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DISPERSION RELATIONS AND QUARK-GLUON STRUCTURE OF HADRONS TOTAL CROSS SECTIONS

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The amplitudes of hadron-hadron forward elastic scattering at high energies are investigated on the basis of analyticity and crossing-symmetry. The universal uniformizing variable for them is proposed. The total cross sections for hyperon-proton scattering are predicted. They are consistent with experimental data.

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1 Introduction

In a series of papers [1], Ohme has shown that for gauge theories the confinement conditions can be formulated so that physical amplitudes do posses the analytical properties and conditions of crossing symmetry established earlier [2]. Below, the Ohme's result is used to construct a model which allows one to determine the quark - gluon structure of hadrons total cross sections on the experimental basis.

2 Universal Riemann surface of the forward scattering amplitude

The notion of universal Riemann surface of forward scattering amplitude for hadron-hadron processes at high energies arises when one introduces the well-known variable

$$\nu = \frac{s-u}{4M\mu},$$

where s, u are usual Mandelstam variables and M, μ are the masses of colliding particles. Thresholds of any elastic hadron-hadron process, corresponding to the direct and cross reactions in the s-plane, transform into the points $\nu = \pm 1$. The thresholds of all inelastic processes (direct and crossed) lie on the cuts $(-\infty, -1], [+1, +\infty)$. They make the Riemann surface of a scattering amplitude as a function of ν infinitely-sheeted. This property of the Riemann surface can be modelled by particular choice of the uniformizing variable, the same for all hadron-hadron processes,

$$w(\nu) = \arcsin(\nu)/\pi. \tag{1}$$

The Riemann surface of function $w(\nu)$ is just what we call the universal Riemann surface of a scattering amplitude. The function $w(\nu)$ is suitable for taking account of the crossing symmetry of amplitudes of hadron-hadron scattering $F^A_\pm$. We choose the latter so that the equality

$$\mathrm{Im} F^A_\pm = \sigma^{\bar{A}p}_{\text{tot}} \pm \sigma^{Ap}_{\text{tot}} \tag{2}$$

be valid on the upper edge of the right-hand cut of ν plane; then, the condition of crossing symmetry is

$$F_\pm(\nu) = \pm F_\pm(-\nu). \tag{3}$$

Besides, the amplitudes obey the condition of reality

$$F^*_\pm(\nu) = -F_\pm(\nu^*). \tag{4}$$

In the w-plane, a physical sheet of the ν-plane is mapped into the strip $|\mathrm{Re} w| \leq 1/2$, whose boundaries are images of cuts of the ν-plane. We call it the physical strip in the w-plane. Let $w = x + iy$. Then, owing to Eqs. (3)–(4), on the boundary of the physical strip we find

$$F^*_\pm(1/2 + iy) = \mp F_\pm(-1/2 + iy). \tag{5}$$

Let us expand the amplitudes $F_\pm(w)$ into Taylor series with the center at the point $w_0 = iy_0$. The parameters of those expansion determine both the real and imaginary parts of amplitudes $F_\pm(w)$. Below, we will use only the imaginary parts of amplitudes (the total cross sections). Corresponding expansions are given in Ref. [3]. From (1) it follows that $y = \ln(\nu + \sqrt{\nu^2 - 1})$. For $s \gg M^2$, we have $y \sim \ln(2p/\mu)$. In this case, the function $(y - y_0) \sim 1/\pi \ln(p/p_0)$ is the argument of expansions in [4]. Here the quantity p_0 has a clear mathematical meaning. It is the center of the expansion into the Taylor series.

3 Quark-gluon structure of hadrons total cross sections

The Taylor series with tree terms were employed to analyze the experimental data on pp, $\bar{p}p$, $K^\pm p$,$\pi^\pm p$ total cross sections [3]. Twenty four coefficients a_m, b_m for F_+, F_- are determined by 300 experimental points. Twelve coefficients b_m display the simple dependence:

$$(b_m)_{pp} : (b_m)_{\pi p} : (b_m)_{Kp} : (b_m)_{np} = 5 : 1 : 2 : 4. \tag{6}$$

The mean ratios of it are as follows:

$$\overline{\left(\frac{b_{pp}}{b_{\pi p}}\right)} = 5.37 \pm 0.22, \qquad \overline{\left(\frac{b_{Kp}}{b_{\pi p}}\right)} = 2.16 \pm 0.12, \qquad \overline{\left(\frac{b_{np}}{b_{\pi p}}\right)} = 4.79 \pm 0.23.$$

They are in good agreement with ratios (6), except for the last one. It differs from (6) by three statistical errors as a result of large $\chi^2/n.d.f.$ for np scattering. Therefore, it is expedient to use it below only for qualitative estimations. On the other hand it is known [5] that the relationships (6) are obtained from the consideration of annihilation components of amplitudes and are proportional to the number of dual diagrams of scattering of a hadron on a proton

$$n_d(Ap) = 2N_{\bar{u}}^A + N_{\bar{d}}^A,$$

where $N_{\bar{u}}^A$ and $N_{\bar{d}}^A$ are numbers of antiquarks $\bar{u}, \bar{d}$ in hadron A.

It is of great interest but difficult to analyze the crossing-even part of the scattering amplitude. The additive quark model (AQM) predicts the following ratios:

$$\sigma_{pp} : \sigma_{\pi p} : \sigma_{Kp} : \sigma_{np} = 3 : 2 : 2 : 3.$$

However, from [3] it is seen that only the coefficients a_1 and a_3 approximately follow that dependence. At the same time, the difference $(a_1)_{\pi p} - (a_1)_{kp} = 8.74 \pm 0.15$ is significant and, together with other coefficients, determines 30 % accuracy of the AQM. The values of coefficients a_2 from [3] do not comport with the AQM predictions, and therefore, they are very important for choosing new models. Some attempts of refining the AQM are known [4,6].

Below, we construct a new model by using the known idea of quarks being confined in a hadron by gluons. Then it is natural to assume that the total cross section of scattering of hadron A on a proton contains a part that describes gluon-gluon interaction. With this in mind, we set

$$a_m = \alpha_m + \beta_m \cdot N_q^A + \gamma_m \cdot N_q^A \cdot N_{\text{ns}}^A \,, \tag{7}$$

where N_q^A is the total number of quarks; N_{ns}^A is the total number of nonstrange quarks in hadron A; and the numbers α_m do not depend on the quark content of hadron A . The numbers α_m determine the fraction of the total cross section corresponding to the gluon-gluon interaction. It is just the gluon degree of freedom of hadrons A and p that is responsible for them. The assumption on a_m corresponds to the hypothesis of Gershtein and Logunov [7]. They argue that the constant of the Froissart limit does not depend on the guark content of hadron A , but it does depend on glueballs and is the same for all processes. The hypothesis has been verified by Prokoshkin [8]. The numbers from the table $(a_m)_{pp}, (a_m)_{\pi p}, (a_m)_{kp}$ determine $\alpha_m, \beta_m, \gamma_m$. Then, the prediction power of the hypothesis (7) can be verified for the values of the total cross sections of hyperon-proton interactions. In [9], the results are presented on the measurement of the total cross sections of $\Sigma^- p$ and $\Xi^- p$. In this case, the theoretical and experimental results at the momentum $101\,\text{GeV}/C$ are as follows:

$$\sigma_{\Xi^- p} = \begin{array}{l} (29.25 \pm 0.5\,\text{mb})_{\text{th}} \\ (29.12 \pm 0.22\,\text{mb})_{\text{ex}} \end{array}, \quad \sigma_{\Sigma^- p} = \begin{array}{l} (34.8 \pm 0.2\,\text{mb})_{\text{th}} \\ (33.3 \pm 0.3\,\text{mb})_{\text{ex}} \end{array}.$$

Similarly, the data [10] on Λp and $\Sigma^- p$ scattering at $20\,\text{GeV}/C$ are in agreement with the theoretical predictions:

$$\sigma_{\Lambda p} = \begin{array}{l} (33.3 \pm 0.5\,\text{mb})_{\text{th}} \\ (34.7 \pm 3\,\text{mb})_{\text{ex}} \end{array}, \quad \sigma_{\Sigma^- p} = \begin{array}{l} (34.2 \pm 0.5\,\text{mb})_{\text{th}} \\ (34 \pm 1\,\text{mb})_{\text{ex}} \end{array}.$$

Recently, the collaboration SELEX has published the data on $\Sigma^- p$ at $p = 609\,\text{GeV}/C$. [11]. The comparison with predictions of the model is:

$$\sigma_{\Sigma^- p} = \begin{array}{l} (35 \pm 7.5\,\text{mb})_{\text{th}} \\ (37 \pm 0.7\,\text{mb})_{\text{ex}} \end{array}.$$

Though the obtained value of the total cross section is not so accurate as in ref. [12], it should be considered satisfactory. In [6,12] the errors of predicted values were not calculated, but they increase rapidly in the region of extrapolation.

4 Conclusion

A uniformizing variable for hadron-hadron forward scattering at high energies was proposed on the basis of analyzing the analytic properties of physical scattering amplitudes [1]. The total cross sections predicted for scattering of strange hadrons on a proton are in good agreement with experiment in a wide energy range. The gluon-gluon part of the total cross sections at momenta $p = 100\,\mathrm{GeV}/C$ amounts to about 10%.

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