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COMPUTATION OF ENERGY LEVELS OF *N* INDUCTIVELY COUPLED SUPERCONDUCTING FLUX QUBITS

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We present our calculations of energy levels for two, three and four inductively coupled flux qubits. In order to measure this system, qubits are weakly coupled to a high-quality tank circuit. Measurements of the current-voltage phase shift vs. external flux for two qubits using impedance measurement technique have been carried out. Our experimental data are in very good agreement with the calculated theoretical curves. By calculating the tangent of the current-voltage phase shift for four inductively coupled flux qubits we have predicted behavior of these systems, that is needed for future measurements because of their high complexity.

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1 Introduction

There are many types of qubits being investigated, such as various optical, molecular, or solid state two level systems [1-4]. Because of their easy integration, the superconducting solid-state qubits are very interesting. In this presentation, we report our study of 2 and more inductively coupled superconducting flux qubits placed in a high-frequency, high-quality resonant tank circuit.

2 Theoretical

We investigate energy levels and the current-voltage phase shift in a high quality tank circuit inductively coupled to N superconducting three-Josephson junction flux qubits (Fig. 1). The RF signal from the resonant circuit is amplified by a cooled amplifier [5] and detected by an rf lock-in nanovoltmeter.

The energy levels of the N-qubit system as a function of flux flow through qubits can be determined from Hamiltonian:

$$H = \sum_{i=1}^{N} \varepsilon_i \sigma_{z,i} + \sum_{i=1}^{N} \Delta_i \sigma_{x,i} + \sum_{i<j}^{N} J_{i,j} \sigma_{z,i} \sigma_{z,j}, \tag{1}$$

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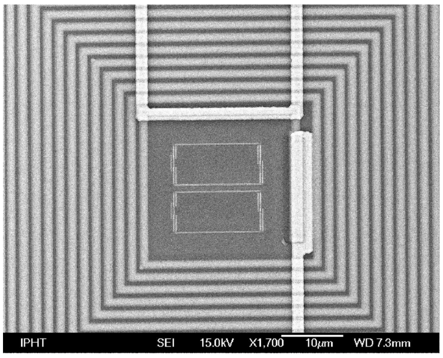

Fig. 1. Two superconducting flux qubits inside the coil of the resonant tank circuit. Using I_{dc2} (top wire) and *dc* current through the coil we are able to change biases of the individual qubits independently.

where the qubit biases are $\varepsilon_i = I_{pers}\Phi_0(f_x - 1/2)$, $f_x = \Phi_x/\Phi_0$ is normalized external magnetic flux, $\sigma_{x,y,z}$ are the Pauli matrices, $J_{i,j}$ is the qubit-qubit coupling energy and I_{pers} are persistent currents.

In order to compare our theoretical results with the experiment, we also need to calculate the current-voltage phase shift of the tank circuit as a function of currents through the coil of the resonant circuit and the *dc* wire (see Fig. 1). From our theoretical model, we can estimate appropriate parameters which can be used in the experiment.

The tangent of current-voltage phase shift in the tank circuit for N qubits has been calculated using the following formula [6,7]:

$$\tan\Theta = -2\frac{Q_T}{L_T}\sum_{\mu<\nu}\frac{\rho_\mu - \rho_\nu}{E_\nu - E_\mu}R_{\mu\nu}, \tag{2}$$

where

$$R_{\mu\nu} = \left(\sum_i \lambda_i \langle\mu|\,\sigma_{z,i}\,|\nu\rangle\right)\left(\sum_i \lambda_i \langle\nu|\,\sigma_{z,i}\,|\mu\rangle\right), \tag{3}$$

$$\rho_\mu = e^{-E_\mu/T}/\left[\sum_\nu e^{-E_\nu/T}\right]. \tag{4}$$

The Q_T is the quality factor of the tank circuit, L_T is the self-inductance of the tank circuit coil, ρ_μ is the population of energy level μ at temperature T, and E_μ are eigenvalues of Hamiltonian (1). Using our simulations, we can estimate the response of this system for various input parameters.

3 Results

From Hamiltonian (1) we have calculated energy levels of the N-qubit system (Fig. 2a) and the second derivative of the energy levels as a function of *dc* currents of the coil and external *dc*

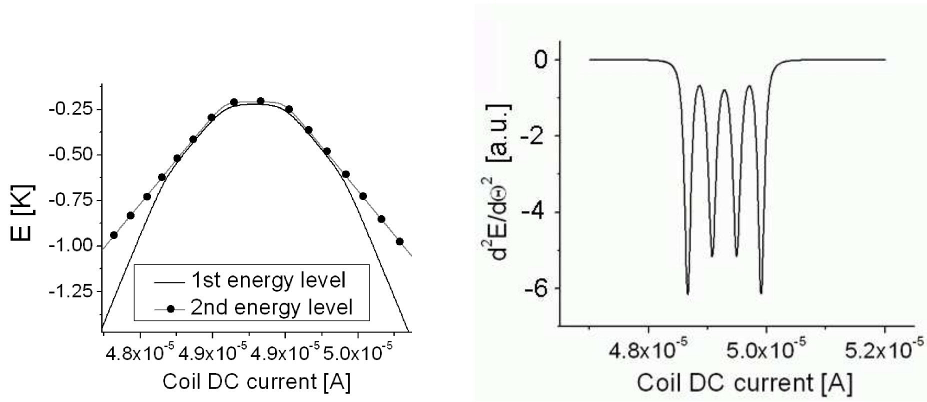

Fig. 2. (a) first two energy levels of a four-qubit system (on the left) calculated from Hamiltonian (1); (b) the second derivative of the first energy level of our four-qubit system (on the right). Coupling energies for all qubits are 0.1 K, tunneling amplitudes are 0.02 K.

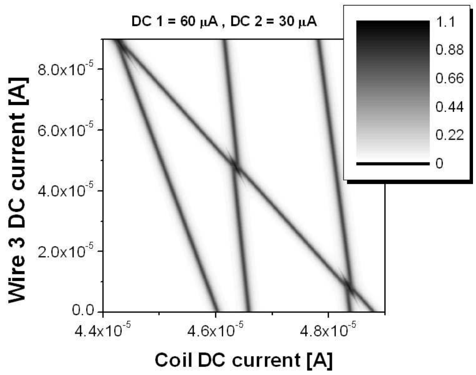

Fig. 3. The $\tan(\theta)$ vs. external flux flow for four qubits. Using additional dc wires to change flux flow through one qubit, we are able to entangle one qubit with every other one; $T = 50$ mK, $J_{ij}/h = 410$ MHz, $\Delta_i/h = 500$ MHz for all qubits.

wires. The dips in the second derivative correspond to the location of degeneracy points of the qubits (Fig. 2b).

Theoretical curves calculated from Equation (2) show very good agreement with experimental data measured by the impedance measurement technique [6]. On the base of previous results, we expect to obtain a good agreement between the calculated theoretical values for 3 and more qubits and the experiment. The work is in progress.

By calculating tan(θ) vs. dc currents through the coil and dc wires for four inductively coupled flux qubits (Fig. 3), we can predict the behavior of these systems, that is necessary for future experimental measurements. We have also shown that we can entangle one qubit with every other one, what is necessary for quantum calculations.

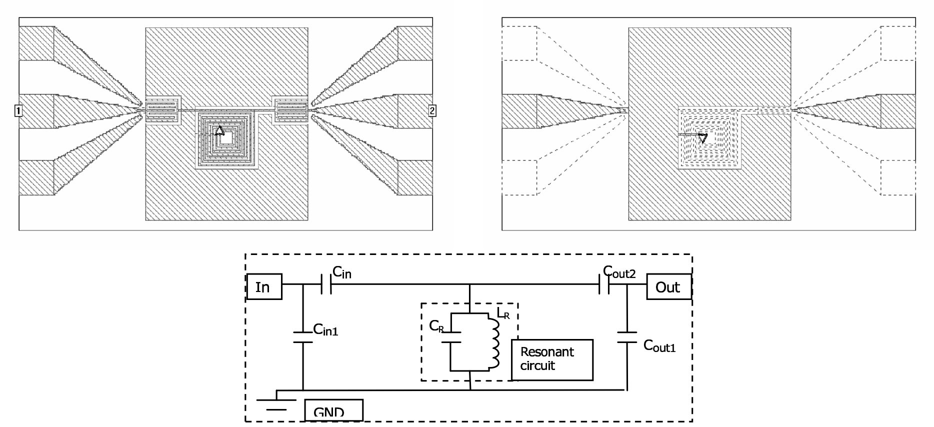

Fig. 4. Model of new high-frequency high-quality resonant tank circuit for frequencies above 1 GHz. Sonnet software two-layer model (the upper panels); schematic (the bottom panel).

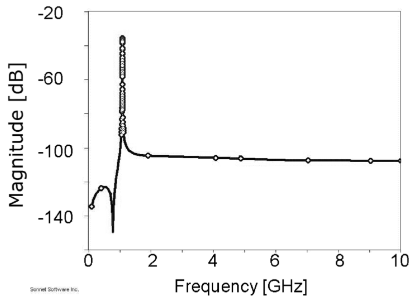

Fig. 5. Response function (S21 parameter) of the tank circuit. Quality factor $Q \approx 10^4$.

To eliminate thermal photons and to make more precise measurements, we have designed a high frequency resonant tank circuit (Fig. 4.), which can be used to measure the flux qubit. The high frequency resonator has been already successfully used to measure a charge qubit [8]. The quality factor of this resonant circuit is about 10^4 with no parasitic resonances (Fig. 5); thus, it should be possible to very precisely measure the change of the current-voltage phase shift at degeneracy points of the qubits.

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