum. This is not easy for all pairs, and impossible for those with propagation vectors parallel or antiparallel to the L direction. The energy gap thus goes to zero in this direction.

The B phase, on the other hand, has pairs with all three spin components, $\Uparrow\Uparrow$, $\Downarrow\Downarrow$ and $\Uparrow\Downarrow$. However, all our work is going to be in the B phase so all we really need to know today is that the gap is constant, as in a conventional superconductor.

2 Generating turbulence

Let us now turn to turbulence in the superfluid. How can we create and detect turbulence? Our basic measuring tool is the vibrating wire resonator shown in Fig. 1 which does everything for us. The device is a loop of a few mm in diameter made of very thin superconducting wire (say $\oslash$ of

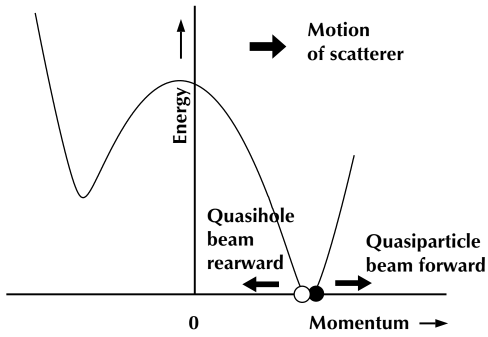

Fig. 2. The dispersion curve for superfluid ^{3}He seen in a frame moving at the Landau critical velocity. The free production of quasiparticles and quasiholes occurs as the minimum in the dispersion curve touches zero energy.

1 to 5 μm). The loop can oscillate perpendicular to its plane and if it exposed to a small vertical magnetic field this mode can be excited by passing a current at the resonant frequency through the loop. The loop moves in the quasiparticles excitation gas in the superfluid and experiences a damping force which limits its velocity. We can measure the velocity by monitoring the voltage across the loop as it moves in the magnetic field. A measure of the damping force gives us the quasiparticle density in the liquid. Since the quasiparticle density varies as $\exp(-\Delta/k_B T)$, which is a very rapid function at the lowest temperatures, a measurement of the damping immediately provides a very accurate value for the temperature.

In the superfluid we have a BCS type dispersion curve for the quasiparticle and quasihole excitations with the familiar "W" shape. An observer moving through the liquid will see this dispersion curve tilted because of the Galilean correction for the relative motion. If we move a scatterer (a vibrating wire, for example) through the liquid, in the frame of the scatterer the dispersion curve tilts until at a critical velocity one of the minima in the "W" touches zero energy. At this point the excitations at this minimum can be freely created by the moving object (since in the scattering frame they have zero energy). This represents the onset of pair breaking and the critical velocity is the Landau critical velocity at which the superfluidity begins to break down. Importantly for us, when the wire velocity reaches this value the emitted excitations take the form of a beam of quasiparticles emitted in the forward direction and a beam of quasiholes in the rearward direction as seen in Fig. 2.

When we move the wire fast the superfluidity breaks down at the surface (and especially near excrescences where the velocity is locally high, see Fig. 3). The breakdown takes the form of the production of vortex loops at the surface which can expand in, and extract energy from, the local high flow field and create tangles of vorticity [2]. This means that without needing a rotation cryostat we can inject small regions of turbulence into the superfluid at precise locations. This is the first tool we need in studying local concentrations of turbulence.

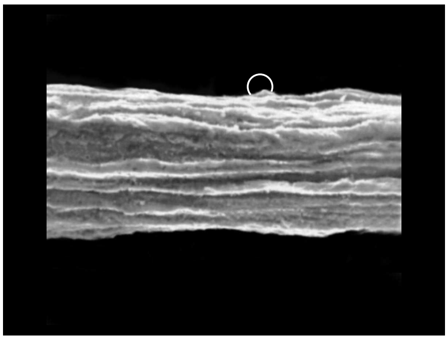

Fig. 3. A micrograph of the surface of a rough wire showing where vortices form at the excrescences.

3 Detecting turbulence

One of the problems of vorticity in general is the difficulty of visualising the process going on in the turbulence liquid. In superfluid ^{3}He we can make use of the unique properties of the quasiparticle gas to actually image the local vorticity. With no vorticity present in a sample of the superfluid the liquid is stationary, but as soon as we introduce turbulence in the form of quantized vortices then we have a rapidly changing flow pattern in the liquid. The dispersion curve for the excitations as described above is thus locally canted by the velocity fields meaning that the excitations are not free to move through the condensate but see a ragged varying potential depending on the local flow field.

For a vibrating wire to exchange energy with the excitation gas, an excitation must come from infinity, be scattered by the wire and then return to infinity. If, on the other hand the wire is surrounded by a turbulent flow field a fraction of the incoming thermal excitations coming from infinity do not have enough energy to traverse the ragged potential and cannot reach the wire. Of those that reach the wire, some cannot return to infinity but re-exchange their momentum with the wires on secondary interactions. Without being too technical, the local ragged flow field effectively shields the vibrating wire from a significant fraction of the incoming thermal excitations and thus the apparent thermal damping on the wire falls. This is quite counterintuitive since a lower damping implies a lower temperature and it took us some time to understand that we were not violating the second law of thermodynamics. Anyway, the simple upshot is that vorticity, or rather the associated flow fields, throw “shadows” on the wire resonators in the “illumination” of the surrounding thermal gas of excitations. The signal is unequivocal, as soon as vorticity appears near a wire the thermal damping falls. We can make quantitative estimates of this effect but for the purposes of the present paper we do not need to go into the details. We can just think of the vibrating wire as a one-pixel cameral looking at the local vorticity [3].

4 Grid turbulence

In an effort to create a more uniform volume of vorticity, we have made an improved turbulence generator by replacing the vibrating wire as a generator by a whole vibrating grid of wires which

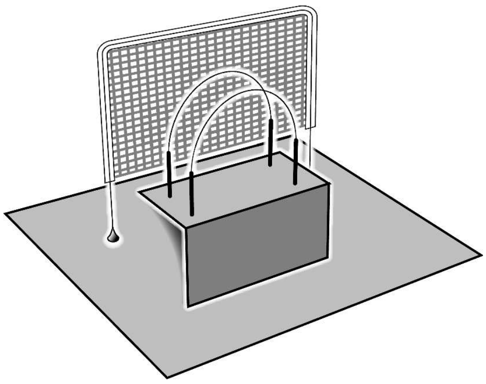

Fig. 4. A vibrating grid with the two detector wires in front.

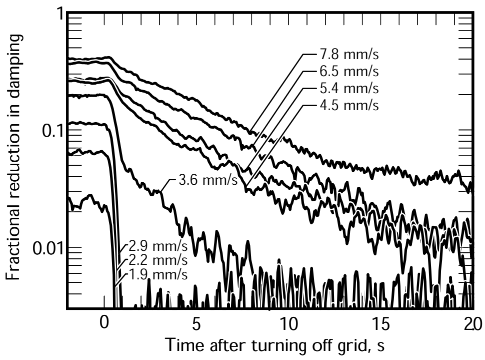

Fig. 5. The decay of vorticity after the grid is switched off, for a number of grid velocities.

should produce turbulence over a wide front. This equipment is shown in Fig. 4. The grid, 5.1 x 2.8 mm, comprises a mesh of $\sim$10 μm square cross-section copper wires 50 μm apart. A 125 μm diameter Ta wire bent into a 5 mm square is glued through the inner wall of the nuclear cooling cell. The mesh is glued to the Ta wire over a layer of electrical insulation. Directly in front of the grid, to act as vorticity detectors are two conventional vibrating wire resonators 2.5 mm diameter loops of 4.5 μm NbTi wire. Resonator No. 1 is 1 mm from the grid and resonator 2 is 2 mm from the grid. An additional remote wire resonator, shielded from the vorticity is used as a thermometer.

When the grid is oscillated, a mass of vorticity is produced which envelopes both the detector wires reducing the damping seen by both. Our first experiment with this system was to observe the rate of decay of the turbulence after the grid oscillation was switched off [4]. The results, shown in Fig. 5, were initially very surprising. What is plotted in the figure is the reduction in damping seen by one of the detector wires (the “vorticity signal”) plotted as a function of time after the vorticity generation is switched off, for various peak velocities of the grid oscillation. At

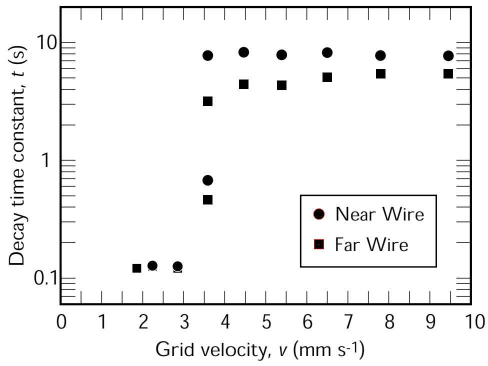

Fig. 6. The time constants for the data of Fig. 5.

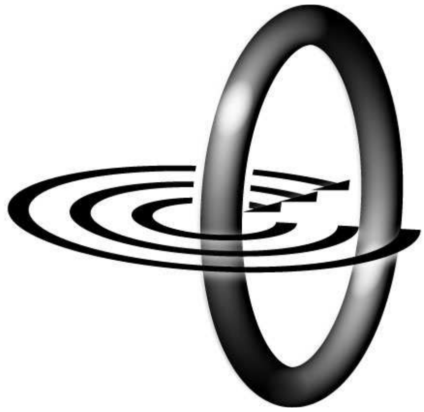

Fig. 7. A vortex ring moving in its own flow field.

high grid velocities ($V_{max} > 3$ mms^{-1}) the vorticity signal decays slowly with a characteristic time constant of order 10 s. However, at grid velocities below 3 mms^{-1}, we see completely different behaviour with the decay occurring so fast that the drop in signal is of order one tenth of a second. Given that the detector wire is only 1 mm from the grid, the vorticity must leave the grid region with a velocity of 10 mms^{-1} or more.

The time constant for this process is shown in Fig. 6 where the sudden jump in behaviour at a grid velocity of 3 mms^{-1} is very clear. What is happening?

The first thing we need to remember is that a single vortex loop as shown in Fig. 7 moves with the condensate. However, the circulation of the condensate around the core means that, crudely put, the core on one side moves in the flow field generated by the core on the other side. The result is that a ring has its own self-induced velocity. The closer the core generating the flow to the other side of the ring, the faster the flow, i.e. the self-velocity increases as the ring diameter decreases. A ring has a self velocity of 10 mms^{-1} when the ring diameter is around 5 μm.

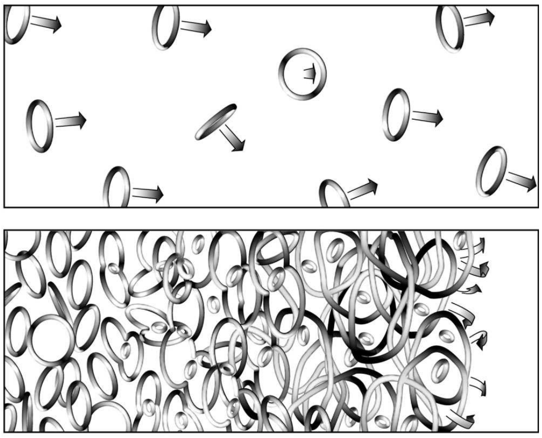

Fig. 8. Schematic view of the behaviour. At low grid velocities a tenuous gas of non-interacting vortex rings is formed as shown in the upper figure. However, in the lower figure, at higher grid velocities the ring density has become high enough that the vortex cores recombine forming a sluggish vortex tangle.

The picture we have is thus one that when the grid moves relatively slowly clouds of vortex rings with diameters of around 5 μm are constantly produced forming a fast-moving tenuous cloud. This cloud is dense enough to prevent excitations reaching the vibrating wire detectors and thus generating vorticity signal but not dense enough for the individual rings to interact. At the critical velocity of $\sim$3 mms^{-1} the rings are now produced in such large numbers that they can no longer avoid each other but intersect, recombine and rapidly form a developed tangle. A tangle has a much more complex flow field and the individual components are constrained to move relatively slowly and cannot disperse rapidly, indeed as we can see from the time constants.

We believe that this behaviour, with the initial production of a gas of vortex rings which only subsequently coalesce into a vortex tangle may be a general feature of vortex creating by agitation in quantum fluids.

A further feature of the behaviour is the precise form of the decay of the vortex tangle as shown in Fig. 8. The decay takes the from expected for a classical Kolmogorov distribution of arising from the dynamic where on large length scales the flow is dissipation-free and dissipation only occurs on very small scales. As the dissipation removes the energy at small scales, the small scale features of the turbulence are lost and a cascade of energy from higher to lower scales is built up. Such behaviour is a feature of all well-behaved classical turbulence.

In the superfluids at very low temperatures there are two main problems with such behaviour. At the lowest temperatures there is no normal fluid to provide viscous dissipation so that the dissipation must become a quantum process, but the exact nature of this process is not clear. Furthermore, in a completely random tangle of quantum vortices there cannot be a range of length scales. The only length scale is the mean distance between neighbouring vortices. Our current efforts are directed to investigating the dissipation processes and how a Kolmogorov spectrum of length scales can be set up.

5 Conclusion

The measurements described here, showing that turbulence is generated in the first instance as a gas of independent vortex rings which only subsequently coalesce to form a developed turbulent tangle came as a surprise. The evolution of vortex tangles has long been a subject of investigation both in a quantum fluid context but also in the cosmological context since the evolution of a network of cosmic strings proceeds essentially according to the same rules. Conventional wisdom suggested that when a reconnection event in the tangle led to the creation of a small ring, then that ring would slowly decay and could be ignored in the subsequent calculation of the evolution. The present result shows the antithesis of this: that small rings can be the precursors of the tangle in the first place and definitely cannot be ignored.

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