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METASTABILITY OF A FLIP-FLOP AND ITS UTILIZATION FOR A CAPACITANCE MEASUREMENT

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Activating a flip-flop circuit by a current fast-rising slope impulse, the circuit occupies one of the two stable states, the stable state ‘one‘ or ‘zero‘. In case of a perfect flip-flop symmetry over a large number of cycles, a noise causes the ratio of ‘ones‘ and ‘ zeros‘ is equal to one – 50% position of a flip-flop. However, any imbalance in the system changes the probability of taking a ‘one’ or a ‘zero’, and thus the ratio of ‘ones’ and ‘zeros’. In this paper, the formula for equivalent voltage of the flip-flop circuit is derived. This voltage is incorporated into the flip-flop as a voltage source to restore 50% position. The formula is derived for the first time. The results are useful for capacitance measurement and in capacitive sensor applications.

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1 Introduction

The first idea to use a flip-flop in sensor-based systems was published in [1]. A flip-flop is depicted in Fig. 1. The flip-flop circuitry consists of two inverter amplifiers. The output of the inverter 1 is connected to the input of the inverter 2, and the output of the inverter 2 is connected to the input of the inverter 1. In this way a positive feedback is formed. It has two stable states, namely ‘one‘ and ‘zero‘ state. The control impulses are not applied to the bases of the transistors, but the circuit is controlled by current impulses, shown in Fig. 1. By fast-rising slope of the control impulse the flip-flop is first brought into an unstable state, which represents a sudden rapid change. The flip-flop will subsequently transfer to one of the two stable states.

The decision of switching over to a ‘one‘ or to a ‘zero‘ is influenced by value asymmetry of the flip-flop. Let us consider the value of capacitor C_1 is larger than C_2, while other parameters are symmetric. When the flip-flop is activated by a current control impulse, the current through capacitor C_2 is smaller than the current through C_1 and the voltage u_1 across C_1 is lower than the voltage u_2 across C_2. Then the voltage u_1 is amplified by the second inverter and passed to the first inverter. This process is repeated and regenerated again, until the differential output voltage $u_1 - u_2$ is so large, that the transistor T_2 is cut off, which results in the stable state ‘one‘. To transform this asymmetry into a voltage, a DC voltage source U_N was incorporated

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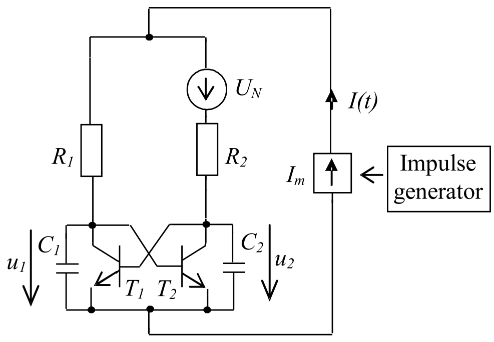

Fig. 1. Circuit diagram of a flip-flop.

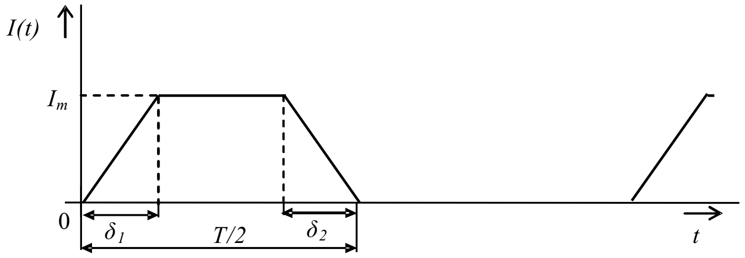

Fig. 2. Current control impulse.

into the flip-flop, as shown in Fig. 1. There exists a certain value of the voltage U_N at which the effect of the asymmetry is fully compensated by the addition of this DC voltage. This voltage is called an equivalent voltage U_{NE} [2]. The formula for the equivalent voltage, at which the effect of the asymmetry in the resistors and in the transistors of the flip-flop is compensated, is derived in [2]. Since capacitances were not included in the model, this formula is valid only if the flip-flop is controlled by slow-rising slope impulses. This control is met if $10R_1C_1 < \delta_1$ and $10R_2C_2 < \delta_1$ [3], according to Fig. 2, where T is the period of the control impulses.

On the other hand, the control by fast-rising slope impulses appears when it holds that $R_1C_1/10 > \delta_1$, $R_2C_2/10 > \delta_1$ [3]. In the following text only the control by fast-rising slope impulses is assumed.

2 Metastable state and effect of the noise

In case the value asymmetry is compensated by the equivalent voltage, the flip-flop will not be able to adopt a stable state. The differential voltage $u_1 - u_2$ is equal to zero. This is known as a metastable state [2,4]. However, the total system noise ensures that the flip-flop reverts to one of two possible but unpredictable stable states after a certain time, depicted in Fig. 3.

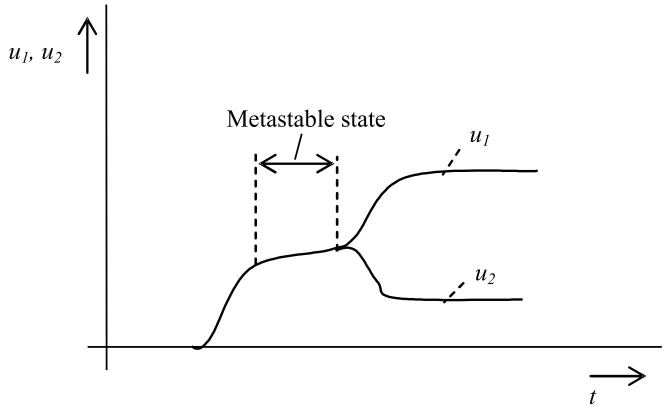

Fig. 3. Metastable state of the flip-flop and consecutive transition into unpredictable stable state.

The total system noise of a flip-flop is derived from two main sources: a thermal noise of the resistors R_1, R_2 and a shot noise of the transistors T_1, T_2 [1]. The authors of [2] showed the noise equivalent voltage of a flip-flop is characterized by a Gaussian distribution with zero mean.

The flip-flop measurements should be repeated many times to infer the effect of value asymmetry from the noise, and the number of 'ones' and 'zeros' in the output must be counted. Then, the thermal and shot noise mean values being zero, the ratio of 'ones' and 'zeros' will be equal to one [1]. This is known as the 50% position of the flip-flop. Fig. 4 shows the dependence of the voltage u_2 on the voltage u_1 over a large number of cycles, when 50% position of the flip-flop is obtained. The arrows 1, 2 indicate a transition into stable state 'one' and 'zero', respectively, while arrows 3, 4 indicate the reset process when the supply current is switched off.

3 Mathematical description of the flip-flop

When the supply current is switched on, the flip-flop in Fig. 1 is described by the equations [3]

$$\frac{du_1}{dt} = -\frac{(u_1 - u_2 - U_N - R_2 I_m + (R_1 + R_2)\,\phi_1)}{(R_1 + R_2)\,C_1}, \tag{1}$$

$$\frac{du_2}{dt} = -\frac{(u_2 - u_1 + U_N - R_1 I_m + (R_1 + R_2)\,\phi_2)}{(R_1 + R_2)\,C_2}, \tag{2}$$

$$\phi_1 = I_S e^{\frac{u_2}{V_T}} + I_S \frac{e^{\frac{u_1}{V_T}}}{\beta}, \tag{3}$$

$$\phi_2 = I_S e^{\frac{u_1}{V_T}} + I_S \frac{e^{\frac{u_2}{V_T}}}{\beta}, \tag{4}$$

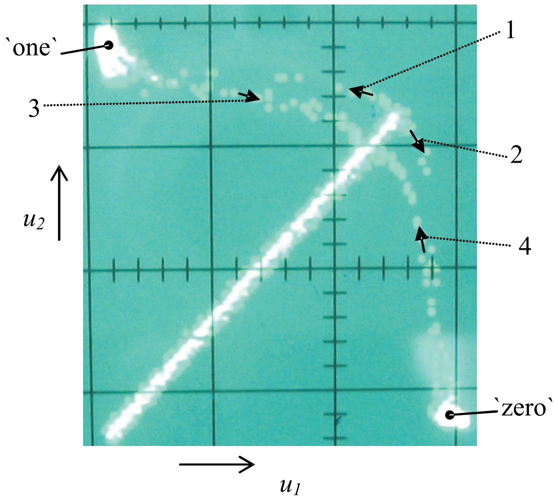

Fig. 4. The 50% position of the flip-flop circuit.

where β is the transistor current gain, I_S is the transistor saturation current, V_T is the thermal voltage, R_1, R_2 are the load resistors, I_m is the amplitude of the current control impulse. It should be noted that in Equations (1)–(4) the effect of a noise is not assumed.

4 Derivation of a formula for the equivalent voltage

Let us consider the influence of a noise is absent and asymmetry is caused only by mismatches in the capacitors of the flip-flop. As explained in Section 2, when $U_N = U_{NE}$, the flip-flop occupies a metastable state. Because, the differential voltage $u_1 - u_2$ is in this state equal to zero, it leads to

$$u_1 = u_2. \tag{5}$$

By taking derivatives of Equation (5) it follows

$$du_1 = du_2. \tag{6}$$

Dividing Equation (1) by Equation (2), by means of (5) and (6), it follows

$$1 = \frac{-U_{NE} - RI_m + 2R\phi}{U_{NE} - RI_m + 2R\phi}\frac{C_2}{C_1}, \tag{7}$$

where $\phi = \phi_1 = \phi_2$ because $u_1 = u_2$ and $R = R_1 = R_2$. The equivalent voltage U_{NE}, which is needed for such a total balance, can be derived from Equation (7). The result is

$$U_{NE}(u) = \frac{(C_1 - C_2)}{(C_1 + C_2)}RI_m - \frac{(C_1 - C_2)}{(C_1 + C_2)}2R\phi, \tag{8}$$

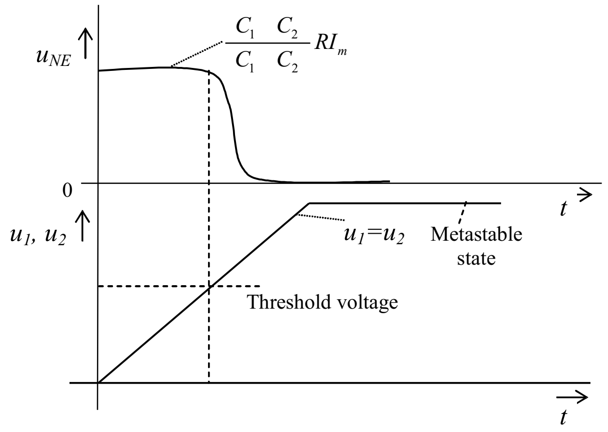

Fig. 5. Equivalent voltage variance needed the flip-flop persists in the metastable state.

where $u = u_1 = u_2$. From the formula (8) it is clear that the equivalent voltage does not have a fixed value, but changes in dependence on the voltage u so that the flip-flop remains in a metastable state. In fact, when the flip-flop is activated by a current impulse, the capacitor currents are much greater than transistor currents only until the voltages across capacitors reach the threshold voltage of the transistors. Then, the transistor currents rapidly increase, while the currents through capacitors rapidly decrease. However, when the transistor currents are much greater than the currents through capacitors, the asymmetry in the capacitors cannot influence the flip-flop value symmetry. The currents through transistors are also equal because of a zero differential voltage in the metastable state. Then, according to formula (8), the current ϕ changes from zero to $I_m/2$ and thus the equivalent voltage changes from

$$U_{NE} = \frac{(C_1 - C_2)}{(C_1 + C_2)} RI_m \tag{9}$$

to zero, as shown in Fig. 5.

Now, let us consider the asymmetry caused by the noise and mismatches in the capacitors of the flip-flop. When the flip-flop is activated by the current control impulse, the voltages across capacitors increase, but because of the noise the differential voltage is non-zero. When, for instance, the voltage u_1 reaches the threshold voltage and is slightly larger than the voltage u_2, because of the positive feedback, the flip-flop goes to a ‘zero‘. An opposite situation will appear when the voltage u_2 is slightly larger than the voltage u_2 in the threshold point. Observing over large number of cycles the 50% position is obtained. Thus, in the threshold point is decided which stable state is chosen. Because in this point the currents through transistors are on the order of a few μA [2] and value of supply current is on the order of a few mA [5], the second expression in Equation (8) is negligible and thus the equivalent voltage is given by Equation (9) for practical purpose. Fig. 6 depicts an example when the stable state ‘one‘ is chosen in the threshold point. The amplitude of the current control impulse was 1.13 mA, load resistances

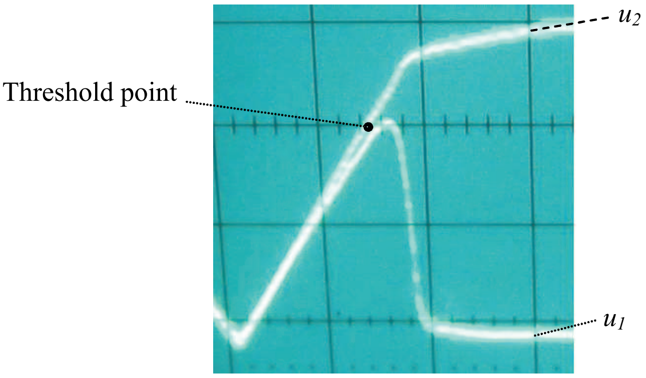

Fig. 6. Example when stable state ‘one‘ was chosen in the threshold point.

R_1,R_2 of the flip-flop were equal to 6.8 kΩ, BC 237 transistors were used, and transistor currents in the threshold point were equal to 3.15 μA.

In case the asymmetry is caused by mismatches in the resistors of the flip-flop, the equivalent voltage can be derived in the same way

$$U_{NE} = \frac{(R_1 - R_2)}{2} I_m. \tag{10}$$

The effect of asymmetry in the saturation currents and current gains of the transistors is not investigated in this letter. It is shown in [2] that asymmetry in the saturation currents and current gains does not have influence on the rise time of the control impulses and thus the equivalent voltage is given by formulae derived in [2].

5 Practical utilization of the formula

Let us consider $C_1 = C + \Delta C$, $C_2 = C$, $\Delta C \ll C$ in Equation (9) and we have

$$U_{NE} = \frac{\Delta C}{2C} R I_m. \tag{11}$$

As can be seen, the equivalent voltage depends linearly on the capacitive difference ΔC. It can be used for the capacitance measurements and in the capacitive sensor based systems. The validity of the formula (11) is proved in [6]. The calculated and measured values are in agreement for ratio $|\Delta C/C| < 1\%$. Above this ratio the formula (11) does not give good results, and Equation (9) must be used. Here, as an example, we report some parameters of a capacitance meter based on the use of the flip-flop.

The value of equivalent voltage is set automatically by using a feedback circuit. More detailed information can be found in [5]. The following table reports some parameters of the capacitance meter according to the standard EN 60 770.

Tab. 1. Parameters of capacitance meter.

Parameter	*Value*
Inaccuracy	+20.5 fF
	-18.5 fF
Measured error	1.2 fF
Non-linearity	+2 fF
	- 2 fF
Hysteresis	0.5 fF
Non-repeatability	0.14 fF
Measurement range	0.1 3.5 pF

6 Conclusions

In this paper, a formula for equivalent voltage of the flip-flop has been derived. To the author's best knowledge, no publication dealing with a derivation of an equivalent voltage for the flip-flop controlled by fast-rising slope impulses has been reported so far. An application of the formula in a capacitance meter is also briefly discussed and some parameters according to the standard EN 60 770 are specified.

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