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THE MOMENTS OF INERTIA OF ACTINIDE NUCLEI AND NEUTRON-PROTON PAIRING

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Moments of inertia of even-even actinide nuclei have been calculated in the framework of BCS cranking model in which the neutron-proton (np) pairing has been taken into account. First, making a perturbative approximation, we assumed that the form of the equations of the BCS theory and usual Bogolyubov transformations are unchanged. Second, considering a new approach for the np pairing strength constant we observed that the results changed dramatically. Since neutrons and protons occupy different major shells in heavy nuclei, well known thought is that interaction between themselves might be weak due to small overlapping of wave functions thus the influence of the np pairing might be so small in the ground state. However, our results showed that the np pairing might be effective on the moments of inertia of actinide nuclei if it has been considered correctly.

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1 Introduction

One of the oldest problems in our understanding of the collective motion of nuclei is the moments of inertia of ground-state rotational bands in well deformed nuclei. They depend in a very sensitive way on collective properties such as deformations and on pairing correlations of this many-body systems. The first microscopic calculations of the moments of inertia have been made by Inglis [1]. The earliest microscopic calculations were based on a mean field of a deformed harmonic oscillator [1–3]. In these calculations, residual interactions were neglected. In this way one found the values of the moments of inertia identical to those of a rigid body with the same shape, in strong disagreement with the experimentally observed values, which were considerably smaller. It was pointed out very early [2, 4] that residual two-body interactions could lower the values of the moments of inertia obtained in the Inglis model. The most important correlations causing such a reduction are pairing correlations [5]. Moments of inertia of doubly even nuclei have been determined with considerable success within the BCS cranking model that was formulated by Belyaev [6].However, the theoretical values obtained in this model are systematically smaller than the experimental ones by 10-40 % [7–15]. In the literature, one can find many attempts in order to reduce the discrepancy between theory and experiment. In the discussion

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on the above discrepancy, the problem originates from the formalism of the BCS model, which is an approximation, essentially. At this point, it should be noted that the BCS model has some fundamental weaknesses. One of them is the nonconservation of particle number and other one is the neglecting of the np interaction. Up to date, there are some works which study the effects of the nonconservation of particle number on the moments of inertia [16, 17]. Recently, Allal and Fellah studied the effects of nonconservation of particle number in the BCS wave functions on the moments of inertia and concluded that the discrepancy between theory and experiment is due to the number-nonconserving effects of the BCS treatment. However, Hagesawa and Tazaki came to opposite conclusion [18]. However, it is possible to see that there is no work addressing the effect of the np interaction on the moments of inertia. Since neutrons and protons occupy different major shells in heavy nuclei, common consideration was that interaction between themselves might be weak due to small overlapping of wave functions, thus the np interaction effect might be so small in the ground state. However, the developments in the nuclear structure physics in the last two decades indicate that it is difficult to make a serious advance without considering the np-pairing in medium and heavy mass nuclei. Recently this topic became is a subject of much debate because of the new nuclei close to N = Z line which are produced artificially using radioactive beams [19], here N (Z) denotes neutron (proton) numbers. In contrast to alike nucleon pairing, the np-pairing may exist in two different channel, $T = 0$ and $T = 1$, where T is the quantum number of isotopic spin. The interplay of $T = 0$ and $T = 1$ channels using generalized BCS formalism for N = Z nuclei have been studied recently on schematic models [20–27]. Almost all works that have been performed up to date, the np-pairing studied via generalized BCS formalism. According to some of these studies the np-pairing interaction effective only in the nuclei with N $\cong$ Z [22, 23]. In the result, an expectation about a contribution from the np-pairing interaction to any nuclear phenomenon in heavy nuclei would be a mistake in respect of the results of these studies. Besides, others claimed that for $N > Z$ nuclei $T = 0$ np-pairing affective and some of the beta and double beta decay observables might be influenced by $T = 0$ np-pairing [28]. However others claimed that $T = 0$ np-pairing does not exist at all [29, 30]. Thus, the generalized versions of the pairing formalism give contradictory consequences. Here, it is beneficial to state there are also some earlier pessimistic views on this issue [31, 32]. In this work, there were objections to mixing of wave functions of the odd-odd nuclei with those of the even-even nuclei by the generalized Bogolyubov transformation. Because the vacuum of the generalized quasi-particles is a mixture of even-even and odd-odd nuclei have very different energy spectrum. In summary, the current situation of the np-pairing is uncertain. Studies mentioned above imply that new approximations and methods are needed in this field. In this work, the effect of the np-pairing on the moments of inertia of even-even actinide nuclei has been investigated in the framework of an approximation. In the total Hamiltonian following the method in [15], taking the new interaction as a small perturbation the effect of the interaction is to keep the form of the equations of the BCS model unchanged, but only the mean field energies renormalized by the np-interaction. In this way, the efect of the np-pairing interaction has been investigated by two different ways.

2 An approximation for the neutron-proton pairing

The Hamiltonian for a deformed even-even nuclei considered here is of the following form

$$H = H_n + H_p + H_{np}. \tag{1}$$

Here, H_n and H_p are the Hamiltonians for neutrons and protons, respectively and H_{np} is the Hamiltonian for the np-pairing interaction. Each of the quantities H_n and H_p has the pairing model expression, for instance in the case of neutrons in the second quantisation representation it is given as

$$H_n = \sum_{\nu\sigma}(E_\nu - \lambda_v)a^+_{\nu\sigma}a_{\nu\sigma} - G_n \sum_{\nu\nu'} a^+_{\nu\sigma}a^+_{\nu-\sigma}a_{\nu'-\sigma}a_{\nu'\sigma}. \tag{2}$$

Similar expression can be written for protons. Here $a^+_{\nu\sigma}(a_{\nu\sigma})$ denotes creation (destruction) operators for single nucleon states. In equation (2), the G_n denotes the strength of the pairing interaction between neutrons. The second quantized version of the np-pairing interaction is

$$H_{np} = \sum_{\nu\widetilde{\pi}\nu'\widetilde{\pi}'\sigma\sigma'} \langle \nu\sigma, \pi - \sigma| V |\nu'\sigma', \pi' - \sigma'\rangle \, a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma'}a_{\nu'\sigma'}. \tag{3}$$

Here, V is the two-nucleon interaction potential and ν and π denote the single particle states of neutrons and protons, respectively. Moreover, $\sigma = \pm 1$ denotes the states conjugated under time reversal. One can show that this Hamiltonian can be written as two parts if the summation runs over the spin state σ' . Because only two probabilities exist, $\sigma' = \sigma$ and $\sigma' = -\sigma$,

$$H_{np} = H^1_{np} + H^2_{np}. \tag{4}$$

The parts of this Hamiltonian are,

$$H^1_{np} = \sum_{\nu\widetilde{\pi}\nu'\widetilde{\pi}'} \langle \nu\sigma, \pi - \sigma| V |\nu'\sigma, \pi' - \sigma\rangle \, a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma}a_{\nu'\sigma} \tag{5}$$

and

$$H^2_{np} = \sum_{\nu\widetilde{\pi}\widetilde{\nu}'\pi'} \langle \nu\sigma, \pi - \sigma| V |\nu' - \sigma, \pi'\sigma\rangle \, a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'\sigma}a_{\nu'-\sigma}. \tag{6}$$

Using the fermion anticommutation relations,

$$\left\{a^+_{\nu\sigma}, a_{\nu'\sigma'}\right\} = \delta_{\nu\nu'}\delta_{\sigma\sigma'}, \qquad \{a_{\nu\sigma}, a_{\nu'\sigma'}\} = 0, \qquad \left\{a^+_{\nu\sigma}, a^+_{\nu'\sigma'}\right\} = 0, \tag{7}$$

respectively, the creation and destruction operators in (5) and (6) could be ordered as follows

$$\begin{aligned} a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma}a_{\nu'\sigma} &= a^+_{\nu\sigma}a_{\nu'\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma}, \\ a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'\sigma}a_{\nu'-\sigma} &= a^+_{\nu\sigma}a_{\nu'-\sigma}a^+_{\pi-\sigma}a_{\pi'\sigma}. \end{aligned} \tag{8}$$

Now, the effect of the np-pairing interaction on the particle occupation probabilities can be investigated. For this purpose, using the idea [15], it is assumed that near the single particle and usual pairing Hamiltonian the np interaction is a weak residual force. It is straightforward to use the Bogolyubov transformation for neutrons and protons separately, in the Hamiltonian (1),

$$a^{+}_{\nu\sigma} = u_\nu \alpha^{+}_{\nu-\sigma} + \sigma \upsilon_\nu \alpha_{\nu\sigma},$$

$$a_{\nu\sigma} = u_\nu \alpha_{\nu-\sigma} + \sigma \upsilon_\nu \alpha^{+}_{\nu\sigma}.$$

Using the variational method, new occupation probabilities of the single quasi-particle model can be obtained. One can easily show that Hamiltonian (6) has no contribution to the occupation probabilities. All contribution comes from the Hamiltonian (5) as a renormalisation to the mean field energies of neutrons is given as [see the appendix]

$$\overline{E}_\nu = E_\nu - G_n \upsilon_\nu^2 + \langle \nu\sigma, \pi-\sigma | \, V \, | \nu'\sigma, \pi' - \sigma \rangle \, \upsilon_\pi^2. \tag{9}$$

Thus, the quasiparticle particle energies and occupation probabilities of the BCS model are changed by np-pairing interaction. In (9), the np-pairing interaction matrix elements can be written in the following form,

$$\langle \nu\sigma, \pi-\sigma | \, V \, | \nu'\sigma, \pi' - \sigma \rangle \cong \delta_{J,0} F\,(nl)\,.$$

Here $F(nl)$ is the angular and radial part of the the matrix elements :

$$F(nl) = \int \Psi^{nl^*}_{\nu\sigma} \Psi^{nl^*}_{\pi-\sigma} V(\overrightarrow{r_n} - \overrightarrow{r_p}) \Psi^{nl}_{\nu'\sigma} \Psi^{nl}_{\pi'-\sigma} r^2 \sin\theta dr d\theta d\varphi.$$

If the two-nucleon interaction potential is assumed as an attractive contact potential for approximation is given as

$$V = V_0 \delta(\vec{r}) = -\left| V_0 \right| \delta(\vec{r}_1 - \vec{r}_2).$$

In this case, interaction matrix elements depends only the radial parts of the wave functions:

$$\langle \nu\sigma, \pi-\sigma | \, V \, | \nu\sigma, \pi - \sigma \rangle \cong - \int \Psi^{*}_{\nu\sigma} \Psi^{*}_{\pi-\sigma} V(\vec{r}_1 - \vec{r}_2) \Psi_{\nu'\sigma} \Psi_{\pi'-\sigma} dr_1 dr_2.$$

If the matrix elements is written in the conventional form, the renormalization term that comes from the np-pairing to the mean field becomes,

$$\langle \nu\sigma, \pi-\sigma | \, V \, | \nu'\sigma, \pi' - \sigma \rangle \, \upsilon_\pi^2 = -G(\nu\widetilde{\pi}\nu'\widetilde{\pi}')\upsilon_\pi^2. \tag{10}$$

Here,

$$G(\nu\widetilde{\pi}\nu'\widetilde{\pi}') = \int \Psi^{*}_{\nu}(r) \Psi^{*}_{\widetilde{\pi}}(r) \Psi_{\nu'}(r) \Psi_{\widetilde{\pi}'}(r) r^2 dr. \tag{11}$$

There is important fact that the np-pairing matrix elements in (11) could not be considered a constant as for the alike nucleons. To assume whole matrix elements as a constant as in [33] is not a correct approximation. Because the overlaps between unlike nucleons at the different levels is much smaller than the ones at the equivalent levels. In fact, even the overlaps between the wave functions of the equivalent levels is smaller than the overlaps between the alike nucleons since the wave functions in the case of the np-pairing are the solutions of two different Schrödinger equations and the wave functions of unlike nucleons have random phases. Consequently,

$$G(\nu \neq \pi) \ll G(\nu = \pi)$$

and the mean field energies in (9) should be

$$\overline{E}_\nu = E_\nu - G_n \upsilon_\nu^2 - G_{np}(\nu = \pi)\upsilon_\pi^2. \tag{12}$$

Thus only the protons (neutrons) on the equivalent levels renormalize the mean field energies of the neutrons (protons). In the result, for a correct calculation only the diagonal matrix elements of the np-pairing interaction should be taken into account as constant, off-diagonal elements should be neglected.

3 Formulae of moments of inertia

In the cranking model, the moments of inertia of a rotating nucleus around the symmetry axis has been given by Inglis [1] as,

$$J_I = 2\hbar^2 \sum_{ex} \frac{|\langle ex|\, j_x \,|gr\rangle|^2}{E_{ex} - E_{gr}}, \tag{13}$$

where $|gr\rangle$ and $|ex\rangle$ denote the ground and excited states with single particle energies E_{gr} and E_{ex}, respectively. The BCS version of the Inglis formula of moments of inertia has been given by Belyaev [6] as,

$$J_B = 2\hbar^2 \sum_{\nu} \frac{|\langle \nu|\, j_x \,|\nu'\rangle|^2}{\varepsilon_\nu + \varepsilon_{\nu'}} (u_\nu \upsilon_{\nu'} - u_{\nu'} \upsilon_\nu)^2. \tag{14}$$

In this work, the occupation probabilities and the quasiparticle energies have been taken as

$$\upsilon_\nu^2 = \frac{1}{2}\left(1 - \frac{E_\nu - \lambda_n - G_n \upsilon_\nu^2 - G_{np}\upsilon_\pi^2}{\varepsilon_\nu}\right), \qquad u_\nu^2 = 1 - \upsilon_\nu^2 \tag{15}$$

and

$$\varepsilon_\nu = \sqrt{(E_\nu - \lambda_n - G_n \upsilon_\nu^2 - G_{np}\upsilon_\pi^2)^2 + \Delta_n^2}. \tag{16}$$

For the calculation of (14), first the ordinary BCS equations without np-pairing have been solved and then the effect of np-pairing on the mean field energies in equation(12) has been taken into account. Finally the modified BCS equations have been solved again self consistently for neutron and proton systems, separately:

$$N = 2\sum_\nu \upsilon_\nu^2, \qquad \Delta_n = G_n \sum_\nu u_\nu \upsilon_\nu.$$

Hence the equations of BCS model are unchanged in form. In addition, in the above formulae E, λ , Δ_n denote single particle energies, chemical potentials and gap parameters, respectively.

4 Calculations and discussion

To obtain the numerical results, the single particle Hamiltonian developed in [34] was used to simplify the matrix elements; the asymptotic basis of eigenvectors $|NN_z\Lambda\Sigma\rangle$ was preferred. Here, the quantum numbers N, N_z, Λ $, \Sigma$ represent the total number of oscillator quanta, the projection of the total number of oscillator quanta on the z-axis, the projection of the angular momentum on the z-axis and the projection of the spin angular momentum on the z-axis, respectively. All of the calculations have been performed using the deformation values in [35]. In addition, all states of the N = 5, 6, and 7 shells for neutrons and protons (85 levels for each) were taken into account. The experimental values of the moments of inertia have been taken from [17]. In order to search the influence of the np-pairing on the moments of inertia of actinide nuclei, calculations are performed in three ways for comparison: In the first one np-pairing has been neglected completely (J_B). In the second one the np-pairing interaction matrix elements have been taken as constant for all components (J_{np1}) and in the third one only diagonal matrix elements have been taken as constant, namely, off-diagonal components have been neglected (J_{np2}).

Moreover, numerical value of the G_{np} strength constant has been taken as the arithmetical mean value of nn and pp-pairing strength constants for approximation.

In Table 1, the first column represents the theoretical values of the moments of inertia without np-pairing, i.e., with J_B in the formula (14). The second column represents calculations of np-pairing with J_{np1}. The third column represents calculations of the np-pairing with J_{np2}.

As is seen from the calculated values in the first and the second column in the table that the np-pairing interaction has a little effect in the case of J_{np1}. According to the third column np-pairing has a profound effect on the theoretical values in the case of J_{np2}. This effect increases the theoretical values of the moments of inertia for almost all nuclei. Only for ^{232}Th, ^{232}U, ^{244}Cm and ^{246}Cm isotopes the results of J_{np2} are lower than those of the J_{np1}.

In Figs. 1 and 2, variation of the ratios of the calculated moments of inertia with the np-pairing to the experimental values versus atomic mass number A, can be seen. In Fig. 1, the theoretical values in the second column (J_{np1}) and in Fig. 2, the values in the third column (J_{np2}) were used. It is clear from the figures that the second approximation changes the general behavior of theoretical values. In Fig. 2 it is seen that ratio has a big fluctuations as the atomic mass number changes. In addition in Fig. 2 ratios change in the band with 0.63-0.99 while in Fig. 1 ratios change in the band with 0.55-0.70. This means that the results of the J_{np2} are closer to the experimental data than those of the J_{np1} are, generally.

Besides, in Figs. 3 and 4. relative deviations of the values of J_{np1} and the J_{np2}, from the values of the J_B have been presented in the table respectively versus atomic mass number A. Relative deviations $(\sigma(J))$ are

$$\sigma(J_{np1}) = \frac{J_{np1} - J_B}{J_B}$$

for figure 3 and

$$\sigma(J_{np2}) = \frac{J_{np2} - J_B}{J_B}$$

for Fig. 4, respectively. $\sigma(J_{np1})$ and $\sigma(J_{np2})$ can be considered as a measure of the contribution of the np-pairing to the BCS cranking model of the moments of inertia. In Fig. 3, the np-pairing

Tab. 1. Theoretical and experimental values of the moments of inertia.

Nucleus	J_B $\left(\frac{2}{\hbar^2}\right) J$ $(\text{MeV})^{-1}$	J_{np1} $\left(\frac{2}{\hbar^2}\right) J$ $(\text{MeV})^{-1}$	J_{np2} $\left(\frac{2}{\hbar^2}\right) J$ $(\text{MeV})^{-1}$	$J_{\exp.}$ $\left(\frac{2}{\hbar^2}\right) J$ $(\text{MeV})^{-1}$
^{224}Ra	37.16	39.18	64.52	68
^{226}Ra	50.78	52.81	64.48	88
^{228}Ra	55.01	56.99	65.70	100
^{224}Th	35.72	37.61	61.27	62
^{226}Th	50.68	52.82	67.34	82
^{228}Th	55.16	57.32	65.18	104
^{230}Th	68.36	70.48	76.99	113
^{232}Th	75.26	76.68	76.29	120
^{234}Th	79.51	80.75	97.94	126
^{230}U	68.63	70.55	78.40	116
^{232}U	76.98	78.89	78.45	126
^{234}U	83.31	84.39	97.81	138
^{236}U	83.43	84.50	98.69	132
^{238}U	84.81	86.31	100.42	134
^{236}Pu	85.96	86.89	98.49	135
^{238}Pu	86.07	86.99	99.37	136
^{240}Pu	87.47	88.82	101.11	140
^{242}Pu	89.33	90.35	103.50	135
^{244}Pu	89.26	89.78	105.14	133
^{242}Cm	89.38	90.50	101.71	142
^{244}Cm	94.51	95.07	90.12	140
^{246}Cm	93.89	93.97	91.18	140
^{248}Cm	95.40	95.55	112.06	138
^{248}Cf	96.30	96.03	113.05	142
^{250}Cf	95.34	95.30	113.10	142
^{252}Cf	94.60	94.43	113.06	136
^{254}Fm	95.19	95.06	114.02	136

becomes weaken as the atomic mass number increases. This is valid for all isotopes groups generally and whole nuclei as a trend. The result can be interpreted easily; the interaction between neutrons and protons reduces because of the Fermi surfaces of the neutrons and protons come to be distant. Moreover, for the last nuclei in the table σ (J_{np1}) give negative result, namely the J_B values are better than those of the J_{np1} are. On the other hand, in Fig. 4 it is difficult to see a general trend as in the case in Fig. 3. There is a dramatic decrease from ^{224}Ra to ^{232}U. After this region the $\sigma(J_{np2})$ is relatively stable except the ^{244}Cm and ^{246}Cm isotopes. For the ^{244}Cm and ^{246}Cm isotopes, the values of J_{np2} are less than the values of J_B and J_{np1}. This result might be related with the microscopic structure of these isotopes in question. In addition it is beneficial to state for the vertical axes of the Figs. 3 and 4, $\sigma(J_{np2})$ values are about ten times

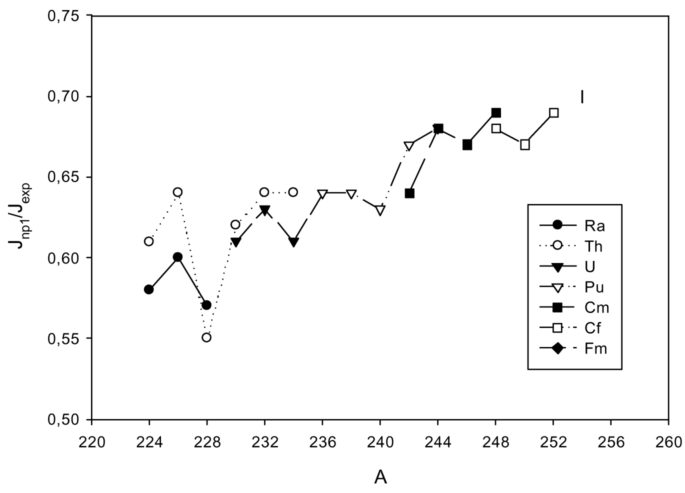

Fig. 1. Variation of the ratios of the calculated moments of inertia with J_{np1} to the experimental data versus atomic mass number A.

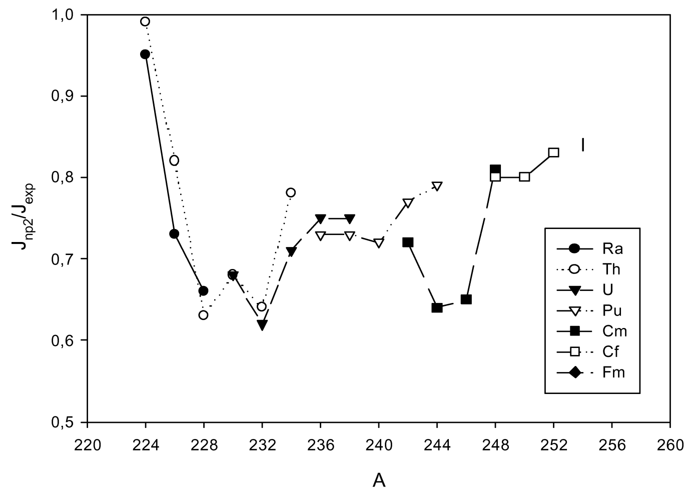

Fig. 2. Variation of the ratios of the calculated moments of inertia with J_{np2} to the experimental data versus atomic mass number A.

bigger than those of the $\sigma(J_{np1})$ are. It means that np-pairing in the case of the J_{np2} is more effective on the theoretical values of the moments of inertia than those in the case of the J_{np1}.

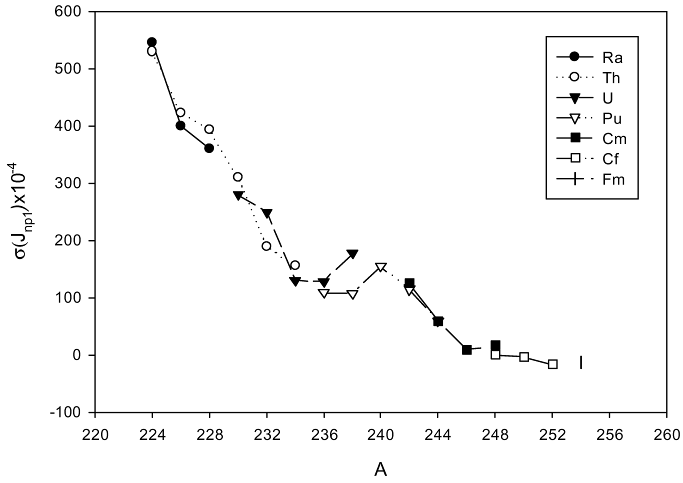

Fig. 3. Variation of the relative deviations of the result of the J_{np1} from those of the J_B versus atomic mass number A.

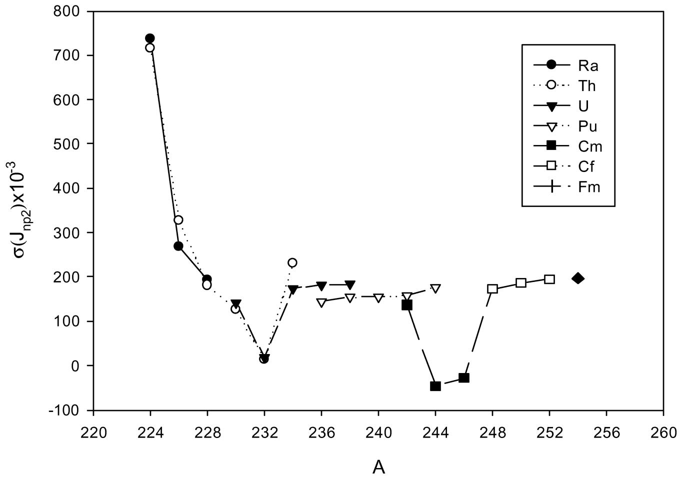

Fig. 4. Variation of the relative deviations of the result of the J_{np2} from those of the J_B versus atomic mass number A.

5 Conclusion

Description of the effect of the np-pairing interaction on the moments of inertia of even-even deformed nuclei in the framework of the BCS model is a new approximation. By taking only the diagonal components as constant and neglecting off-diagonal components for the np-pairing strength change the situation in the problem. Therefore the approximation can be used for other nuclear phenomena. According to this result, np-pairing could be effective in nuclear phenomena not only for the nuclei with N $\cong$ Z but also for the nuclei with N $>$ Z. Therefore it should be take into account for heavy nuclei, too.

It is clear from the literature that the modifications performed on mean fields or pairing correlations between alike nucleons have failed to obtain a good agreement between theory and experiment for the moments of inertia of deformed nuclei. All of the works on this problem imply that the residual correlations that have not been yet considered should be searched. Our calculations show that the residual np-pairing interaction might be one of the candidates for such correlations.

Appendix: Occupation probabilities with the np-pairing

To search the contribution of np-pairing to the occupation probabilities of BCS model using quasi-particle method, first the expectation value of total Hamiltonian (1) in the quasiparticle vacuum should be found as follows

$$\begin{aligned} \langle 0|\, H\, |0\rangle &= \Bigg\langle \sum_{\tau\sigma}(E_\tau-\lambda_\tau)a^+_{\tau\sigma}a_{\tau\sigma} - G_\tau\sum_{\tau\tau'} a^+_{\tau\sigma}a^+_{\tau-\sigma}a_{\tau'-\sigma}a_{\tau'\sigma} \\ &+ \sum_{\nu\nu'\pi\pi'\sigma\sigma'} \langle \nu\sigma,\pi-\sigma|\, V\, |\nu'\sigma',\pi'-\sigma'\rangle\, a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma'}a_{\nu'\sigma'} \Bigg\rangle , \end{aligned} \tag{A1}$$

where $|0\rangle$ stands for the quasiparticle vacuum which is described by $\alpha_{\tau\sigma}\,|0\rangle = 0$. In addition, in (A1) $\tau = \upsilon, \pi$, i.e., there are two different sums for neutrons and protons. By using (5),(6) and (7) the third part of (A1) can be written such that

$$\begin{aligned} &\Bigg\langle \sum_{\nu\nu'\pi\pi'\sigma\sigma'} \langle \nu\sigma,\pi-\sigma|\, V\, |\nu'\sigma',\pi'-\sigma'\rangle\, a^+_{\nu\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma'}a_{\nu'\sigma'} \Bigg\rangle \\ &= \Bigg\langle \sum_{\nu\widetilde{\pi}\nu'\widetilde{\pi}'} \langle \nu\sigma,\pi-\sigma|\, V\, |\nu'\sigma,\pi'-\sigma\rangle\, a^+_{\nu\sigma}a_{\nu'\sigma}a^+_{\pi-\sigma}a_{\pi'-\sigma} \Bigg\rangle \\ &+ \Bigg\langle \sum_{\nu\widetilde{\pi}\widetilde{\nu}'\pi'} \langle \nu\sigma,\pi-\sigma|\, V\, |\nu'-\sigma,\pi'\sigma\rangle\, a^+_{\nu\sigma}a_{\nu'-\sigma}a^+_{\pi-\sigma}a_{\pi'\sigma} \Bigg\rangle . \end{aligned} \tag{A2}$$

The expectation value of the second quantization operators in (A1) can be found by using Bogolyubov transformations and fermion anticommutation relations as follows,

$$\begin{aligned} \langle 0|\, a^+_{\tau\sigma}a_{\tau'\sigma}\, |0\rangle &= \langle 0|\, (u_\tau\alpha^+_{\tau-\sigma}+\sigma\upsilon_\tau\alpha_{\tau\sigma})(u_{\tau'}\alpha_{\tau',-\sigma}+\sigma\upsilon_{\tau'}\alpha^+_{\tau'\sigma})\, |0\rangle \\ &= \upsilon_\tau\upsilon_{\tau'}\,\langle 0|\, \alpha_{\tau\sigma}\alpha^+_{\tau'\sigma}\, |0\rangle = \upsilon_\tau\upsilon_{\tau'}\delta_{\tau\tau'}, \end{aligned}$$

$$\begin{aligned} \langle 0|\, a^+_{\tau-\sigma}a_{\tau'\sigma}\, |0\rangle &= \langle 0|\, (u_\tau\alpha^+_{\tau-\sigma}+\sigma\upsilon_\tau\alpha_{\tau\sigma})(u_{\tau'\sigma}\alpha_{\tau'\sigma}-\sigma\upsilon_{\tau'}\alpha^+_{\tau'-\sigma})\, |0\rangle \\ &= \upsilon_\tau\upsilon_{\tau'}\,\langle 0|\, \alpha^+_{\tau'-\sigma}\alpha_{\tau\sigma}\, |0\rangle = 0 \end{aligned}$$

$$
\begin{aligned}
\langle 0|\, a^{+}_{\tau\sigma} a^{+}_{\tau-\sigma} a_{\tau'-\sigma} a_{\tau'\sigma}\, |0\rangle &= \langle 0|\, (u_\tau \alpha^{+}_{\tau-\sigma} + \sigma \upsilon_\tau \alpha_{\tau\sigma})(u_\tau \alpha^{+}_{\tau\sigma} - \sigma \upsilon_\tau \alpha_{\tau-}) \\
&\times \ (u_{\tau'} \alpha_{\tau'\sigma} - \sigma \upsilon_{\tau'} \alpha^{+}_{\tau'-\sigma})(u_{\tau'} \alpha_{\tau'-\sigma} + \sigma \upsilon_{\tau'} \alpha^{+}_{\tau'\sigma})\, |0\rangle \\
&= u_\tau \upsilon_\tau u_{\tau'} \upsilon_{\tau'} - \upsilon^2_\tau \upsilon^2_{\tau'} \langle 0|\, \alpha_{\tau\sigma} \alpha_{\tau-\sigma} \alpha^{+}_{\tau'-\sigma} \alpha^{+}_{\tau'\sigma}\, |0\rangle \\
&= u_\tau \upsilon_\tau u_{\tau'} \upsilon_{\tau'} - \delta_{\tau\tau'} \upsilon^4_\tau,
\end{aligned}
$$

$$\langle 0|\, a^{+}_{\nu\sigma} a_{\nu'\sigma} a^{+}_{\pi-\sigma} a_{\pi'-\sigma}\, |0\rangle = \langle 0|\, a^{+}_{\nu\sigma} a_{\nu'\sigma}\, |0\rangle \langle 0|\, a^{+}_{\pi-\sigma} a_{\pi'-\sigma}\, |0\rangle = \upsilon_\nu \upsilon_{\nu'} \upsilon_\pi \upsilon_{\pi'} \delta_{\nu\nu'} \delta_{\pi\pi'},$$

$$\langle 0|\, a^{+}_{\nu\sigma} a_{\nu'-\sigma} a^{+}_{\pi-\sigma} a_{\pi'\sigma}\, |0\rangle = \langle 0|\, a^{+}_{\nu\sigma} a_{\nu'-\sigma}\, |0\rangle \langle 0|\, a^{+}_{\pi-\sigma} a_{\pi'\sigma}\, |0\rangle = 0.$$

Thus, (A1) becomes

$$
\begin{aligned}
\langle 0|\, H\, |0\rangle &= \sum_{\tau=\nu,\pi} (2(E_\tau - \lambda_\tau) - G_\tau \upsilon^2_\tau)\upsilon^2_\tau - G_\tau (\sum_{\tau=\nu,\pi} u_\tau \upsilon_\tau)^2 \\
&+ \sum_{\nu\pi} \langle \nu\sigma, \pi - \sigma|\, V\, |\nu'\sigma, \pi' - \sigma\rangle\, \upsilon^2_\pi \upsilon^2_\nu.
\end{aligned}
$$

Carrying out the variation, for example for neutrons,

$$\frac{\partial}{\partial \upsilon_\nu} \langle 0|\, H\, |0\rangle = 0. \tag{A3}$$

Using the normalization

$$u^2_\nu + \upsilon^2_\nu = 1 \tag{A4}$$

we obtain

$$2(\overline{E}_\nu - \lambda_n) u_\nu \upsilon_\nu = \Delta_n (u^2_\nu - \upsilon^2_\nu), \tag{A5}$$

where

$$\overline{E}_\nu = E_\nu - G_n \upsilon^2_\nu + \langle \nu\sigma, \pi - \sigma|\, V\, |\nu'\sigma, \pi' - \sigma\rangle\, \upsilon^2_\pi, \tag{A6}$$

$$\Delta_n = G_n \sum_\nu u_\nu \upsilon_\nu. \tag{A7}$$

Squaring (A5) and (A4) and eliminating u^4_ν and υ^4_ν, gives

$$u^2_\nu \upsilon^2_\nu = \frac{\Delta^2_n}{4[(\overline{E}_\nu - \lambda_\nu)^2 + \Delta^2_n]} = \frac{1}{4}\left(1 - \frac{(\overline{E}_\nu - \lambda_n)^2}{(\overline{E}_\nu - \lambda_n)^2 + \Delta^2_n}\right)$$

from which we obtain

$$u_\nu^2 = \frac{1}{2}\left(1 + \frac{\overline{E}_\nu - \lambda_n}{[(\overline{E}_\nu - \lambda_n)^2 + \Delta_n^2]}\right) \quad \text{and} \quad \upsilon_\nu^2 = \frac{1}{2}\left(1 - \frac{\overline{E}_\nu - \lambda_n}{[(\overline{E}_\nu - \lambda_n)^2 + \Delta_n^2]}\right). \tag{A8}$$

Naturally, similar expressions are valid for protons, too. As is well known that in the standard BCS model (without np-pairing), single particle energies of neutrons (protons) are changed only by the effect of the pairing between neutrons (protons), i.e.

$$E'_\nu = E_\nu - G_n \upsilon_\nu^2. \tag{A9}$$

It is clear from (A6); the single particle energies are changed by the effect of the pairing between neutrons and also the np-pairing. Thus np-pairing renormalizes the single particle energies. Consequently, the quasiparticle energies and occupation probabilities are modified by the effect of the np-pairing.

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