

MULTIPHOTON EXCHANGES IN PAIR PRODUCTION PROCESSES IN HEAVY ION COLLISIONS**E. Bartoš^{*}, S. R. Gevorkyan[†], E. A. Kuraev[†], N. N. Nikolaev[‡]**^{*} *Dept. of Theoretical Physics, Comenius University, 84248 Bratislava, Slovakia*[†] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*[‡] *Institute für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*

Received 30 November 2004, in final form 5 January 2005, accepted 11 January 2005

The multiple exchange of virtual photons emitted by heavy ions with the created lepton pairs are considered. The main attention is devoted to the case when each of the ions emits two virtual photons. We have found rather complicated expression for the “effective” Green function of intermediate lepton state. The explicit result for the amplitude and the relevant cross section are obtained for the case when the transferred momenta to the ions are small in comparison with the transversal to beam axes momenta of created electron and positron. We found the double logarithmical enhancement of the amplitude under consideration, which disappears when the number of emitted photons by each ion exceed two.

PACS: 13.60.Hb, 25.75.Dw

The exact theory of Coulomb distortions of the spectrum of ultrarelativistic lepton pairs photoproduced in the Coulomb field of the nucleus has been developed by Bethe and Maximon [1]. It is based on the description of leptons by exact solutions of the Dirac equation in the Coulomb field (see e.g., the textbook [2]). In the Feynman diagram language one has to sum multiphoton exchanges between produced electrons and positrons and the target nucleus. For ultrarelativistic leptons this reduces to the eikonal factors in the impact parameter representation. In the momentum space the same eikonal form leads to simple recurrence relations between the $(n + 1)$ and n -photon exchange amplitudes [3], the incoming photon can be either real or virtual. The similar reasons allow one to cast the pair production cross section in the dipole representation. They have also been behind the color dipole perturbative Quantum ChromoDynamics (pQCD) analysis of nuclear distortions and the derivation of nonlinear k_{\perp} -factorization for multijet hard processes in Deep Inelastic Scattering (DIS) off nuclei [4]. The process of lepton pair production in the Coulomb fields of two colliding ultrarelativistic heavy ions was intensively investigated in the past years [5–12]. Such activity is mainly connected with new possibilities opened with operation of such facilities as RHIC and LHC. Despite the high activity in this area the issue of correct allowance for final state interaction of produced leptons with the colliding ion Coulomb fields is lacking yet.

The main motivation of the present paper is a further investigation of multiple exchanges and their impact on the lepton pair yield in the ultrarelativistic heavy ion collisions, an issue which

Tab. 1. The coefficients for formula (2). The parentheses denote index permutation, e. g., (12) \equiv 12 + 21.

n	R_{ijkl}	a_n	b_n	c_n	d_n
1	$R_{(12)(34)}$	q_-	$q_- - q_1$	—	—
2	$R_{(34)(12)}$	$q_1 - q_+$	q_+	—	—
3	R_{1324}	q_-	$q_- - k_1$	$q_- - k_1 - k_2$	$q_- - q_1 - k_2$
4	R_{1423}	q_-	$q_- - k_1$	$q_- - q_2 + k_2 - k_1$	$-q_+ + k_2$
5	R_{2314}	q_-	$q_- - q_1 + k_1$	$q_- - q_1 + k_1 - k_2$	$-q_+ + q_2 - k_2$
6	R_{2413}	q_-	$q_- - q_1 + k_1$	$-q_+ + k_1 + k_2$	$-q_+ + k_2$
7	R_{4231}	$q_- - q_2 + k_2$	$-q_+ + k_1 + k_2$	$-q_+ + k_1$	q_+
8	R_{3241}	$q_- - k_2$	$q_- - q_1 + k_1 - k_2$	$-q_+ + k_1$	q_+
9	R_{4132}	$q_- - q_2 + k_2$	$q_- - q_2 + k_2 - k_1$	$-q_+ + q_1 - k_1$	q_+
10	R_{3142}	$q_- - k_2$	$q_- - k_1 - k_2$	$-q_+ + q_1 - k_1$	q_+
11	$R_{3(12)4}$	$q_- - k_2$	$-q_+ + q_2 - k_2$	—	—
12	$R_{4(12)3}$	$q_- - q_2 + k_2$	$-q_+ + k_2$	—	—

is useful not only in understanding the electromagnetic processes, but has a wide application in QCD.

We did not consider the case when one of the ions radiates a single photon and other one radiates an arbitrary number of photons absorbed by a created pair [12]. The photon exchanges between the ions also were not taken into account [11].

We are interested in the process of lepton pair production in the collision of two relativistic nuclei A, B with charge numbers Z_1, Z_2 with kinematical invariants and the Sudakov parametrization for all 4-momenta in terms of [10].

The final result for the pair production by 4-photons (2 photons emitted by one ion and two by other) reads

$$M_{(2)}^{(2)} = \frac{is}{(2!)^2} (16\pi\alpha^2 Z_1 Z_2)^2 N_1 N_2 \int \frac{d^2\mathbf{k}_1}{\pi} \frac{d^2\mathbf{k}_2}{\pi} \frac{\bar{u}(q_-) R_{(2)}^{(2)} \hat{p}_s^2 v(q_+)}{\mathbf{k}_1^2 \mathbf{k}_2^2 (q_1 - \mathbf{k}_1)^2 (q_2 - \mathbf{k}_2)^2}, \quad (1)$$

where k_1, k_2, q_1, q_2, q_- and q_+ are the momenta of the particles in the process under consideration.

$$R_{(2)}^{(2)} = \sum_{n=1}^2 \frac{[\hat{a}_n \hat{b}_n]_{\perp}}{\beta_- \mathbf{b}_n^2 + \beta_+ \mathbf{a}_n^2} - \sum_{n=3}^{10} \frac{[\hat{a}_n \hat{b}_n \hat{c}_n \hat{d}_n]_{\perp}}{2[\beta_- \mathbf{b}_n^2 \mathbf{d}_n^2 + \beta_+ \mathbf{a}_n^2 \mathbf{c}_n^2]} \left(1 + i \frac{(-1)^{n+1}}{\pi} \ln \frac{\beta_- \mathbf{b}_n^2 \mathbf{d}_n^2}{\beta_+ \mathbf{a}_n^2 \mathbf{c}_n^2} \right) + \sum_{n=11}^{12} i \frac{(-1)^{n+1}}{\pi} \frac{[\hat{a}_n \hat{b}_n]_{\perp}}{\beta_- \mathbf{b}_n^2 + \beta_+ \mathbf{a}_n^2} \ln \frac{\beta_- \mathbf{b}_n^2}{\beta_+ \mathbf{a}_n^2}, \quad (2)$$

with coefficients given in Tab. 1. One can verify that the following condition is satisfied:

$$R_{(2)}^{(2)} = 0 \quad \text{if} \quad \mathbf{k}_1 = 0 \quad \text{or} \quad \mathbf{k}_2 = 0 \quad \text{or} \quad \mathbf{k}_1 = \mathbf{q}_1 \quad \text{or} \quad \mathbf{k}_2 = \mathbf{q}_2. \quad (3)$$

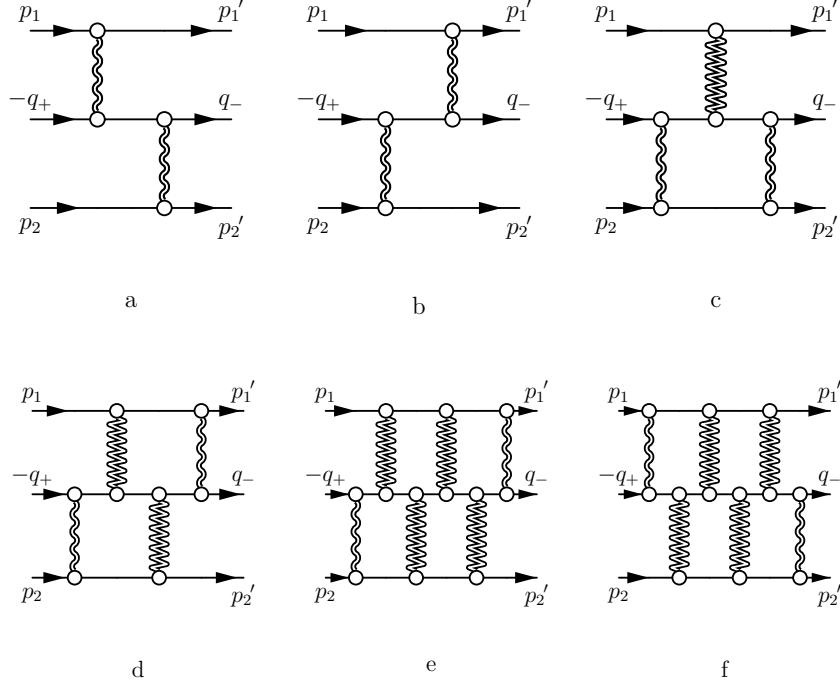


Fig. 1. The Feynman diagrams for the amplitudes with many photon exchanges. The double photon line represents any number of exchanged photons, the double zigzag line represents only the odd number of exchanged photons and “blob” represents all photon permutations.

This property is crucial for the infrared convergence in integrations over $d^2\mathbf{k}_i$.

The above picture can be generalized for the case of multiple photon exchanges ($m, n > 2$). In this case, one has to replace any single photon exchange by an infinite set of photons, multiplying the amplitude by the factors of type $\exp\{i\varphi_i(\mathbf{q}^2)\}$ with the phase $\varphi_i(\mathbf{q}^2) = \pm\alpha Z_i \ln \frac{\mathbf{q}^2}{\lambda}$.

Using the same technique as in [13] one can see that the amplitude relevant to Fig. 1a and Fig. 1b can be cast in the form

$$\tilde{R}_{(1)}^{(1)} = B e^{-i[\varphi_1(\mathbf{q}_1^2) - \varphi_2(\mathbf{q}_2^2)]} + \tilde{B} e^{i[\varphi_1(\mathbf{q}_1^2) - \varphi_2(\mathbf{q}_2^2)]}. \quad (4)$$

The interaction of the electron and the positron with Coulomb field differs only by signs. Though this expression is infrared unstable in the case $Z_1 \neq Z_2$ the regularization parameter λ enters it in a standard way.

It can be shown that the terms of higher order with any even number of photons from same nuclei attached to the lepton world line between two photons from other nuclei (Fig. 1c) do not contribute to the amplitude of the process under consideration.

The general structure of the amplitude corresponding to Fig. 1c can be constructed using the

lowest order truncated amplitude (without single photon propagators) $R_{(2)}^{(1)}$

$$\begin{aligned}\tilde{R}_{(2)}^{(1)} &= \frac{\cos \varphi_1(\mathbf{q}_1^2)}{q_1^2} \bar{R}_{(2)}^{(1)} e^{i[\varphi_2(\mathbf{k}^2) - \varphi_2((\mathbf{q}_2 - \mathbf{k})^2)]}, \\ \bar{R}_{(2)}^{(1)} &= \frac{1}{i\pi} \frac{(\hat{q}_- - \hat{q}_2 + \hat{k})_\perp (-\hat{q}_+ + \hat{k})_\perp}{\beta_-(\mathbf{q}_+ - \mathbf{k})^2 + \beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k})^2} \ln \frac{\beta_+(\mathbf{q}_- - \mathbf{q}_2 + \mathbf{k})^2}{\beta_-(\mathbf{q}_+ - \mathbf{k})^2}\end{aligned}\quad (5)$$

The further generalization is obvious. For instance, we cite the expression corresponding to the diagram depicted on Fig. 1d

$$\tilde{R}_{(2)}^{(2)} = \cos \varphi_1(\mathbf{k}_1^2) e^{-i\varphi_1((\mathbf{q}_1 - \mathbf{k}_1)^2)} \cos \varphi_2(\mathbf{k}_2^2) e^{i\varphi_2((\mathbf{q}_2 - \mathbf{k}_2)^2)} R_{1324}. \quad (6)$$

From the above consideration we conclude that the general structure of the matrix element $M_{(n)}^{(m)}$, corresponding to m photon exchanges from one ion and n exchanges from other one, schematically reads

$$\begin{aligned}M_{(n)}^{(m)} &= isN_1N_2(Z_1\alpha)^m(Z_2\alpha)^n \frac{\pi^2}{16n!m!} \\ &\times \int \frac{d^2k_1}{\pi} \dots \frac{d^2k_{m-1}}{\pi} \frac{d^2\kappa_1}{\pi} \dots \frac{d^2\kappa_{n-1}}{\pi} \frac{1}{\mathbf{k}_1^2 \dots \mathbf{k}_m^2} \frac{1}{\boldsymbol{\kappa}_1^2 \dots \boldsymbol{\kappa}_n^2} \bar{u}(q_-) \bar{R}_{(n)}^{(m)} \frac{\hat{p}_2}{s} v(q_+),\end{aligned}\quad (7)$$

where m and n obey the condition $|m - n| \leq 1$. At this stage, we omitted phase factors in the structure $R_{(n)}^{(m)}$ (for clearly understanding the problem), so it can be written in the form

$$\bar{R}_{(n)}^{(m)} = \bar{R}_{(1)}^{(1)} + \bar{R}_{(2)}^{(1)} + \bar{R}_{(1)}^{(2)} + \bar{R}_{(2)}^{(2)} + \bar{R}_{(3)}^{(2)} + \bar{R}_{(2)}^{(3)} + \bar{R}_{(3)}^{(3)R} + \bar{R}_{(3)}^{(3)L} \dots \quad (8)$$

$$\bar{R}_{(1)}^{(2)} = \frac{1}{i\pi} \frac{(\hat{q}_- - \hat{k})_\perp (-\hat{q}_+ + \hat{q}_1 - \hat{k})_\perp}{\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k})^2 + \alpha_+(\mathbf{q}_- - \mathbf{k})^2} \ln \frac{\alpha_+(\mathbf{q}_- - \mathbf{k})^2}{\alpha_-(-\mathbf{q}_+ + \mathbf{q}_1 - \mathbf{k})^2},$$

$$\bar{R}_{(3)}^{(2)} = \bar{R}_{(2)}^{(3)} = 0,$$

$$\bar{R}_{(3)}^{(3)R} = \frac{1}{c_1 + c_2} \left[3\zeta_2 + \frac{1}{2} \ln^2 \frac{c_1}{c_2} \right], \quad \zeta_2 = \frac{\pi^2}{6},$$

$$c_1 = \beta_-(\mathbf{q}_- - \mathbf{k}_1)^2 (\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2 - \boldsymbol{\kappa}_1)^2 (-\mathbf{q}_+ + \mathbf{q}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)^2,$$

$$c_2 = \beta_+ \mathbf{q}_-^2 (\mathbf{q}_- - \mathbf{k}_1 - \boldsymbol{\kappa}_1)^2 (\mathbf{q}_- - \mathbf{k}_1 - \mathbf{k}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)^2,$$

$$\bar{R}_{(4)}^{(3)} = \frac{1}{d_1 + d_2} \left[3\zeta_2 + \frac{1}{2} \ln^2 \frac{d_1}{d_2} \right]$$

$$d_1 = \beta_+ (\mathbf{q}_- - \boldsymbol{\kappa}_1)^2 (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \mathbf{k}_1)^2 (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3 - \mathbf{k}_1 - \mathbf{k}_2)^2,$$

$$d_2 = \beta_- (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \mathbf{k}_1)^2 (\mathbf{q}_- - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \mathbf{k}_1 - \mathbf{k}_2)^2 (-\mathbf{q}_+ + \mathbf{q}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3)^2. \quad (9)$$

Here $\bar{R}_{(2)}^{(2)}$ is only the second term in the right-hand side in (2) and the index $R(L)$ denotes two possible configurations of photons for $\bar{R}_{(3)}^{(3)R}$ (Fig. 1e) and $\bar{R}_{(3)}^{(3)L}$ (Fig. 1f).

In such a way, the general algorithm for construction of an arbitrary term is transparent. Unfortunately, we cannot obtain the compact expression for the whole amplitude. The reason is the increasing nonlinearity of the propagators with the order of interaction.

1 Conclusions

We obtained the expression for the amplitude $2\gamma + 2\gamma \rightarrow 1^+1^-$ and show that its contribution is dominant in a wide angle limit. Our principal finding is that the amplitude is manifestly of non-Born nature, which is suggestive of the complete failure of linear k_{\perp} -factorization even in the Abelian case.

We have shown that the terms in perturbation series of the amplitude for the process of lepton pair production in the Coulomb fields of two relativistic nuclei relevant to the closed two photon loops are logarithmically enhanced in this case, while in higher order terms such enhancement is absent. We presented the algorithm which allows one to construct the full amplitude in all orders. The obtained results can be useful in application to the QCD process of production of high k_{\perp} jets, the issue which will be investigated elsewhere.

Acknowledgement: We are grateful to the participants of the seminar at BLTP JINR, Dubna, INP Novosibirsk for critical comments and discussions. E. K. and E. B. acknowledge the support of INTAS grant No. 00366, RFFI grant No. 03-02-17077 and Grant Program of Plenipotentiary of Slovak Republic at JINR, grant No. 02-0-1025-98/2005.

References

- [1] H. Bethe, L. Maximon: *Phys. Rev.* **93** (1954) 768 ; H. Davies, H. Bethe, L. Maximon: *ibid.* **93** (1954) 788
- [2] L. D. Landau, E. M. Lifshitz: *Quantum Mechanics*, (N) a uka, Moscow 1989
- [3] D. Yu. Ivanov, K. Melnikov: *Phys. Rev. D* **57** (1998) 4025
- [4] N. N. Nikolaev, W. Schafer, B. G. Zakharov, V. R. Zoller: *JETP Letters* **97** (2003) 441
- [5] A. J. Baltz, L. McLerran: *Phys. Rev. C* **58** (1998) 1679
- [6] B. Segev, J. C. Wells: *Phys. Rev. A* **57** (1849) 1998
- [7] U. Eichmann, J. Reinhardt, W. Greiner: *Phys. Rev. A* **61** (2000) 062710
- [8] D. Yu. Ivanov, A. Schiller, V. G. Serbo: *Phys. Lett. B* **457** (1999) 155
- [9] R. N. Lee, A. I. Milstein: *Phys. Rev. A* **61** (2000) 032103
- [10] E. Bartoš et al.: *Phys. Rev. A* **66** (2002) 042720
- [11] E. Bartoš et al.: *Phys. Lett. B* **538** (2002) 45
- [12] S. R. Gevorkyan, E. A. Kuraev: *J. Phys. G* **29** (2003) 1227
- [13] E. Bartoš, S. R. Gevorkyan, E. A. Kuraev: *Yad. Phys.* **67** (2004) 8