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ON THE ANOMALOUS ACCELERATION IN THE SOLAR SYSTEM

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We study an impact of the cosmological environment on the solar gravitational system by the imbedding formalism of Gautreau. It turns out that the cosmic mean-mass density and the cosmological constant give negligibly small contribution to the gravity potentials. On the other hand, the cosmic acceleration beyond the Robertson-Walker geometry can considerably influence the curvature of spacetime in the solar system. The resulting anomalous constant acceleration towards the Sun is order of magnitude smaller than that measured by Pioneer 10 and 11. However, it is larger than the second order terms of potentials, thus well within the sensitivity of new gravity probes such as the LATOR mission.

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The general relativity (GR) corrections to Newtonian laws of planetary orbits are small but included in many body dynamical calculations. Due to the great advances in theoretical calculations by numerical codes and permanent technological achievements and improvements, numerous attempts are planned to measure tiny corrections to even higher accuracies as a check of the theory of gravity. In this paper we want to emphasize the importance of including some cosmological corrections to metric potentials and investigating whether they are observable and distinguishable from the higher order parametrized post-Newtonian (PPN) terms.

We are faced with a problem of imbedding a Schwarzschild mass (in our case the solar mass) into cosmological fluid. The approach of Einstein and Straus is to cut out a spherical vacuum region and to put therein a Schwarzschild mass, and then to join smoothly two different geometries at the boundary surface. This approach looks quite unnatural because it excludes the expected overlapping of gravity forces. Therefore, we choose a different approach which allows merging of gravity fields, namely, that of Gautreau [1]. He relies on the introduction of geodesic time coordinates, which makes possible defining cosmic fluid sources and imbedding of the Schwarzschild mass.

We deal with the so called flat (zero curvature) cosmic spacetime geometry, favoured by current observations. In adition to expansion, we include acceleration into the line element.

Small amounts of acceleration or vorticity in cosmological models are allowed by current cosmological data. Moreover, recent claims on the large-scale asymmetry deduced from WMAP

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data could be explained by the presence of the vorticity (rotation) of the Universe [2]. On the other hand, even a small amount of acceleration, contributing to the cosmic geometry beyond that of Robertson-Walker, can strongly influence the evolution of the cosmic mass density contrast at small redshifts [3]. In the Einstein-Cartan cosmology with a spinning cold dark matter particle one can derive relationship between Hubble's expansion (H_0), vorticity (ω_0) and acceleration (Σ) parameters owing to the additional algebraic relation between the torsion of spacetime and the spin of matter [4]:

$$\Sigma H_0 = \omega_0 \frac{2}{\sqrt{3}}, \qquad \Sigma = \mathcal{O}(10^{-3}). \tag{1}$$

In addition, the Einstein-Cartan nonsingular cosmology can solve the problem of the primordial mass density contrast [5] and the flatness problem [4].

Let us neglect the vorticity and assume a flat cosmic geometry with acceleration:

$$\begin{aligned} &ds^2 = dt^2 - S^2(t)[(1-\Sigma)dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)] - 2\sqrt{\Sigma}S(t)drdt, \qquad (2)\\ &w^\mu \equiv u_\nu \nabla^\nu u^\mu \equiv \dot{u^\mu}, \qquad w^\mu w_\mu = -\Sigma(\frac{\dot{S}}{S})^2,\\ &u^\mu = \text{velocity vector}, \qquad w^\mu = \text{acceleration vector}. \end{aligned}$$

The first task is to make a transition to the coordinates where the Gautreau imbedding formalism could be applied. We perform a general coordinate transformation (GCT) to the physical radial coordinate $R = rS(t)$:

$$\begin{aligned} g_{tt} &= 1 + 2\sqrt{\Sigma}\frac{\dot{S}}{S}R + (-1+\Sigma)(\frac{\dot{S}}{S})^2 R^2,\\ g_{tR} &= -\sqrt{\Sigma} + (1-\Sigma)\frac{\dot{S}}{S}R, \qquad (3)\\ g_{RR} &= -1 + \Sigma. \end{aligned}$$

To reach a diagonal form of the curvature coordinates [1], we apply next GCT to the preceding metric [6]:

$$\begin{aligned} &ds^2 = C(t,R)dt^2 - D(t,R)dr^2 - 2E(t,R)dtdR - R^2(d\theta^2 + sin^2\theta d\phi^2), \qquad (4)\\ &dt' = \eta(t,R)[C(t,R)dt - E(t,R)dR],\\ &\quad \Longrightarrow ds^2 = \eta^{-2}C^{-1}dt'^2 - (D + C^{-1}E^2)dR^2 - R^2 d\Omega^2,\\ &\quad \text{condition}: \frac{\partial}{\partial R}[\eta(t,R)C(t,R)] = -\frac{\partial}{\partial t}[\eta(t,R)E(t,R)]. \end{aligned}$$

The functions $C(t,R)$, $D(t,R)$, and $E(t,R)$ are defined directly by Eqs. (3) and (4) and they are displayed below. Now we can find η function by solving its condition having a perfect differential dt' in power series expansion in the small variable $RH_0 = \mathcal{O}(10^{-14})$, $H_0 = H(today),\ H \equiv \frac{\dot{S}}{S}$:

$$\begin{aligned} &C(t,R) = 1 + 2\sqrt{\Sigma}HR + (-1+\Sigma)H^2R^2,\ D(t,R) = 1-\Sigma,\\ &E(t,R) = \sqrt{\Sigma} - (1-\Sigma)HR, \end{aligned}$$

$$\eta(t,R) = \eta_0(1 - 2\sqrt{\Sigma}HR + (1+\Sigma - \frac{q+1}{2}(1+\Sigma))H^2R^2 + ...), \tag{5}$$
$$\frac{\ddot{S}}{S} \equiv -q(\frac{\dot{S}}{S})^2.$$

The constant η_0 can be removed by a redefinition of the time coordinate $T \equiv \eta_0^{-1}t'$(we keep only the terms linear in HR in the curvature coordinates (T, R) form of the metric):

$$ds^2 = B(T,R)dT^2 - A^{-1}(T,R)dR^2 - R^2d\Omega^2,$$
$$A = (D + C^{-1}E^2)^{-1}, \qquad B = \eta^{-2}C^{-1}. \tag{6}$$

Gautreau introduces geodesic time (τ, R) coordinates to write Einstein field equations and perform the imbedding by the energy-momentum tensor (for details see [1]):

$$ds^2 = d\tau^2 - [dR - (1-A)^{1/2}d\tau]^2 - R^2d\Omega^2,$$
$$T_{\mu\nu} = T_{\mu\nu}(\text{cosmic fluid}) + T_{\mu\nu}(\text{Schwarzschild mass}).$$

The complete modification of the Schwarzschild geometry within cosmic mean mass-density $\rho_{m,0}$, the cosmological constant ρ_Λ [1] and with the relic cosmic acceleration becomes (Eqs. (5) and (6)):

$$A(T,R) = \frac{B(T,R)}{\tau_{,T}^2} = 1 - 2G_NM/R - H_0^2R^2(\Omega_m + \Omega_\Lambda) + 2\sqrt{\Sigma}H_0R, \tag{7}$$

$$\rho_{m,0} = \Omega_m\rho_{c,0}, \qquad \rho_\Lambda = \Omega_\Lambda\rho_{c,0}, \qquad G_N\rho_{c,0} = \frac{3}{8\pi}H_0^2. \tag{8}$$

We employ the fact that for small R we have $\tau_{,T} \to 1$, and for large R the Einstein-de Sitter model ($\Omega_\Lambda = 0$) leads to [1]:

$$T = \tau[1 + \frac{1}{2}(2R/3\tau)^2]^{3/2},$$
$$\frac{\partial\tau}{\partial T}\,|_{\text{today}} = 1 + \frac{1}{3}(\frac{R}{\tau_U})^2 + ... = 1 + \mathcal{O}(10^{-28}),\ \tau_U \simeq H_0^{-1},\ R \simeq 10^{12}\text{m}. \tag{9}$$

The inclusion of the positive or negative cosmological constant can change our estimate of the small correction only for, roughly, one order of magnitude [6], thus confirming our ignorance of any correction to $\tau_{,T}$.

Let us make numerical evaluations and comparisons of the cosmological contributions to the Schwarzschild metric relevant for solar system dynamics. We denote Hubble's constant by H_0, the solar mass by $M_\odot$, the typical planetary distance from the Sun by $\bar{R}$; a_M is the Newtonian acceleration of the Sun, a_Σ is the anomalous acceleration of the solar system caused by cosmological acceleration, $a_{\rho+\Lambda}$ is the acceleration due to the cosmic mass density and the cosmological constant:

$$H_0 = h_0 \times 100\text{km/Mpc/s}, \qquad h_0 = 0.71,$$
$$M_\odot = 2 \times 10^{33}\text{g}, \qquad \bar{R} = 10^{12}\text{m},$$
$$a_M = G_NM_\odot/\bar{R}^2, \qquad a_\Sigma = \sqrt{\Sigma}cH_0,$$
$$a_{\rho+\Lambda} = -\bar{R}H_0^2, \qquad \Omega_m + \Omega_\Lambda = 1, \qquad \Sigma = 10^{-3},$$

$$a_\Sigma/a_M = \mathcal{O}(10^{-7}), \qquad a_{\rho+\Lambda}/a_\Sigma = \mathcal{O}(10^{-13}),$$
$$(\frac{G_N M_\odot}{\bar{R}})^2 = 2.22 \times 10^{-18} < \sqrt{\Sigma} H_0 \bar{R} = 2.42 \times 10^{-16}. \qquad (10)$$

We see that the cosmic mean-mass density and the cosmological constant contributions to the solar system dynamics are negligible. The cosmic relic acceleration term could be important for a precise PPN analysis appearing as a constant effective acceleration towards the Sun comparable or greater than the second order PPN terms [7]. The LATOR mission has sensitivity large enough to measure the effect. The Pioneer 10 and 11 anomalous acceleration acts with the same sign and R-scaling as the cosmic acceleration relic, but it is more than one order of magnitude larger [8]:

$$a_\Sigma = \sqrt{\Sigma} c H_0 = 0.22 \times 10^{-10} m s^{-2} \quad \text{for} \quad \Sigma = 10^{-3}, \qquad (11)$$
$$a(\text{Pioneer}) = 8.74 \times 10^{-10} \text{ms}^{-2}. \qquad (12)$$

The large acceleration parameter $\Sigma = \mathcal{O}(1)$ is cosmologically forbidden [3, 4]. The Pioneer anomalous acceleration is not consistent with planetary orbits data (see the second paper of ref. (8) for details). Thus, only forthcoming precise observations could resolve the puzzle of the constant anomalous universal acceleration in the solar system.

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