acta physica slovaca vol. 54 No. 6, 525 – 533 December 2004

VIBRATING WIRE: A PROBE FOR THE ENERGY GAP MEASUREMENT IN SUPERFLUID ^{3}He-B

M. Človečko, T. Hanušin, M. Kráľ, P. Skyba
Centre of Low Temperature Physics, Institute of Experimental Physics, Slovak Academy of Sciences, Watsonova 47, 04353 Košice, Slovakia

Received 30 June 2004, accepted 29 November 2004

We present properties of a vibrating wire in vacuum and in superfluid ^{3}He-B at temperatures well below transition temperature, where the properties of the superfluid ^{3}He-B are comparable (analogous) with that of quantum vacuum. It is shown that the wire can be used to measure such fundamental physical quantity as the energy gap in a spectrum of quasiparticle excitations of the superfluid ^{3}He-B.

PACS: 67.57.-z, 67.57.Bc

1 Introduction

The vibrating wire is a simple device widely used in the study of quantum liquids, like superfluid ^{4}He and superfluid ^{3}He. Usually it is a superconducting wire — very thin fibre — bowed into a semicircle (or other shape e.g. rectangular) and immersed in the liquid. With an aid of steady magnetic field **B** orientated along a plane of the wire loop and ac-current I_0 driven via the wire, the wire is lead through a resonance in consequence of acting Lorentz force. The motion of the wire per unit of mass and wire length is described by following equation:

$$d^2x/dt^2 + \gamma dx/dt + \omega_0^2 x = f \exp(i\omega t), \tag{1}$$

where parameter $f = I_0 B/m$ describes the driving force per mass, $m = \pi r^2 \rho_w$ is the mass per unit length of wire with r and ρ_w being radius and density of the wire, respectively. The second term in equation (1) characterizes a damping force of the fluid acting against the wire motion and it is assumed that is linear with the wire velocity v. The constant γ ($\gamma = \gamma_2 + i\gamma_1$) is the damping constant, where γ_2 refers to the dissipative component of the damping force. The γ_1 characterizes its reactive component associated with the fluid backflow around the wire and effectively gives the wire a greater mass. The last term in equation (1) is the restoring force of the wire, where ω_0 is the wire resonance frequency in vacuum. The steady state solution of equation (1) is well known and it leads to a Lorentz shape of the absorption and dispersion curves in dependence on the frequency.

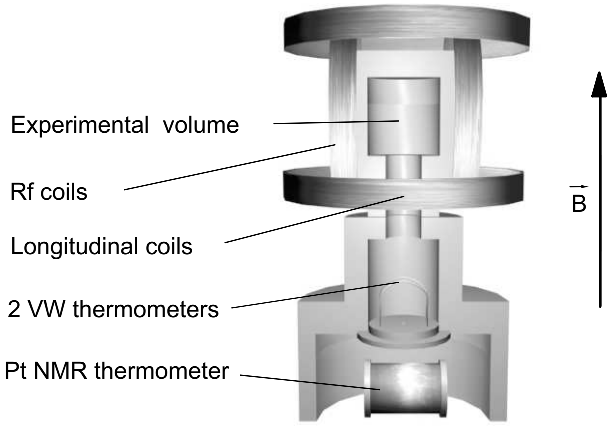

Fig. 1. The 3-D cross section of the experimental cell.

The width of the absorption curve $2\pi\Delta f_2 = \gamma_2$ is related to the total damping force per unit of mass acting on the wire motion as:

$$F = 2\pi\Delta f_2 v. \tag{2}$$

On the other hand, the frequency shift of the resonance frequency from its value in vacuum Δf_1 is associated with γ_1 as $\gamma_1 = 4\pi\Delta f_1$.

The vibrating wire moves in the direction which is almost perpendicular to the applied magnetic field $\mathbf{B}$, so an additional voltage U_i is induced. Its magnitude can be found by applying the Faraday's law in form $U_i = kBlv$, where l is the wire length and the constant k characterizes the geometric shape of the wire. Solving the equation (1), one can get the expression for maximum value of in-phase velocity in the form $v = BI_0/m\gamma_2$. Combining last two equations together, one gets a quadratic dependence of the induced voltage on magnetic field $U_i \sim B^2$. The induced voltage U_i is amplified by a preamplifier and then measured by a phase sensitive detector (so called Lock-in amplifier). A reference signal for phase sensitive detector is taken from generator supplying the wire with current. By sweeping the frequency of the generator, the resonance characteristics of the vibrating wire is measured.

An aim of this paper is to show that such simple mechanical resonator, as the vibrating wire is, can be very useful tool to study physical properties of quantum system like the superfluid ^{3}He-B.

2 Experiment

The experiments were performed in an experimental cell made of epoxy resin Stycast 1266 (see Fig. 1) mounted on Košice nuclear demagnetization stage [1]. Two vibrating wires were made from one filament (125 μm in diameter) NbTi fiber bowed to a semicircle of radius ~ 2 mm, and glued inside the cell. Both wires were thermally anchored to the copper nuclear stage. First, the properties of the wires in vacuum i.e. in an empty experimental cell were studied. Then the cell was filled up with the liquid ^{3}He and properties of the wires immersed in superfluid ^{3}He-B were investigated. The temperature of the nuclear stage and superfluid ^{3}He were measured independently by NMR thermometers made from: (i) Pt wires thermally connected to the stage and (ii) Pt powder immersed in liquid. Both thermometers were calibrated against transition temperature T_c of ^{3}He into superfluid state during demagnetization or warming up processes several times. Absorption and dispersion signals from the wires in vacuum were measured at temperature ~ 1 mK and in magnetic field up to 25 mT. The temperature dependence of the vibrating wire width Δf_2 in superfluid ^{3}He-B was measured at magnetic field corresponding to the resonance frequency 125 kHz for Pt NMR thermometer. This field is small in comparison with the value of the critical field (~ 330 mT) and the energy gap distortion due to magnetic field can be neglected.

3 Properties of the vibrating wire in a vacuum

At very low temperatures the superfluid ^{3}He-B behaves like vacuum from the viewpoint of excitations and damping of the vibrating wire due to interaction with them is comparable with an intrinsic damping of the wire itself. Therefore, it is important to know properties of the vibrating wire in a vacuum. Similar study of the nonlinear acoustic properties of various vibrating wires in vacuum and superfluid ^{3}He-B is presented in [2].

As first the dependence of induced voltage on the value of magnetic field was measured ($U_i \sim B^2$). Figure 2 shows the values of induced voltage U_i measured in dependence on B and confirms expected quadratic dependence on magnetic field. The same dependence was observed for the second wire resonating at frequency $\sim 3500 Hz$. The dependence of frequency width Δf_2 on the excitation amplitude (or the wire velocity) is presented on Fig. 3. As can be seen from this dependence, Δf_2 shifts to higher values as the excitation i.e. the wire velocity increases. This is inconsistent with the linear model (see equation (2)) according to which the width Δf_2 should be constant. Where can a physical origin of the nonlinear intrinsic damping come from?

There are at least two processes responsible for the energy dissipation in oscillating wire. One of the mechanisms of energy dissipation is the presence of vortices in vibrating wires. The wires are usually made from superconductors of the second type and they may be in a mixed state with vortices inside. The vortices are regions of the normal phase in superconductor allowing the penetration of the magnetic field through the wire. This mechanism dominates especially at magnetic fields above B_{c1} and/or higher temperature due to a temperature dependence of $B_{c1}(T)$:

$$B_{c1}(T) = B_{c1}(0)[1 - (T/T_{wc})^2], \tag{3}$$

where T_{wc} is the transition temperature of the wire into the superconducting state and $B_{c1}(0)$ is the threshold of magnetic field of the transition into the mixed state at $T = 0$. The energy is dissipated due to eddy current heating in normal regions determined by vortices. Owing to AC current flow through the wire, the Magnus force acts on the pinned

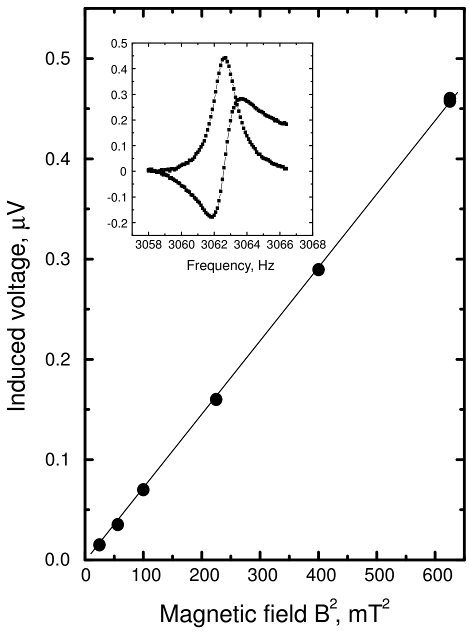

Fig. 2. The dependence of the induced voltage by the vibrating wire on magnetic field. Inset shows typical induced resonance signals measured during the frequency sweep.

vortices causing their viscous oscillations. This leads to the additional energy dissipation increase thus to the intrinsic damping of the wire. However, at very low temperature and low magnetic field (at values below B_{c1}) the NbTi wire is in pure superconducting state without the presence of any vortices and the magnetic field penetrates only on the distance of order of the London penetration depth from the wire surface.

The origin of another channel for the energy dissipation can be found if one considers the deformations of the lattice during oscillations. When an atom is moving closer to its neighbor, the potential energy of the atom is rising up faster than when it is moving away. This gives rise to an asymmetrical restoring force acting on the atom. A fundamental property of such non-linear system is the presence of higher harmonics in the frequency spectrum, and in this particular case, it is the presence of second harmonics (see Fig. 3). Moreover, due to asymmetrical shape of the potential energy, the atom will oscillate around a new equilibrium position and the distance between two atoms will be time dependent. The oscillations of atoms produce additional thermal phonons which are responsible for the energy dissipation. The energy dissipation can occur (i) via local phonon-phonon scattering which leads to thermalization corresponding of "a new lattice constant" and (ii) by ballistic transport of phonons between "hot" i.e. deformed region of the wire and "cold" i.e. non deformed part of the wire. We believe that this mechanism dominates at ultra low temperatures and low magnetic fields and is responsible for the

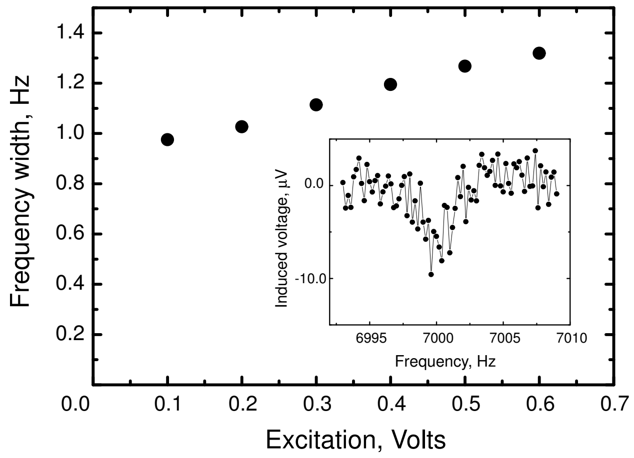

Fig. 3. The dependence of Δf_2 on the excitation voltage. measured at T = 1 mK and B = 15 mT. Inset: the presence of the second harmonics in oscillations of the vibrating wire documents the nonlinear properties of the resonator.

intrinsic damping of the wire.

4 Vibrating wire: a detector of excitations in superfluid ^{3}He-B

When the wire is immersed in superfluid ^{3}He-B then we can observe two phenomena. First, the wire has an effectively greater mass due to the reactive component associated with the liquid backflow. This results in a shift of the resonance frequency to lower values. Secondly, the liquid itself damps the wire motion due to presence of excitations what increases its damping in comparison with the case in vacuum. Generally, the motion of the wire in superfluid ^{3}He-B is restricted due to a mutual interaction between the excitations and the wire. A total damping force acting on the wire, in general, consists of three terms: $F = F_I + F_C + F_T$, where F_I is the intrinsic damping force of the wire discussed above, F_C is the damping force due to pair breaking when the wire velocity is above the critical and the wire works like a heater rather then thermometer, and F_T is thermal damping force due to collision of the wire with existing excitations (quasiparticles and quasiholes). The last term F_T per unit area can be expressed as [3]:

$$F_T = p_f \langle n v_g \rangle \left[1 - \exp\left(-\frac{p_f v}{kT} \right) \right], \tag{4}$$

where v_g and n are the group velocity and the number of excitations per unit volume, respectively, v is the wire velocity and $\langle n v_g \rangle = n(p_f) kT \exp(-\Delta/kT)$ represents quasiparticle flux with $n(p_f)$ being the density of states in momentum space, k is the Boltzman constant and Δ is the energy gap. The origin of this non-linearity with wire velocity arises

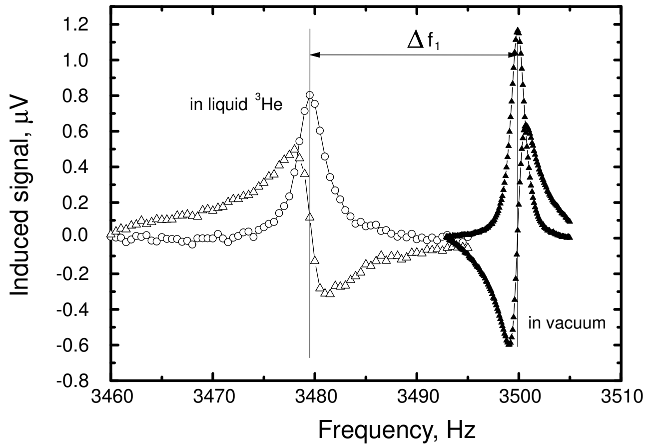

Fig. 4. Typical induced signal from vibrating wire measured in vacuum and liquid ^{3}He. The frequency shift from the resonance frequency in vacuum and increase of damping due to presence of the liquid is clearly seen.

from Andreev scattering of excitations from the flow field around the moving wire [4]. At low velocity the damping force per unit area of the wire becomes linear in velocity v:

$$F_T = \frac{p_f^2 v}{kT}\langle n v_g\rangle \sim v p_f^2 \exp\left(-\frac{\Delta(T)}{kT}\right). \tag{5}$$

For low temperatures the damping force F_T at constant velocity depends only on temperature. Assuming that the intrinsic damping of the wire is negligible i.e. $F_I = 0$ and the term F_C is close to zero because the wire velocity v is always lower than the critical velocity v_c, then the total damping force acting on the wire will come only from F_T. Combining equations (2) and (5) one can get the final result:

$$\Delta f_2 = A \exp\left(-\frac{\Delta(T)}{kT}\right), \tag{6}$$

where A is a constant which needs to be determined from experiment. Thus the vibrating wire operating in linear regime works as a thermometer directly measuring the temperature of superfluid ^{3}He-B. Now, one can logarithm the equation (6) and using the relation $\Delta(T) = \Delta(0)\sqrt{(1 - T/T_c)}$, together with the BCS value of the energy gap $\Delta(0) = \Delta_{BCS} = 1.76kT_c$, he gets:

$$\ln(\Delta f_2) = \ln(A) - \frac{\Delta(0)}{kT}\sqrt{1 - \frac{T}{T_c}} = \ln(A) - 1.76\frac{T_c}{T}\sqrt{1 - \frac{T}{T_c}}. \tag{7}$$

In our experiments we were able to measure the temperature of superfluid ^{3}He-B independently by Pt NMR thermometer and therefore we can consider the term

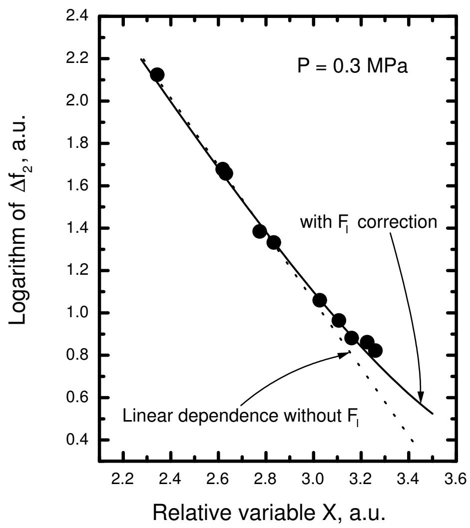

Fig. 5. The dependence of Δf_2 on temperature with and without correction of the intrinsic damping of the vibrating wire.

$T_c\sqrt{1 - T/T_c}/T$ as an independent variable X. As a result, we get a linear dependence of $\ln(\Delta f_2)$ on variable X:

$$\ln(\Delta f_2) = \ln(A) - 1.76X. \tag{8}$$

Figure 5 shows the dependence of $\ln(\Delta f_2)$ on the variable X. One can see that the measured data are not linear dependent on X. At lower temperatures i.e. higher values of X the intrinsic damping of the wire starts to be comparable with that from the interaction with excitations and therefore the intrinsic damping can not be neglected. The total damping force F in this case has two components $F = F_T + F_I$ and it corresponds to the total width Δf_2 measured. The width Δf_2 consists of two parts represented by thermal Δf_T and the intrinsic Δf_I damping, respectively. By logarithm of this expression and developing the terms on right side in Taylor series around of $a = \Delta f_T$ (we assume $\Delta f_T > \Delta f_I$), we finally get $\ln(\Delta f_2) = \ln(\Delta f_T) + \Delta f_I/\Delta f_T$. The thermal damping Δf_T is equal to $A\exp(-\Delta(T)/kT)$ and substituting it into the equation above we receive:

$$\ln(\Delta f_2) = \ln(A) - \frac{\Delta(T)}{kT} + \frac{\Delta f_I}{A\exp(-\Delta(T)/kT)}. \tag{9}$$

The last term on the right side is the correction connected with intrinsic damping force F_I. As can be seen from Fig. 5, the application of this dependence with correction on the intrinsic damping provides reasonable agreement with the measured data, confirming thus the argument about an important role of the intrinsic damping at very low temperatures. Having the possibility of independent temperature measurements on one side and the measurements of the excitation density using the wire on the other, within the simple model presented above, we have a tool to investigate such fundamental property of the superfluid like the value of the energy gap.

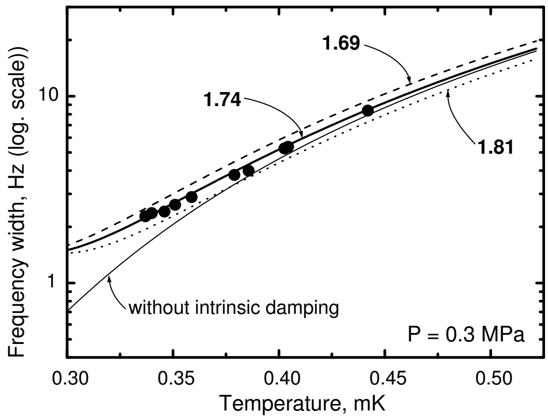

Fig. 6. The dependence of Δf_2 on temperature including the correction of intrinsic damping of the vibrating wire. The solid line represents the fit of data using the value $\Delta(0)/kT_c = 1.74$ for pressure 0.3 MPa. Another lines represent the dependencies using the values $\Delta(0)/kT_c$ equal to 1.69 (dashed line) and 1.81 (dotted line) for pressures of 0 and 1.1 MPa, respectively. It is obvious that these dependencies do not fit the experimental data well. A thinner solid line shows the temperature dependence of Δf_2 for an ideal vibrating wire.

The theory of superfluid ^{3}He was developed in a weak coupling limit using the BCS theoretical approach [5]. In this approach the value of the energy gap for ^{3}He is $\Delta_{BCS} = 1.76kT_c$ and does not depend on pressure. From experiments, however, it was obvious that the weak-coupling limit needed to incorporate the strong-coupling corrections [6]. For example, the result of weak coupling theory suggests that the B-phase is the lowest in free energy at all temperatures which contradicts to the phase diagram showing the creation of the A-phase at higher pressures first (for details see [7]). Therefore, the incorporation of strong-coupling corrections resulted in a pressure dependence of the energy gap $\Delta(0)$. The method presented above gives us a possibility to determine the value $\Delta(0)/kT_c$ for various pressures. Indeed, instead of using the BCS value $\Delta(0)/kT_c = 1.76$, we can take this ratio as a free parameter and fit our data by equation (9). The results are presented in Fig. 6. The solid line represents the fit of the data using the value $\Delta(0)/kT_c = 1.74$. This value is in reasonable agreement with the data measured by R. Movshovich et al. [8] and N. Masuhara et al. [9]. Another lines represent the dependencies using the values $\Delta(0)/kT_c$ equal to 1.69 (dashed line) and 1.81 (dotted line) for pressures of 0 and 1.1 MPa, respectively. It is obvious that these dependencies do not fit the experimental data.

In summary, we showed that simple mechanical resonator - vibrating wire - can be used as the probe for measurement of the superfluid ^{3}He-B energy gap providing that the correction on the wire intrinsic damping is taken into account and temperature of the superfluid ^{3}He-B is measured by an independent thermometer.

Acknowledgement: We thank Š. Bicák, V. Pavlík and G. Pristáš for their technical backup, and we appreciate the liquid nitrogen support provided by U.S. Steel Košice s.r.o. This work is supported by grant agency APVT under grant 51-020102 and partially by grant agency VEGA under grant 2/3196/23.

References

[1] P. Skyba et al., Cryogenics **36** (1997) 293.
[2] R. König, P. Esquanzi, F. Pobell, J. Low Temp. Phys. **90** Nos. 1/2 (1993) 55.
[3] S. N. Fisher, et al., Phys. Rev. Lett. **63** (1989) 2566.
[4] M. P. Enrico, et al., Phys. Rev. Lett. **70** (1993) 1846.
[5] A. J. Leggett, Rev. Mod. Phys. **47** (1975) 331.
[6] J. W. Serene and D. Rainer, Phys. Rep. **101** (1983) 221.
[7] D. Vollhardt and P. Wölfle: The Superfluid Phases of Helium 3, Taylor & Francis (1990).
[8] R. Movshovich et al., Phys. Rev. Lett. **64** (1990) 431.
[9] N. Masuhara et al., Phys. Rev. Lett. **85** (2000) 2537.