# WHITE-LIGHT SPECTRAL INTERFEROMETRY TO MEASURE THE EFFECTIVE THICKNESS OF OPTICAL ELEMENTS OF KNOWN DISPERSION

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Received 20 September 2004, in final form 1 April 2005, accepted 4 April 2005

A spectral-domain white-light interferometric technique of measuring the effective thickness of optical elements of known dispersion is presented when a Michelson interferometer with a cube beam splitter is not dispersion balanced and when the spectral interference fringes are resolved over a wide wavelength range. The technique uses processing one of the recorded spectral interferograms by an adequate method to retrieve the unwrapped phase function, the ambiguity of which is removed by a simple procedure based on linear dependence of the optical path difference between interferometer beams on the refractive index of optical elements. The effective thickness of optical elements is given by the slope of the linear dependence. The technique is used to measure the effective thickness of a cube beam splitter alone or combined with a thin plate made of BK7 glass.

PACS: 06.30.Bp, 07.60.Ly, 42.62.Eh

#### 1 Introduction

Spectral-domain white-light interferometric techniques based on channelled spectrum detection have been widely used for distance and displacement measurements [1–5], in optical profilometry [6–8] and for dispersion characterizing optical specimens [9–11]. However, an optical configuration with white-light channelled spectrum forming and detection operates in a limited distance range with the minimum distance given by the spectral bandwidth of a white-light source and the maximum distance given by the spectrometer resolving power [2].

Recently, we have demonstrated experimentally that a new technique of dispersive whitelight interferometry employing a low-resolution spectrometer and based on resolving the spectral interference fringes in a narrow wavelength range in the vicinity of the so-called equalization wavelength can be used in a wider range of distances [12, 13]. The new technique, which needs no phase retrieving procedure to be applied, has been used to measure the effective thickness of fused-silica beam splitter [12] or mirror positions and displacements when dispersion in a two-beam interferometer is known [13]. The beam splitter effective thickness is determined from the slope of linear dependence of the measured mirror displacement on the group refractive

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index of fused silica corresponding to the determined equalization wavelength. The equalization wavelength is in this case a wavelength at which the group optical path difference (OPD) between beams in the interferometer is zero.

Most recently, we have extended the use of spectral interferometry for measuring mirror distances and displacements of a slightly dispersive Michelson interferometer when dispersion is known and the spectral interference fringes are resolved over a wide wavelength range [14]. We have confirmed that in contrary to standard spatial-domain white-light interferometry employing dispersion balanced interferometers our measurement techniques are characterized by the range of measurable distances dependent on the amount of dispersion in the interferometer [13–15]. We have also demonstrated [15] that processing of the recorded spectral interferograms using a least-squares method gives distances with resolution comparable to vertical resolutions of standard spatial-domain white-light profilometers [16]. The results are important from the point of view of the most recent considerations of dispersion errors in spatial-domain white-light interferometers not compensated for dispersion [17].

In this paper, a spectral-domain white-light interferometric technique of measuring the effective thickness of optical elements of known dispersion is presented when a Michelson interferometer with a cube beam splitter is not compensated for dispersion and when the spectral interference fringes are resolved over a wide wavelength range. The cube beam splitter is represented by a plate beam splitter [12] of the effective thickness. The technique uses processing one of the recorded spectral interferograms by the Fourier transform method (FTM) [18] to retrieve the unwrapped phase function, the ambiguity of which is removed by using a simple procedure based on linear dependence of the OPD between interferometer beams on the refractive index of optical elements. The effective thickness of optical elements is given by the slope of the linear dependence. We utilize the technique to measure the effective thickness of optical elements made of BK7 glass, namely a cube beam splitter alone or combined with a thin plate.

## 2 Experimental configuration

The experimental set-up used in the application of spectral-domain white-light interferometry to measure the effective thickness of optical elements of a slightly dispersive Michelson interferometer is shown in Fig. 1. It consists of a white-light source, a halogen lamp with launching optics, an optical fibre and a collimating lens, a bulk-optic Michelson interferometer with a cube beam splitter and a transparent thin plate, a micropositioner connected to one of the mirrors, a microscope objective, micropositioners, a detecting optical fibre of a 6  $\mu$ m core diameter, a miniature fibre-optic spectrometer S2000, an A/D converter and a personal computer. The beam splitter and the plate are made of BK7 glass and the plate has a thickness of approximately 170  $\mu$ m. The resolution of the fibre-optic spectrometer S2000 (Ocean Optics, Inc.), the design of which was reported previously [19], is in our case given by the effective width of the light beam from a core of the read optical fibre: we used the read optical fibre of a 50  $\mu$ m core diameter to which a Gaussian response function corresponds. The spectral interferograms were recorded at room temperature in the rigid positions of the interferometer with known positions of mirror 2.



Fig. 1. Experimental set-up with a slightly dispersive Michelson interferometer to measure the effective thickness of optical elements of known dispersion.

## **3** Experimental method

Let us consider the mutual interference of two beams from a broadband source at the output of the uncompensated (dispersive) Michelson interferometer with a cube beam splitter and a transparent thin plate of the thickness t. If the geometrical path lengths of the light rays in dispersive glass of the beam splitter are not the same for both interferometer arms, the beam splitter can be represented by a plate beam splitter of the effective thickness  $t_{ef}$  [12]. The spectrum recorded at the output of the interferometer by a spectrometer can be expressed as [18]

$$I(\lambda) = I^{(0)}(\lambda) \{ 1 + V_I(\lambda) \cos[(2\pi/\lambda)\Delta_{\rm M}(\lambda)] \},\tag{1}$$

where  $I^{(0)}(\lambda)$  is the reference spectrum,  $V_I(\lambda)$  is the wavelength-dependent overall visibility of the spectral interference fringes and  $\Delta_M(\lambda)$  is the wavelength-dependent OPD between two beams in the Michelson interferometer. When the beam splitter and the plate are made of the same material, the OPD  $\Delta_M(\lambda)$  in eq. (1) is given by

$$\Delta_{\rm M}(\lambda) = 2L + 2n(\lambda)(t_{\rm ef} - t), \tag{2}$$

where 2L is the difference of path lengths between the interfering beams in the air and  $n(\lambda)$  is the wavelength-dependent refractive index of the material.

Consider now that the recorded spectrum  $I(\lambda)$  given by eq. (1) is processed by the FTM or by the phase-locked loop method [18] and is represented in the form

$$I(\lambda) = a(\lambda) + b(\lambda) \cos[\Phi(\lambda)], \tag{3}$$

where  $a(\lambda) = I^{(0)}(\lambda)$  and  $b(\lambda) = I^{(0)}(\lambda) V_I(\lambda)$  are background and envelope spectra respectively, and  $\Phi(\lambda) = (2\pi/\lambda) \Delta_M(\lambda)$  is the unwrapped phase function, which is known with the

ambiguity of  $m2\pi$ , where m is an integer. Using eq. (2), the relation for the OPD  $\Delta_M(\lambda)$  between interfering beams

$$\Delta_{\rm M}(\lambda) = [\Phi(\lambda)/(2\pi) + m]\lambda = 2L + 2n(\lambda)(t_{\rm ef} - t) \tag{4}$$

has to be fulfilled. It results from eq. (4) that knowing the unwrapped phase function  $\Phi(\lambda)$  and the refractive index dispersion  $n(\lambda)$ , the interference order m and thus the difference of path lengths between the interfering beams in the air 2L and the overall effective thickness  $t_{\rm ef} - t$  can be determined. The interference order m of such a value has to be chosen so that the OPD  $\Delta_{\rm M}(\lambda)$ between beams in the interferometer is linearly dependent on the refractive index  $n(\lambda)$  [14].

#### 4 Experimental results and discussion

First, the effective thickness  $t_{ef}$  of the cube beam splitter made of BK7 glass was measured. Mirror 2 of the Michelson interferometer was adjusted to resolve the spectral interference fringes in the spectral range as wide as possible. Mirror 2 was displaced manually by using the micropositioner with a constant step of 5 or 10  $\mu$ m and the spectral region from 450 to 850 nm was chosen as that in which the spectral interference fringes should be fully observable. It was revealed that due to the limited resolving power of the spectrometer S2000 [19] the OPDs between beams of the interferometer can be adjusted in the ranges approximately from -80 to  $-5 \ \mu m$  and from 5 to  $80 \,\mu\text{m}$ . As an example, Fig. 2 shows by the dots the recorded spectral interferogram corresponding to the adjusted OPD between beams of the interferometer of approximately  $-10 \ \mu m$ . By processing the spectral interferogram using the FTM, the background spectrum  $a(\lambda)$ , the envelope spectrum  $b(\lambda)$  and the unwrapped phase function  $\Phi(\lambda)$  were determined. The spectrum (3) corresponding to all the determined parameters is shown in Fig. 2 by the solid line. Very good agreement between the numerically determined spectrum and the recorded spectrum is apparent. Knowing both the unwrapped phase function  $\Phi(\lambda)$  and the refractive index dispersion  $n(\lambda)$  of the beam splitter material, the unknown interference order m in eq. (4) and thus the difference of path lengths between the interfering beams in the air 2L and the beam splitter effective thickness  $t_{\rm ef}$  were determined by a simple procedure [14]. It is illustrated in Fig. 3, in which the OPD  $\Delta_{\rm M}(\lambda)$  between beams in the interferometer, which is given by eq. (4), is evaluated from the known unwrapped phase  $\Phi(\lambda)$  and such a choice of the interference order m so that the dependence of the OPD  $\Delta_{\rm M}(\lambda)$  on the refractive index  $n(\lambda)$  of BK7 glass at room temperature [20] deviates minimally from linear dependence. Fig. 3 shows the dependence by the solid curve together with the corresponding linear function characterized by parameters that give the beam splitter effective thickness  $t_{\rm ef} = -49.363 \ \mu {\rm m}$  with a standard deviation of 0.030  $\mu {\rm m}$  and the position of the interferometer mirror  $L = 69.750 \ \mu m$  with a standard deviation of 0.045  $\mu m$ . The results obtained demonstrate that the interferometer with the cube beam splitter used in this case is not dispersion balanced.

Second, the overall effective thickness  $t_{ef} - t$  of the cube beam splitter combined with the thin plate made of BK7 glass was measured. Once again, mirror 2 of the Michelson interferometer was adjusted to resolve the spectral interference fringes in the spectral region from 450 to 850 nm. As an example, Fig. 4 shows by the dots the recorded spectral interferogram corresponding to the adjusted OPD between beams of the interferometer of approximately  $-8 \ \mu m$ . By processing the spectral interferogram using the FTM, the background spectrum  $a(\lambda)$ , the envelope spectrum

 $b(\lambda)$  and the unwrapped phase function  $\Phi(\lambda)$  were determined. The spectrum (3) corresponding to all the determined parameters is shown in Fig. 4 by the solid line. Once again, very good agreement between the numerically determined spectrum and the recorded spectrum is apparent.



Fig. 2. The spectral interferogram recorded for the first case and for the adjusted OPD between beams of approximately  $-10 \ \mu m$  (dots) together with the numerically determined spectral interferogram (solid line).



Fig. 3. The OPD between interfering beams (solid curve) for the first case as a function of the refractive index of BK7 glass together with a linear fit (dashed line).

Knowing both the unwrapped phase function  $\Phi(\lambda)$  and the refractive index dispersion  $n(\lambda)$  of the material of the beam splitter and the plate, the unknown interference order m in eq. (4) and thus the difference of path lengths between the interfering beams in the air 2L and the



Fig. 4. The spectral interferogram recorded for the second case and for the adjusted OPD between beams of approximately  $-8 \ \mu m$  (dots) together with the numerically determined spectral interferogram (solid line).



Fig. 5. The OPD between interfering beams (solid curve) for the second case as a function of the refractive index of BK7 glass together with a linear fit (dashed line).

overall effective thickness  $t_{\rm ef} - t$  were determined. The procedure used is illustrated in Fig. 5, in which the OPD  $\Delta_{\rm M}(\lambda)$  between beams in the interferometer, which is given by eq. (4), is evaluated from the known unwrapped phase  $\Phi(\lambda)$  and such a choice of the interference order m so that the dependence of the OPD  $\Delta_{\rm M}(\lambda)$  on the refractive index  $n(\lambda)$  of BK7 glass deviates minimally from linear dependence. Fig. 5 shows the dependence by the solid curve together with the corresponding linear function characterized by parameters that give the overall effective thickness  $t_{\rm ef} - t = -220.993 \,\mu{\rm m}$  with a standard deviation of 0.060  $\mu{\rm m}$  and the position of the

interferometer mirror  $L = 331.026 \,\mu\text{m}$  with a standard deviation of 0.090  $\mu\text{m}$ . The beam splitter and overall effective thicknesses determined give for the thickness t of the plate made of BK7 glass a value of  $171.630 \,\mu\text{m}$ , which agree well with that provided by the manufacturer.

#### 5 Conclusion

A spectral-domain white-light interferometric technique was presented for measuring the effective thickness of optical elements of known dispersion. It uses a Michelson interferometer with a cube beam splitter not compensated for dispersion that gives rise to the spectral interference fringes resolved over a wide wavelength range. The technique utilizes processing one of the recorded spectral interferograms by the FTM to retrieve the unwrapped phase function, the ambiguity of which is removed by using a simple procedure. It is based on linear dependence of the OPD between interferometer beams on the refractive index of optical elements. The effective thickness of optical elements is simply given by the slope of the linear dependence. We used the technique to measure the effective thickness of a cube beam splitter alone or combined with a thin plate made of BK7 glass. The results obtained are important from the point of view of both the implementation of white-light spectral interferometry to measure distances and displacements and the evaluation of dispersion errors in spatial-domain white-light interferometers not compensated for dispersion.

Acknowledgement: This research was partially supported by the Grant Agency of the Czech Republic (project no. 202/03/0776).

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