THE GREEN FUNCTION METHOD FOR THE TRANSVERSE HEISENBERG MODEL

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The magnetic properties of the two-dimensional anisotropic Heisenberg model with spin S = 1 are investigated by the Green function method. The Heisenberg model with the exchange anisotropy in the (x, y)-plane and in an external magnetic field $\mathbf{H} = (H^x, 0, 0)$ is considered as a special model of a non-collinear magnetic system. The Hamiltonian for the anisotropic Heisenberg model smoothly interpolates between that of the isotropic Heisenberg model and the Ising model with the variation of the exchange anisotropy parameter. The orientation of the magnetization is determined by the expectation values $\langle S^x \rangle$ and $\langle S^z \rangle$ of the spin components from which the total magnetization and the orientation angle θ are obtained as a function of the temperature and of the transverse magnetic field for various parameters of the exchange anisotropy. We present the results of $\langle S^x \rangle$, $\langle S^z \rangle$, and θ obtained within Heisenberg and Ising models.

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1 Introduction

There is no spontaneous magnetization of the system at nonzero temperatures in the absence of any external magnetic field that for ferromagnetic exchange interactions between pairs of nearest neighbor spins in the two-dimensional (2D) Heisenberg spin systems [1,2]. This behavior is marked in contrast to the ferromagnetic-paramagnetic phase transition seen in the corresponding Ising spin systems with associated critical temperature T_c. However, if the Hamiltonian of the Heisenberg spin system is modified through the introduction of an anisotropy in the exchange interaction, Ising-like phase behavior is observed for sufficiently small values of the exchange interaction parameter in (x, y)-plane. But as the magnitude of the exchange anisotropy parameter is reduced to zero, the contributions to the interaction energy from the components of the spin-spin interaction in the $x$and y directions also reduce to zero. Thus the Hamiltonian for the anisotropic Heisenberg model smoothly interpolates between that of the isotropic Heisenberg model and the Ising model with the variation of the exchange anisotropy parameter.

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In the present work, we investigate the influence of the anisotropy of the exchange interaction in (x, y)-plane on the properties of the quantum spin system with the transverse magnetic field in the framework of the many-body Green function theory. From this investigation we can compare the results for the transverse and longitudinal magnetization obtained within the transverse Heisenberg and the Ising model.

The transverse Ising model was introduced for the first time by de Gennes as a pseudo-spin model for hydrogen-bonded ferroelectrics [3]. Since then the model has been applied to several physical systems, such as the cooperative Jahn-Teller systems and ferromagnets with strong uniaxial anisotropy in a transverse magnetic field [4]. The elementary spin excitation in the structurally disordered anisotropic Heisenberg model and the Ising model in the transverse field have been investigated as a special case of the anisotropic model by the Green function method in [5]. Many results about elementary excitation spectra and thermodynamics of the system were obtained within this method. Recently, the many-body Green function method has been developed for treating the reorientation of the magnetization of the ferromagnetic monolayers and thin films [6] by including the exchange anisotropy in the longitudinal direction. The purpose of this paper is to extend the treatment of [6] to the transverse Heisenberg model for the square lattice with spin S = 1 including the exchange anisotropy in (x,y) plane. The transverse Heisenberg model represents one of the simplest noncolinear magnetic systems. However, the requisite noncommutativity of operators in the Hamiltonian creates a potentially difficult technical problem. The direction of the magnetic field and that of the chosen axis do not coincidence. Correspondingly, the spectrum of elementary excitations contains a finite gap everywhere except the orientational phase transition point [7].

2 Theory

We consider the 2D Heisenberg model consisting of the anisotropic exchange interaction J between nearest neighbor lattice sites on a square lattice occupied by spin $S = 1$ and an external magnetic field, $\mathbf{H} = (H^x, 0, 0)$

$$\mathcal{H} = -\frac{1}{2}\sum_{\langle ij\rangle} J_{ij}[(1-\Lambda)S_i^- S_j^+ + S_i^z S_j^z] - \frac{1}{2}\sum_{<ij>} G_{ij} S_i^z S_j^z - \frac{1}{2}h^x \sum_i (S_i^+ + S_i^-). \quad (1)$$

The notation $S_i^{\pm} = S_i^x \pm i S_i^y$ and the external magnetic field $h^x = g\mu_B H^x$ (g is the Lande factor and μ_B is the Bohr magneton) is introduced and G represents the anisotropy constant. For G negative, the second term represents the role of the demagnetizing field [8]. We note that there is no divergence difficulty for finite values of G in calculation of the components of magnetization in the case of 2D ferromagnet. On the other hand, for G positive, the second term may represent the anisotropic exchange interaction in z direction. The parameter Λ determines the strength of the exchange anisotropy in the (x, y)-plane. Thus the Hamiltonian (1) smoothly interpolates between that of the Heisenberg model and of the Ising model changing the parameter Λ from 0 to 1, respectively.

By solving the equations of motion for the Green functions we calculate the z and x components of the magnetization directly, what allows the immediate determination of the orientation angle. We study the reorientation of the magnetization induced by the transverse external magnetic field and the influence of the exchange anisotropy on the magnetization reorientation. For

$\Lambda \to 1$ we have Ising-like model with a strength anisotropic exchange interaction in the z-direction that prefers ordering of spins in z-direction (the magnetic fluctuation is suppressed) and naturally, for the reorientation of magnetization we need higher transverse external magnetic field.

We define the following Fourier transform of the Green functions

$$G_{ij(\eta)}^{(\alpha,lm)}(\omega) = \langle\langle S_i^\alpha; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)}, \tag{2}$$

where $\alpha = +, -, z$ and η refers to the commutator ($\eta = -1$) or anticommutator ($\eta = +1$) Green functions, respectively, and i, j denote lattice sites.

The functions $G_{ij(\eta)}^{(\alpha,l)}(\omega)$ are determined from equations of motion

$$\omega G_{ij(\eta)}^{(\pm,lm)}(\omega) = A_{ij(\eta)}^{(\pm,lm)}\delta_{ij} \pm \sum_{k\neq i}[(J_{ik} + G_{ik})(\langle\langle S_k^z S_i^\pm; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)} \\ - (1-\Lambda)\langle\langle S_i^z S_k^\pm; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)})] \mp h^x \langle\langle S_i^z; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)}, \tag{3}$$

$$\omega G_{ij(\eta)}^{(z,lm)}(\omega) = A_{ij(\eta)}^{(z,lm)}\delta_{ij} + \frac{1}{2}\sum_{k\neq i} J_{ik}(1-\Lambda)\langle\langle (S_i^- S_k^+ - S_k^- S_i^+); (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)} \\ - \frac{1}{2}h^x \langle\langle S_i^+; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)} + \frac{1}{2}h^x \langle\langle S_i^-; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)}. \tag{4}$$

The inhomogeneities are given by

$$A_{ij(\eta)}^{(\alpha,lm)} = \langle [S_i^\alpha, (S_j^z)^l (S_j^-)^m]_\eta \rangle, \tag{5}$$

where angular brackets denote the canonical ensemble average.

The higher-order Green functions occurring on the right-hand side need to be decoupled in order to obtain a closed set of equations. In order to close the chain of equations we apply a generalized Tyablikov approximation - (or Random Phase Approximation (RPA), where the product of the fluctuations is neglected $(S_i^\alpha - \langle S_i^\alpha \rangle)(S_k^\beta - \langle S_k^\beta \rangle)$ at $i \neq k$)

$$\langle\langle S_i^\alpha S_k^\beta; (S_j^z)^l (S_j^-)^m \rangle\rangle_{\omega(\eta)} \cong \langle S_i^\alpha \rangle G_{kj(\eta)}^{(\beta,lm)}(\omega) + \langle S_k^\beta \rangle G_{ij(\eta)}^{(\alpha,lm)}(\omega) \tag{6}$$

with $\alpha, \beta = +, -, z;\ i \neq k$.

Additional 2D Fourier transform to the space $\boldsymbol{q} = (q_x, q_y)$, $\boldsymbol{q}$ being the in-plane wave vector, yields the following set of equations of motion

$$\Delta[\Omega(\mathbf{q})] \cdot \mathbf{g}_\eta(\mathbf{q}, \Omega) = \mathbf{A}_\eta, \tag{7}$$

where $\Omega = \omega/J$, $\mathbf{g}_\eta(q, \Omega) = J\mathbf{G}_\eta(\mathbf{q}, \Omega)$ and $\mathbf{g}_\eta(\mathbf{q}, \Omega)$, $\mathbf{A}_\eta$ are the matrices given by

$$\mathbf{g}_\eta(\mathbf{q}, \Omega) = \begin{pmatrix} g_\eta^{(+,lm)}(\mathbf{q}, \Omega) \\ g_\eta^{(-,lm)}(\mathbf{q}, \Omega) \\ g_\eta^{(z,lm)}(\mathbf{q}, \Omega) \end{pmatrix}, \quad \mathbf{A}_\eta = \begin{pmatrix} A_\eta^{(+,lm)} \\ A_\eta^{(-,lm)} \\ A_\eta^{(z,lm)} \end{pmatrix} \tag{8}$$

and

$$\Delta[\Omega(\mathbf{q})] = \left(\begin{array}{lll} \Omega - A & 0 & C \\ 0 & \Omega + A & -C \\ \frac{1}{2}B & -\frac{1}{2}B & \Omega \end{array} \right), \tag{9}$$

where we have used

$$\begin{array}{l} A = \langle S^z \rangle [4(1+\Gamma) - (1-\Lambda)\gamma_{\mathbf{q}}], \\ B = \chi^x + \langle S^x \rangle (1-\Lambda)[4-\gamma_{\mathbf{q}}], \\ C = \chi^x + \langle S^x \rangle [4(1-\Lambda) - (1+\Gamma)\gamma_{\mathbf{q}}], \\ \gamma_{\mathbf{q}} = 2(\cos q_x + \cos q_y), \Gamma = G/J, \chi^x = h^x/J. \end{array} \tag{10}$$

From (7), the Green functions $g_\eta^{(+,lm)}(\mathbf{q},\Omega)$, $g_\eta^{(-,lm)}(\mathbf{q},\Omega)$, and $g_\eta^{(z,lm)}(\mathbf{q},\Omega)$ are

$$g_\eta^{(\alpha,lm)}(\mathbf{q},\Omega) = \frac{|\Delta_\eta^{(\alpha,lm)}[\Omega(\mathbf{q})]|}{|\Delta[\Omega(\mathbf{q})]|}, \tag{11}$$

where $|\Delta_\eta^{(\alpha,lm)}[\Omega(\mathbf{q})]|$ is the determinant obtained by replacing the column α of the determinant $|\Delta[\Omega(\mathbf{q})]|$ by matrix $\mathbf{A}_\eta$. The Green functions $g_\eta^{(\alpha,lm)}(\mathbf{q},\Omega)$ have poles Ω_i that can be obtained by solving $|\Delta[\Omega(\mathbf{q})]| = 0$: $\Omega_1 = 0, \Omega_2 = \Omega_{\mathbf{q}}, \Omega_3 = -\Omega_{\mathbf{q}}, \Omega_{\mathbf{q}} = \sqrt{A^2 + BC}$.

In this case, $g_\eta^{(\alpha,lm)}(\mathbf{q},\Omega)$ can be expressed as

$$g_\eta^{(\alpha,lm)}(\mathbf{q},\Omega) = \frac{R_\eta^{(\alpha,lm)}\{\Omega_1(\mathbf{q})\})}{\Omega - \Omega_1(\mathbf{q})} + \frac{R_\eta^{(\alpha,lm)}\{\Omega_2(\mathbf{q})\}}{\Omega - \Omega_2(\mathbf{q})} + \frac{R_\eta^{(\alpha,lm)}\{\Omega_3(\mathbf{q})\}}{\Omega - \Omega_3(\mathbf{q})}, \tag{12}$$

where

$$R_\eta^{(\alpha,lm)}\{\Omega_i(\mathbf{q})\} = \frac{|\Delta_\eta^{(\alpha,lm)}[\Omega_i(\mathbf{q})]|}{\prod\limits_{j \neq i} [\Omega_i(\mathbf{q}) - \Omega_j(\mathbf{q})]}. \tag{13}$$

In order to calculate the correlation functions $C_{\mathbf{q}}^{(\alpha,lm)} = \langle (S^z)^l (S^-)^m S^\alpha \rangle_{\mathbf{q}}$ in the case of a vanishing eigenvalue momentum space, we use the spectral theorem [9]

$$C_{\mathbf{q}}^{(\alpha,lm)} = \frac{i}{2\pi} \lim_{\varepsilon \to 0} \int\limits_{-\infty}^{\infty} \frac{1}{e^{\Omega/T^*} + 1} \left(g_{\eta=+1}^{(\alpha,lm)}(\mathbf{q}, \Omega + i\varepsilon) - g_{\eta=+1}^{(\alpha,lm)}(\mathbf{q}, \Omega - i\varepsilon) \right), \tag{14}$$

where $T^* = kT/J$. Using the relation between the anticommutator and the commutator

$$A_{\mathbf{q},\eta=+1}^{(\alpha,lm)} = A_{\mathbf{q},\eta=-1}^{(\alpha,lm)} + 2C_{\mathbf{q}}^{(\alpha,lm)} \tag{15}$$

one obtains the following set of equations

$$C_{\mathbf{q}}^{(+,lm)} - \frac{C}{A} C_{\mathbf{q}}^{(z,lm)} = A_{\eta=-1}^{(+,lm)} \left(\frac{\Omega_{\mathbf{q}}}{2A} \coth \left(\frac{\Omega_{\mathbf{q}}}{2T^*} \right) - \frac{1}{2} \right) + \frac{C}{2A} A_{\eta=-1}^{(z,lm)}, \tag{16}$$

$$-C_{\mathbf{q}}^{(-,lm)} + \frac{C}{A} C_{\mathbf{q}}^{(z,lm)} = A_{\eta=-1}^{(-,lm)} \left(\frac{\Omega_{\mathbf{q}}}{2A} \coth \left(\frac{\Omega_{\mathbf{q}}}{2T^*} \right) + \frac{1}{2} \right) - \frac{C}{2A} A_{\eta=-1}^{(z,lm)}, \tag{17}$$

$$C_{\mathbf{q}}^{(+,lm)} - C_{\mathbf{q}}^{(-,lm)} = \frac{1}{2}(A_{\eta=-1}^{(-,lm)} - A_{\eta=-1}^{(+,lm)}) - \frac{\Omega_{\mathbf{q}}}{B} \coth\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) A_{\eta=-1}^{(z,lm)}. \tag{18}$$

The relations (16) - (18) are derived in detail in Appendix A.

Now we apply the following Fourier transform in Eqs. (16) and (17)

$$\begin{aligned} &\langle (S^z)^l (S^-)^m S^\alpha \rangle = \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi C_{\mathbf{q}}^{(\alpha,lm)} dq_x dq_y, \\ &\Phi_1(T^*) = \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi \tfrac{\Omega_{\mathbf{q}}}{A} \coth(\Omega_{\mathbf{q}}/2T^*) dq_x dq_y, \end{aligned} \tag{19}$$

and obtain

$$\begin{aligned} &C^{(+,lm)} - \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi \tfrac{C}{A} C_{\mathbf{q}}^{(z,lm)} dq_x dq_y = -\tfrac{1}{2} A_{\eta=-1}^{(+,lm)} \\ &+\tfrac{1}{2} A_{\eta=-1}^{(+,lm)} \Phi_1(T^*) + \tfrac{1}{2} A_{\eta=-1}^{(z,lm)} \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi \tfrac{C}{A} dq_x dq_y, \end{aligned} \tag{20}$$

$$\begin{aligned} &-C^{(-,lm)} + \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi \tfrac{C}{A} C_{\mathbf{q}}^{(z,lm)} dq_x dq_y = \tfrac{1}{2} A_{\eta=-1}^{(-,lm)} \\ &+\tfrac{1}{2} A_{\eta=-1}^{(-,lm)} \Phi_1(T^*) - \tfrac{1}{2} A_{\eta=-1}^{(z,lm)} \tfrac{1}{\pi^2} \int_0^\pi \int_0^\pi \tfrac{C}{A} dq_x dq_y. \end{aligned} \tag{21}$$

By adding Eqs. (20) and (21), one obtains

$$C^{(+,lm)} - C^{(-,lm)} - \frac{1}{2}\left(A_{\eta=-1}^{(-,lm)} - A_{\eta=-1}^{(+,lm)}\right) = \frac{1}{2}\left(A_{\eta=-1}^{(-,lm)} + A_{\eta=-1}^{(+,lm)}\right) \Phi_1(T^*). \tag{22}$$

The Fourier transform of Eq. (18) gives

$$C^{(+,lm)} - C^{(-,lm)} - \frac{1}{2}\left(A_{\eta=-1}^{(-,lm)} - A_{\eta=-1}^{(+,lm)}\right) = -A_{\eta=-1}^{(z,lm)} \Phi_2(T^*), \tag{23}$$

where

$$\Phi_2(T^*) = \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \frac{\Omega_{\mathbf{q}}}{B} \coth(\Omega_{\mathbf{q}}/2T^*) dq_x dq_y. \tag{24}$$

Eqs. (22) and (23) are sufficient to determine the observables. To elucidate these equations we derive the explicit expressions for spins S = 1. For this spin one needs Eqs. (22) and (23) for (l= 0, m = 1), (l= 1, m = 1), (l= 0, m = 2) and (l= 0, m = 3). This yields 8 equations for ten unknown quantities: $\langle S^-\rangle$, $\langle S^z\rangle$, $\langle (S^z)^2\rangle$, $\langle S^-S^-\rangle$, $\langle S^zS^-\rangle$, $\langle S^zS^-S^-\rangle$, $\langle S^-S^z\rangle$, $\langle S^-(S^z)^2\rangle$, $\langle S^-S^-S^z\rangle$, and $\langle S^-S^-(S^z)^2\rangle$.

Only 6 (out of the ten) unknown quantities are independent: $\{\langle XY\rangle\} \equiv \{\langle S^-\rangle$, $\langle S^z\rangle$, $\langle (S^z)^2\rangle$, $\langle (S^-)^2\rangle$, $\langle S^zS^-\rangle$, $\langle S^-(S^z)^2\rangle\}$. They are determined by the following six equations

$$2 - \langle (S^z)^2 - \langle (S^-)^2\rangle = \begin{cases} \langle S^z\rangle \Phi_1(T^*), \\ \langle S^-\rangle \Phi_2(T^*), \end{cases} \tag{25}$$

$$\frac{1}{2}(\langle S^z\rangle + \langle (S^z)^2\rangle + \langle (S^-)^2\rangle - 2) = \left\{ \begin{array}{l} \frac{1}{2}(\langle (S^-)^2\rangle + 3\langle (S^z)^2 - \langle S^z\rangle - 2)\Phi_1(T^*), \\ \langle S^z S^-\rangle \Phi_2(T^*), \end{array} \right. \tag{26}$$

$$2\langle S^-\rangle - \langle (S^-)^2\rangle - \langle S^-(S^z)^2\rangle = \left\{ \begin{array}{l} (2\langle S^z S^-\rangle - \langle S^-\rangle)\Phi_1(T^*), \\ 2\langle (S^-)^2\rangle \Phi_2(T^*). \end{array} \right. \tag{27}$$

From these equations one obtains the following expressions for expectation values $\langle S^-\rangle$ and $\langle S^z\rangle$

$$\langle S^z\rangle = \frac{4\Phi_1\Phi_2^2}{\Phi_1^2 + \Phi_2^2 + 3\Phi_1^2\Phi_2^2}, \qquad \langle S^-\rangle = \frac{4\Phi_1^2\Phi_2^2}{\Phi_1^2 + \Phi_2^2 + 3\Phi_1^2\Phi_2^2}. \tag{28}$$

These equations have to be solved numerically. The integrals Φ_i (i = 1, 2) are actually rather complicated because the detailed shape of the Brillouine zone must be taken into account and the actual energy of magnons must be used. To reduce the necessary computational time, the trick described by Colpa [10] has been used, and the double integrals $\Phi_i(T^*)$ $(i = 1, 2)$ can be rewritten into only one integral

$$\begin{aligned} \Phi_i(T^*) &= \frac{1}{\pi^2}\int_0^{\pi}\int_0^{\pi} f_i(\Omega_{\mathbf{q}})\coth(\Omega_{\mathbf{q}}/2T^*)dq_x dq_y \\ &= \frac{1}{\pi^2}\int_{-2}^{2} f_i(\Omega_{\mathbf{q}})\coth\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) K_{ell}\left(\frac{4-\kappa^2}{4}\right)d\kappa, \end{aligned} \tag{29}$$

where K_{ell} is the elliptic integral of the first kind, $\kappa = \cos q_x + \cos q_y$, $f_1 = \Omega_{\mathbf{q}}/A$, and $f_2 = \Omega_{\mathbf{q}}/B$.

We restricted ourselves to an external transverse magnetic field confined to the $x-$direction. Therein, it is sufficient to deal with the x- and z- components of the magnetization ($\langle S^y\rangle = 0$). The total magnetization $m(T^*)$ and the equilibrium polar angle ϑ of the magnetization are determined

$$m(T^*) = \sqrt{\langle S^x\rangle^2 + \langle S^z\rangle^2}, \qquad \vartheta = \arctan\frac{\langle S^x\rangle}{\langle S^z\rangle}. \tag{30}$$

3 Results

We study the behaviour of the longitudinal $\langle S^z\rangle$ and the transverse $\langle S^x\rangle$ components of the magnetization and the polar angle of the total magnetization ϑ with h^x and T for the square lattice with spin S = 1 for two typical values of the anisotropy parameter Λ: a) $\Lambda = 0$ (the Heisenberg model), b) $\Lambda = 1$ (the Ising model). Figure 1 shows typical results for $\langle S^z\rangle$ and $\langle S^x\rangle$ and the polar angle of the total magnetization ϑ as a functions of the temperature in the case when the applied transverse field is $\chi^x \equiv h^x/J = 0.08$. The exchange anisotropy parameter $\Gamma = 0.1$. Indices 1 and 2 refer to the cases a) and b), respectively. It is clear from the figure that the longitudinal component of the magnetization $\langle S^z\rangle$ vanishes at the reorientation temperature T_{R}. The transverse component $\langle S^x\rangle$ remains constant until the component $\langle S^z\rangle$ has dropped to

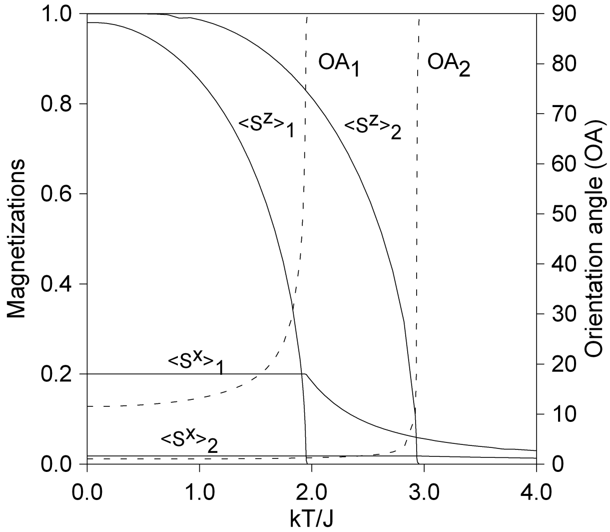

Fig. 1. The components of the magnetization $\langle S^x \rangle$ and $\langle S^z \rangle$ and the equilibrium orientation angle ϑ of the total magnetization as functions of the reduced temperature *kT/J* for a fixed transverse magnetic filed $\chi^x = 0.08$, for the anisotropy parameters Γ = 0.1 when a) $\Lambda = 0.0$ (index 1, the Heisenberg model), b) $\Lambda = 1.0$ (index 2, the Ising model).

zero, and an in-plane magnetization ($\vartheta = 90$) is reached and then it decreases with a long tail in the Heisenberg model. On the other hand, in the Ising model the transverse component $\langle S^x \rangle$ decreases very slowly. For the same applied magnetic field, the reorientation temperature $T_{\rm R}$ is considerably higher in the case $\Lambda = 1$ (Ising model). The result obtained is qualitatively clear: the reorientation temperature increases with Λ. (In the Ising limit, $\Lambda = 1$, the RPA corresponds to the mean filed treatment and the mean filed Curie temperature is higher than the RPA Curie temperature.) We emphasize the long tail in particular of the transverse $\langle S^x \rangle$ components of the magnetization at the large temperatures within the Heisenberg model, which is absent within the Ising model. This behaviour is due to the strong effect of external magnetic fields on the properties of two-dimensional Heisenberg magnets [11]

In Fig. 2 the transverse magnetization $\langle S^x \rangle$ is plotted for Γ = 0.1 and at fixed transverse magnetic field: a)$\chi^x = 0.15$ when $\Lambda = 0.0$ (index 1, the Heisenberg model), b) $\chi^x = 1.5$ when $\Lambda = 1.0$ (index 2, the Ising model). Fluctuations modify the behaviour of the system, so that the transition point shifts and the character of the magnetization behaviour changes. The reorientation temperature calculated within the Heisenberg model and the Ising model are fairly similar (of course, without long tail for the higher temperatures in the Ising model) when the transverse field in the Ising model is 10 times higher than in the Heisenberg model.

In Figure 3 we plot the components of the magnetization, $\langle S^z \rangle$, $\langle S^x \rangle$ and the polar angle of the total magnetization ϑ as functions of the transverse magnetic field $\chi^x \equiv h^x/J$ at the reduced temperature *kT/ J* = 1.0 for the anisotropy parameter Λ = 0.0 and for the anisotropy parameter G/J = 0.1. The field-induced magnetic reorientation is characterized by decreasing $\langle S^z \rangle$ and increasing $\langle S^x \rangle$. The magnetization reaches the in-plane direction ($\langle S^z \rangle$ = 0) at field

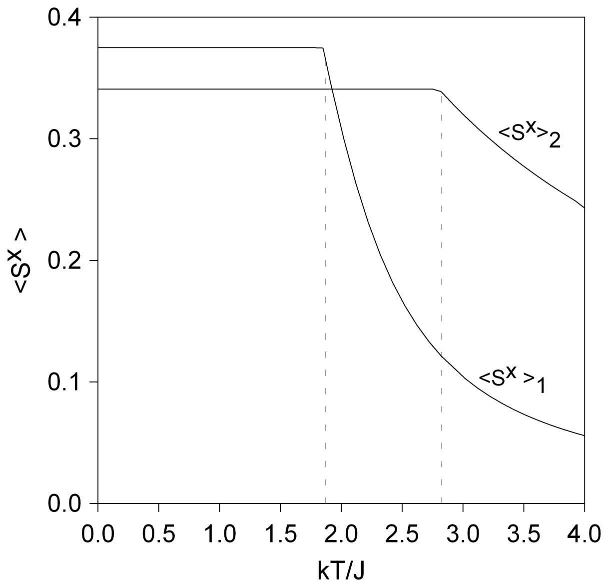

Fig. 2. The transverse magnetization $\langle S^x \rangle$ for Γ = 0.1 and fixed transverse magnetic filed: a)$\chi^x = 0.15$ when a) $\Lambda = 0.0$ (index 1, the Heisenberg model), b) $\chi^x = 1.5$ when $\Lambda = 1.0$ (index 2, the Ising model) are shown as a function of the reduced temperature *kT/J*.

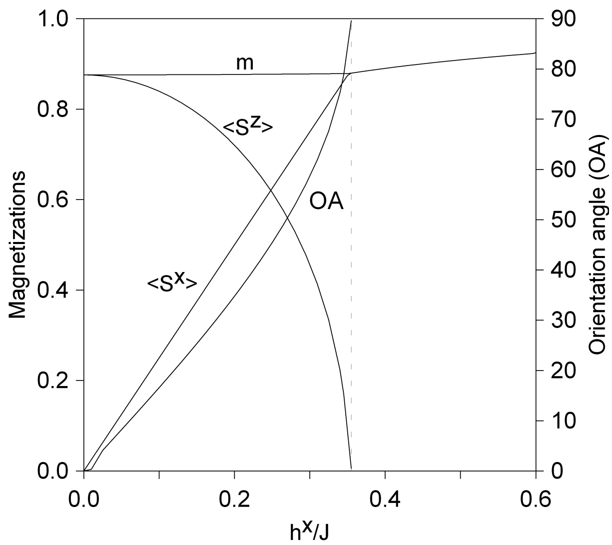

Fig. 3. The components of the magnetization $\langle S^x \rangle$ and $\langle S^z \rangle$ and the equilibrium reorientation angle ϑ as functions of the reduced transverse magnetic field h^x/J for $\Gamma = 0.1$ at a fixed reduced temperature *kT/J* = 1.0, when $\Lambda = 0.0$ (the Heisenberg model).

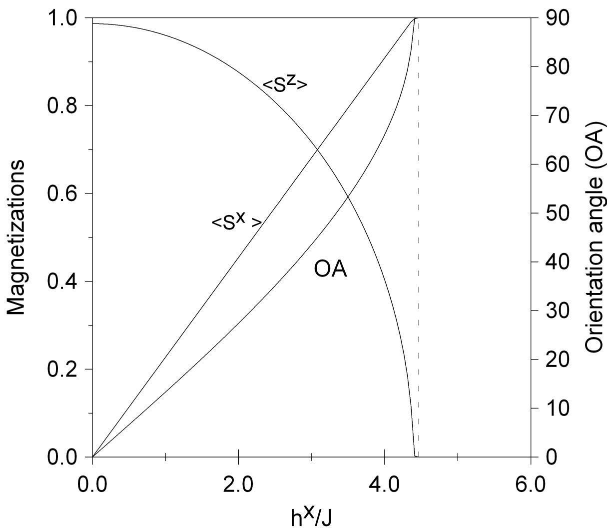

Fig. 4. As in Fig. 3, but for the Ising model.

strength h_R^x depending on the temperature. In Fig. 4 we show the corresponding results obtained within the Ising model (Λ = 1) with the same parameters for *kT/J* and for G/J as in the Fig. 3. The behaviour of the magnetizations in both cases are qualitatively fairly similar. However, the strength of the reorientation field h_R^x calculated within Ising model [12] is ten times larger than within the Heisenberg model.

As follows from Fig. 5, quantum effects $(\Lambda \to 0)$ shift the strength of the reorientation magnetic field to smaller values. The curves refer to different values of parameter Λ: curve a: $\Lambda = 0.0$; curve b: $\Lambda = 0.5$ and curve c: $\Lambda = 1.0$. Thus, this contracts the region where the ordered phase in the z direction exists, and leads to a nontrivial quantum renormalization of the critical field value h_R^x [13].

4 Conclusion

In the present paper, we have applied Green function theory for the calculation of the magnetic properties of the ferromagnetic monolayer with an anisotropy in the exchange interaction in the (x, y)-plane with applied a transverse external magnetic field. Considered model smoothly interpolates between that of the Heisenberg model and the Ising model with the variation of the exchange anisotropy parameter Λ. We have investigated the square lattice for spin $S = 1$ and calculated the longitudinal and transverse components of the magnetization, which allows an immediate determination of the orientation angle. The results are sensitive to a variation of the strength of the anisotropy in the exchange interaction. The magnitude of the reorientation temperature $T_{\rm R}$ and the reorientation magnetic field $\chi_{\rm R}^x$ increase when the anisotropy parameter Λ increases. In Ising-like model, both quantities are larger than for the Heisenberg model. Comparable result for the reorientation temperature within both models we obtain when the transverse

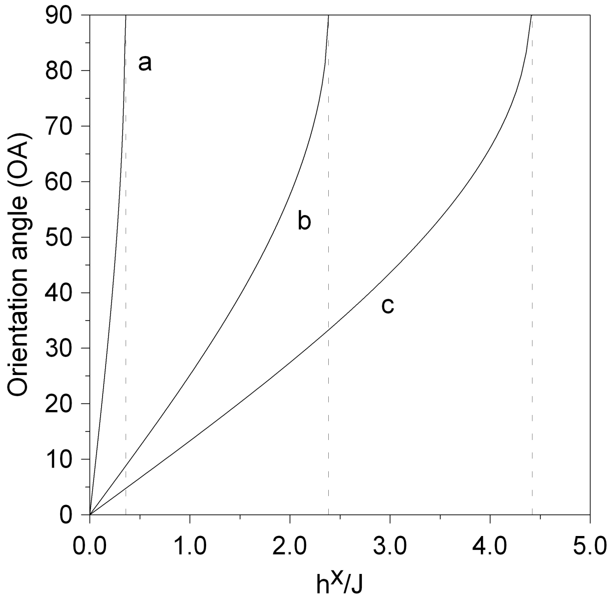

Fig. 5. The equilibrium reorientation angle ϑ as a function of the reduced transverse magnetic field h^x/J at the reduced fixed temperature *kT/J* = 1.0 for $\Gamma = 0.1$. The curves refer to different values of the parameter Λ: curve a: Λ = 0.0; curve b: Λ = 0.5 and curve c: Λ = 1.0.

magnetic field is just ten times greater for the Ising model than for the Heisenberg model. Quantum corrections modify the behaviour of the system, so that the transition point shifts and the character of the magnetization behaviour changes.

Appendix

In this Appendix we derive the relations (16) – (18) in detail. Using (12), (13) and (14), one obtains for the correlation function $C_{\mathbf{q}}^{(z,lm)}$

$$C_{\mathbf{q}}^{(z,lm)} = \sum_{i=1}^{3} \frac{R_{\eta=+1}^{(z,lm)}(\Omega_i)}{e^{\Omega_i/T^*}+1} = \sum_{i=1}^{3} \frac{1}{e^{\Omega_i/T^*}+1} \frac{|\Delta_{\eta=+1}^{(z,lm)}(\Omega_i)|}{\prod\limits_{j\neq i}([\Omega_i - \Omega_j)}, \tag{A1}$$

where $\Omega_1 = 0$, $\Omega_2 = \Omega_{\mathbf{q}}$, $\Omega_3 = -\Omega_{\mathbf{q}}$, $\Omega_{\mathbf{q}} = \sqrt{A^2+BC}$. For the determinant $\left|\Delta_{\eta=+1}^{(z,lm)}(\Omega_i)\right|$ we get

$$\begin{aligned} \left|\Delta_{\eta=+1}^{(z,lm)}(\Omega_i)\right| &= \begin{vmatrix} \Omega_i - A & 0 & A_{\eta=+1}^{(+,lm)} \\ 0 & \Omega_i + A & A_{\eta=+1}^{(-,lm)} \\ \frac{1}{2}B & -\frac{1}{2}B & A_{\eta=+1}^{(z,lm)} \end{vmatrix} \\ &= (A_{\eta=-1}^{(z,lm)} + 2C_{\mathbf{q}}^{(z,lm)})(\Omega_i^2 - A^2) \end{aligned} \tag{A2}$$

$$
\begin{aligned}
&+ \quad \frac{B}{2}(A_{\eta=-1}^{(-,lm)} + 2C_{\mathbf{q}}^{(-,lm)})(\Omega_i - A) \\
&- \quad \frac{B}{2}(A_{\eta=-1}^{(+,lm)} + 2C_{\mathbf{q}}^{(+,lm)})(\Omega_i + A),
\end{aligned}
$$

where we have used (15). If we insert (A2) into (A1), the correlation function $C_{\mathbf{q}}^{(z,lm)}$ can be written as

$$
\begin{aligned}
C_{\mathbf{q}}^{(z,lm)} &= \sum_{i=1}^{3} \frac{1}{e^{\Omega_i/T^*}+1} \frac{|\Delta_{\eta=+1}^{(z,lm)}(\Omega_i)|}{\prod\limits_{j\neq i}(\Omega_i - \Omega_j)} \\
&= C_{\mathbf{q}}^{(z,lm)} - C_{\mathbf{q}}^{(-,lm)} \frac{B}{2\Omega_{\mathbf{q}}} \tanh\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) + C_{\mathbf{q}}^{(+,lm)} \tanh\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) \qquad \text{(A3)} \\
&- A_{\eta=-1}^{(-,lm)} \frac{B}{4\Omega_{\mathbf{q}}} \tanh\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) + \frac{1}{2} A_{\eta=-1}^{(z,lm)} + A_{\eta=-1}^{(+,lm)} \frac{B}{4\Omega_{\mathbf{q}}} \tanh\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right).
\end{aligned}
$$

After small manipulation, we find

$$
C_{\mathbf{q}}^{(+,lm)} - C_{\mathbf{q}}^{(-,lm)} = \frac{1}{2}(A_{\eta=-1}^{(-,lm)} - A_{\eta=-1}^{(+,lm)}) - \frac{\Omega_{\mathbf{q}}}{B} \coth\left(\frac{\Omega_{\mathbf{q}}}{2T^*}\right) A_{\eta=-1}^{(z,lm)}, \qquad \text{(A4)}
$$

which corresponds to Eq. (18).

Likewise we derive the relation (17). Using (12), (13) and (14), one obtains the correlation function

$$
C_{\mathbf{q}}^{(-,lm)} = \sum_{i=1}^{3} \frac{R_{\eta=+1}^{(-,lm)}(\Omega_i)}{e^{\Omega_i/T^*}+1} = \sum_{i=1}^{3} \frac{1}{e^{\Omega_i/T^*}+1} \frac{|\Delta_{\eta=+1}^{(-,lm)}(\Omega_i)|}{\prod\limits_{j\neq i}(\Omega_i - \Omega_j)}. \qquad \text{(A5)}
$$

For the determinant $\left|\Delta_{\eta=+1}^{(-,lm)}(\Omega_i)\right|$ we obtain

$$
\begin{aligned}
\left|\Delta_{\eta=+1}^{(-,lm)}(\Omega_i)\right| &= \begin{vmatrix} \Omega_i - A & A_{\eta=+1}^{(+,lm)} & C \\ 0 & A_{\eta=+1}^{(-,lm)} & -C \\ \frac{1}{2}B & A_{\eta=+1}^{(z,lm)} & \Omega_i \end{vmatrix} \\
&= (A_{\eta=-1}^{(z,lm)} + 2C_{\mathbf{q}}^{(z,lm)})C(\Omega_i^{-}A) + (A_{\eta=-1}^{(-,lm)} + 2C_{\mathbf{q}}^{(-,lm)}) \qquad \text{(A6)} \\
&\times \quad [\Omega_i(\Omega_i - A) - \frac{CB}{2}] - \frac{CB}{2}(A_{\eta=-1}^{(+,lm)} + 2C_{\mathbf{q}}^{(+,lm)}).
\end{aligned}
$$

We obtain Eq. (17) by inserting (A6) into (A5).

Finally, using Eqs. (12), (13) and (14), one obtains the correlation function

$$
C_{\mathbf{q}}^{(+,lm)} = \sum_{i=1}^{3} \frac{R_{\eta=+1}^{(+,lm)}(\Omega_i)}{e^{\Omega_i/T^*}+1} = \sum_{i=1}^{3} \frac{1}{e^{\Omega_i/T^*}+1} \frac{|\Delta_{\eta=+1}^{(+,lm)}(\Omega_i)|}{\prod\limits_{j\neq i}(\Omega_i - \Omega_j)}. \qquad \text{(A7)}
$$

For the determinant $\left|\Delta^{(+,lm)}_{\eta=+1}(\Omega_i)\right|$ we get

$$\begin{aligned} \left|\Delta^{(+,lm)}_{\eta=+1}(\Omega_i)\right| &= \begin{vmatrix} A^{(+,lm)}_{\eta=+1} & 0 & C \\ A^{(-,lm)}_{\eta=+1} & \Omega_i + A & -C \\ A^{(z,lm)}_{\eta=+1} & -\frac{1}{2}B & \Omega_i \end{vmatrix} \\ &= -(A^{(z,lm)}_{\eta=-1} + 2C^{(z,lm)}_{\mathbf{q}})C(\Omega_i^+ A) \\ &- \ (A^{(-,lm)}_{\eta=-1} + 2C^{(-,lm)}_{\mathbf{q}})\frac{CB}{2} \\ &+ \ [(\Omega_i(\Omega_i + A) - \frac{CB}{2}](A^{(+,lm)}_{\eta=-} + 2C^{(+,lm)}_{\mathbf{q}}). \end{aligned} \quad \text{(A8)}$$

Inserting (A8) into (A7) yields the relation (16).

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