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NONLINEAR THERMODYNAMIC MAGNETIC FIELD AND SPECIFIC HEAT OF TWO-BAND SUPERCONDUCTORS IN THE GINZBURG-LANDAU THEORY

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Temperature dependence of thermodynamic magnetic field for superconducting magnesium diboride MgB_2 is studied in the vicinity of T_c using the two-band Ginzburg-Landau theory. The results are in good agreement with calculations from experimental data. In addition, the two-band Ginzburg-Landau theory gives a smaller specific heat jump than a single-band Ginzburg-Landau theory and nonlinear temperature dependence below T_c.

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1 Introduction

Much attention has been focused on the recently discovered superconductor MgB_2 [1], because of the highest superconducting transition temperature of about $T_c = 39$K for a binary compound. Two-band characteristic of the superconducting state in MgB_2 is clearly evident in the recently performed tunnel measurements [2, 3]. At microscopic level, the two-band Eliashberg model of superconductivity in MgB_2 was first proposed by Shulga et al. [4]. The calculations of specific heat using the first principles of the two-band Eliashberg theory were given by Golubov et al. [5]. More recently, the two-band Ginzburg-Landau model was applied to study the temperature dependence of different physical quantities near T_c for bulk magnesium diboride MgB_2 and nonmagnetic borocarbides $LuNi_2B_2C$ and YNi_2B_2C [6–8].

It is generally known that one of the important characteristics of superconductors is the electronic specific heat. Its temperature behavior is well described in the framework of the Bardeen-Cooper-Schrieffer (BCS) theory. Accordingly to the isotropic BCS theory, the jump in specific heat at T_c is constant and it is $(C_S - C_N)/C_N = 1.43$, where C_S and C_N are the specific heats in the superconducting and normal state respectively. The Eliashberg theory, assuming a strong electron-phonon coupling, is expected to give a value greater than 1.43. Several groups have carried out measurements of specific heat on magnesium diboride MgB_2 [9–11]. The measured

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specific heat shows a small jump at T_c, which is not explainable by the use of the standard BCS and the Eliashberg theories [12].

In this paper, we apply two-band Ginzburg-Landau (G-L) theory to determine on the one hand the temperature dependence of thermodynamic magnetic field $H_{cm}(T)$ and on the other hand the value of the specific heat jump at T_c. The temperature dependence of $H_{cm}(T)$ is essential for the assessment of the behavior of specific heat at temperatures close to T_c. We show that the presence of two-order parameter in the theory gives a non-linear temperature dependence of thermodynamic magnetic field $H_{cm}(T)$ and the differences of specific heats $C_S(T) - C_N(T)$.

2 Basic Equations

In the presence of two order parameters in a superconductor, Ginzburg-Landau free energy functional can be written as [6–8, 13]

$$
\begin{aligned}
F[\Psi_1, \Psi_2] &= \int \mathrm{d}V \left\{ \frac{\hbar^2}{4m_1} \left| \left(\nabla - \frac{2\pi i A}{\Phi_0} \right) \Psi_1 \right|^2 + \alpha_1(T)\Psi_1^2 + \frac{\beta_1}{2}\Psi_1^4 \right. \\
&+ \frac{\hbar^2}{4m_2} \left| \left(\nabla - \frac{2\pi i A}{\Phi_0} \right) \Psi_2 \right|^2 + \alpha_2(T)\Psi_2^2 + \frac{\beta_2}{2}\Psi_2^4 + \varepsilon(\Psi_1\Psi_2^* + c.c.) \\
&+ \left. \varepsilon_1 \left[\left(\nabla + \frac{2\pi i A}{\Phi_0} \right) \Psi_1^* \left(\nabla - \frac{2\pi i A}{\Phi_0} \right) \Psi_2 + c.c. \right] + H^2/8\pi \right\}. \qquad (1)
\end{aligned}
$$

Here, m_i denotes the effective mass of the carriers belonging to the band i with $(i = 1, 2)$. The coefficient α is given as $\alpha_i = \gamma_i(T - T_{ci})$, which depends on temperature linearly, γ is the proportionality constant, while the coefficient β is independent of temperature and $\vec{H}$ is the external magnetic field ($\vec{H} = \vec{\nabla} \times \vec{A}$). The quantities ε and ε_1 describe the interband mixing of two order parameters and their gradients, respectively.

Minimization of the free energy functional with respect to the order parameters yields G-L equations for two-band superconductors in one dimension $\vec{A} = (0, Hx, 0)$ as in [6–8]

$$
-\frac{\hbar^2}{4m_1} \left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^4} \right) \Psi_1 + \alpha_1(T)\Psi_1 + \varepsilon\Psi_2 + \varepsilon_1 \left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^4} \right) \Psi_2 + \beta_1\Psi_1^3 = 0, \qquad (2a)
$$

$$
-\frac{\hbar^2}{4m_2} \left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^4} \right) \Psi_2 + \alpha_2(T)\Psi_2 + \varepsilon\Psi_1 + \varepsilon_1 \left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^4} \right) \Psi_1 + \beta_2\Psi_2^3 = 0, \qquad (2b)
$$

where $l_s^2 = \hbar c/2eH$ is the so-called magnetic length (there are misprints in the corresponding equations of Refs. [6–8], l_s^2 must be replaced with $(l_s^2)^2$). Without loss of generality, we present the case when Ψ and A depend only on single coordinate x in deriving the last equations. By linearizing the above equations in the absence of any external magnetic field, the critical temperature T_c can obtained from

$$
\alpha_1(T_c)\alpha_2(T_c) = \varepsilon^2. \qquad (3)
$$

Based on the last expression, the interband interaction will lead to enhancement of critical temperature in comparison with T_{c1} and T_{c2}. Considering $\Psi_j(r) = |\Psi_j(r)| \exp(i\phi_j(r))$, with $\phi_j(r)$ the phase of order parameters and $|\Psi_j(r)|$ the modulus, we can obtain the equilibrium values for $|\Psi_j(r)|$ in the absence of any external magnetic field as

$$|\Psi_{10}|^2 = -\frac{\alpha_2^2(T)(\alpha_1(T)\alpha_2(T) - \varepsilon^2)}{\varepsilon^2\beta_2\alpha_1(T) + \beta_1\alpha_2^3(T)}. \tag{4a}$$

Due to symmetric character of Eqs. (2a) and (2b), the solution for $|\Psi_{20}|^2$ can be obtained by replacement of symbols "1→2" and "2→1", i.e.

$$|\Psi_{20}|^2 = -\frac{\alpha_1^2(T)(\alpha_1(T)\alpha_2(T) - \varepsilon^2)}{\varepsilon^2\beta_1\alpha_2(T) + \beta_2\alpha_1^3(T)}, \tag{4b}$$

The phase difference of order parameters at equilibrium can be given as

$$\cos(\phi_1 - \phi_2) = 1; \qquad \text{if} \quad \varepsilon<0, \tag{5a}$$

$$\cos(\phi_1 - \phi_2) = -1; \qquad \text{if} \quad \varepsilon>0. \tag{5b}$$

3 Results and discussions

The free-energy difference between the normal and the superconducting states can be written using Eq. (1) as

$$\Delta F = -\frac{\beta_1}{2}|\Psi_{10}|^4 - \frac{\beta_2}{2}|\Psi_{20}|^4 - 2\varepsilon|\Psi_{10}||\Psi_{20}|. \tag{6}$$

The last term in Eq. (6) is related to the interband mixing and it leads to increasing of free-energy differences and consequently of critical temperature. On the other hand, the thermodynamic magnetic field is related to the free energy difference by

$$-\frac{H_{cm}^2}{8\pi} = \Delta F. \tag{7}$$

Here, we use the notation H_{cm} for the thermodynamic critical field of a bulk superconductor, which is different from that for thin films, which will be of subject future investigations. Using Eqs. (4), (6), and (7), one can obtain the following formula with an appropriate manipulation of the thermodynamic magnetic field.

$$\begin{aligned} H_{cm}(T) = & - \sqrt{4\pi}\frac{\alpha_1(T)\alpha_2(T) - \varepsilon^2}{\varepsilon^2\beta_1\alpha_2(T) + \beta_2\alpha_1^3(T)} \\ & \times \sqrt{\beta_1\varepsilon^4 + \beta_2\alpha_1^4(T) - 2\varepsilon^2\alpha_1(T)\frac{\varepsilon^2\beta_1\alpha_2(T) + \beta_2\alpha_1^3(T)}{\alpha_1(T)\alpha_2(T) - \varepsilon^2}}. \end{aligned} \tag{8}$$

We now introduce a dimensionless parameter of the form $h_{cm} = H_{cm}(T)/H_{cm}(0)$, where $H_{cm}(0) = \sqrt{4\pi}T_c(\gamma_1/\sqrt{\beta_1} + \gamma_2/\sqrt{\beta_2})$ and we then have a normalized form of the thermodynamic magnetic field

$$\begin{aligned} h_{cm}(\theta) = & - \frac{\sqrt{D}}{1+\sqrt{D}}\left[\theta^2 + (2-\tau_{c1}-\tau_{c2})\theta\right] \\ & \times \frac{\sqrt{D(\varepsilon^*)^4 + (\tau-\tau_{c1})^4 - 2(\varepsilon^*)^2 \frac{(\varepsilon^*)^2 D(\tau-\tau_{c2}) + (\tau-\tau_{c1})^3}{\theta^2+(2-\tau_{c1}-\tau_{c2})\theta}}}{(\varepsilon^*)^2 D(\tau-\tau_{c2}) + (\tau-\tau_{c1})^3}. \end{aligned} \tag{9}$$

Here, all the parameters are dimensionless and they can be expressed as

$$\begin{aligned} D &= \frac{\beta_1\gamma_2^2}{\beta_2\gamma_1^2}, \\ \theta &= \tau - 1, \\ \tau &= \frac{T}{T_c}, \\ \tau_{c1,c2} &= \frac{T_{c1,c2}}{T_c}, \\ (\varepsilon^*)^2 &= \frac{\varepsilon^2}{\gamma_1\gamma_2 T_c^2}. \end{aligned} \tag{10}$$

The differences in specific heat between the superconducting and normal state $C_N - C_S$ can be written as [12]

$$C_N - C_S = \frac{T}{4\pi}\left[\left(\frac{\partial H_{cm}}{\partial T}\right)^2 + H_{cm}\frac{\partial^2 H_{cm}}{\partial T^2}\right]. \tag{11}$$

At $T = T_c$, $H_{cm} = 0$ and we have the Ruthgers formula for the specific heat jump at T_c,

$$\frac{\Delta C}{T_c} = \frac{1}{4\pi}\left(\frac{\partial H_{cm}}{\partial T}\right)^2_{T_c}. \tag{12}$$

Temperature dependence of the thermodynamic magnetic field H_{cm} in the single band superconductors [12] is given by

$$H_{cm} = -\sqrt{\frac{4\pi}{\beta}}\alpha(T). \tag{13}$$

Consequently, in single band superconductors specific heat below T_c reveal linear behavior. Due to Eq. (11) and nonlinear character of thermodynamic magnetic field $H_{cm}(T)$ (see Eq. (9) and (10)) in two band superconductors, specific heat show also nonlinear behavior. Such conclusion is in qualitative agreement with first principles calculations in [5]. Another important result is related to the specific heat jump at T_c. For the normalized specific heat jump at T_c, we obtain

$$\Delta c = \left(\frac{\partial h_{cm}}{\partial \theta}\right)^2_{\theta=0}, \tag{14}$$

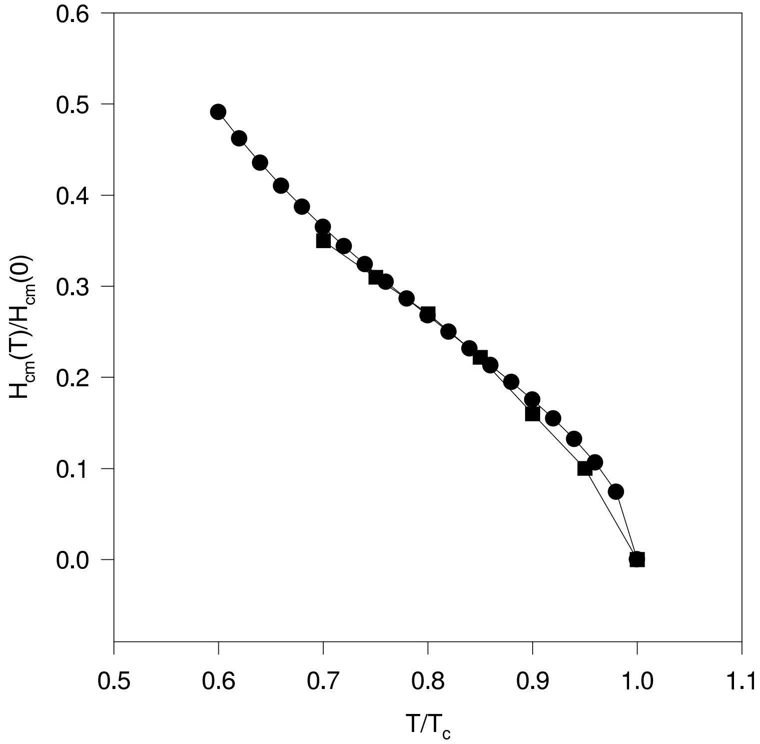

Fig. 1. Temperature dependence of thermodynamic magnetic field (the circles represent the two-band GL theory and the squares represent the empirical data).

where $\Delta c = \Delta C/\Delta C_0$ and $\Delta C_0/T_c = H^2_{cm}(0)/4\pi T^2_c$.

Note that the thermodynamic magnetic field $H_{cm}(T)$ is not directly measurable quantity. Fortunately, it can be calculated from specific heat measurements. In Fig. 1, we plot the temperature dependence of $H_{cm}(T)$ using Eqs. (9) and (10) (circles). To produce the data in Fig. 1, we have used the following parameters: $D = 1.35, T_{c1} = 20\text{K}, T_{c2} = 10\text{K}, T_c = 40\text{K}$, $(\varepsilon^*)^2 = 3/8$, similarly as in Refs. [6–8]. It is necessary to remark that it was reported that the critical temperature one of groups of superconducting electrons is 10K [14]. It confirms correct character of our fitting parameters presented above. Empirical data forH_{cm} (squares) were extracted from the results of Bouquet et al. [9] with H_{cm}(0)=0.45 T. Similar results were also observed experimentally in the work of Wang et al. [11]. It is worth noting that isotropic single band approximation gives a linear temperature dependence of h_{cm} (see Eq. (13)). In contrast to single band approach, the two-band superconductivity model yields nonlinear temperature dependence near T_c and as a result to nonlinear behavior of specific heat.

Substituting the calculated value of $\partial h_{cm}/\partial\theta$ at the critical temperature into Eq. (14), we can estimate the specific heat jump at T_c. The estimated value for the jump is 0.64, which is very small compared to unity calculated from the single-band G-L theory. However, the value is consistent with the experimental data in [11], accordingly to which $\Delta C/C_N = 0.8$ is smaller than the single band BCS value of 1.43. Such conclusion is in agreement with two-band BCS calculations conducted by Moskalenko et al. [15].

4 Conclusion

We show that the presence of two order parameters in the Ginzburg-Landau theory gives a non-linear temperature dependence of thermodynamic magnetic field and specific heat. The strength of the non-linearity is mainly dependent on the interaction coupling between the order parameters of two separate bands. The results of the calculations are in a good agreement with experimental data for bulk MgB_2. We conclude that the two-band G-L theory explains the reduced magnitude of the specific heat jump and the small slope of the thermodynamic magnetic field in MgB_2.

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