

**PION NUCLEON COUPLING CONSTANT, GOLDBERGER-TREIMAN  
DISCREPANCY AND  $\pi N \sigma$  TERM**

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Received 12 July 2004, accepted 17 September 2004

We start by studying the Goldberger-Treiman discrepancy (GTd)  $\Delta = (2.259 \pm 0.591)\%$ . Then we look at the  $\pi N \sigma$  term, with the dimensionless ratio  $\sigma_N/2m_N = 3.35\%$ . Finally we return to predicting (via the quark model) the  $\pi N$  coupling constant, with GTd  $\Delta \rightarrow 0$  as  $m_q \rightarrow m_N/3$ .

PACS: 12.39.Ki, 12.39.Mk, 13.25.-k, 14.40.-n

Given the recent new value of the  $\pi NN$  coupling constant [1]

$$g_{\pi NN}^2/4\pi = 13.80 \pm 0.12 \quad \text{or} \quad g_{\pi NN} = 13.169 \pm 0.057, \quad (1)$$

along with the observed axial current coupling [2]

$$g_A = 1.267 \pm 0.004, \quad (2)$$

combined with the measured pion decay constant [2]

$$f_\pi = (92.42 \pm 0.26)\text{MeV}, \quad (3)$$

the Goldberger-Treiman discrepancy (GTd) is then

$$\Delta = 1 - \frac{m_N g_A}{f_\pi g_{\pi NN}} = (2.259 \pm 0.591)\%. \quad (4)$$

Here we have used the mean nucleon mass  $m_N = 938.9$  MeV and have computed the overall mean square error.

To verify this GTd in Eq. (4), we employ the constituent quark loop with imaginary part [3]

$$\text{Im}f_\pi(q^2) = \frac{3g_{\pi qq}}{2} \frac{4\hat{m}}{8\pi} \left(1 - \frac{4\hat{m}^2}{q^2}\right)^{1/2} \Theta(q^2 - 4\hat{m}^2). \quad (5)$$

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This follows from unitarity with the inclusion of a factor of 3 from colour. Following ref. [3] using the quark level Goldberger-Treiman relation (GTr)  $f_\pi g_{\pi qq} = \hat{m}$ , the GTd to fourth order in  $q'^2$  predicts for a once-subtracted dispersion relation assuming a quark-level GTr:

$$\frac{f_\pi(q^2) - f_\pi(0)}{f_\pi(0)} = \frac{q^2}{\pi} \int_{4\hat{m}^2}^{\infty} \frac{dq'^2 \left(1 - \frac{4\hat{m}^2}{q'^2}\right)^{1/2}}{q'^2(q'^2 - q^2)} \frac{3g_{\pi qq}^2}{4\pi}, \quad (6)$$

or for  $q^2 = m_\pi^2$ , the integral in Eq. (6) gives a discrepancy for  $f_\pi$

$$\bar{\Delta} = \frac{f_\pi(m_\pi^2)}{f_\pi(0)} - 1 = \frac{3g_{\pi qq}^2}{2\pi^2} \left[1 - r \tan^{-1} \left(\frac{1}{r}\right)\right] \quad (7)$$

for  $r^2 = \frac{4\hat{m}^2}{m_\pi^2} - 1 \geq 0$ . Since  $m_\pi^2/4\hat{m}^2 \ll 1$ , a Taylor series expansion leads to

$$1 - r \tan^{-1} \left(\frac{1}{r}\right) = \frac{1}{3r^2} - \frac{1}{5r^4} + \dots = \frac{m_\pi^2}{12\hat{m}^2} \left(1 + \frac{1}{10} \frac{m_\pi^2}{\hat{m}^2} + \dots\right)$$

and a discrepancy [4]

$$\bar{\Delta} = \frac{f_\pi(m_\pi^2)}{f_\pi(0)} - 1 = \frac{m_\pi^2}{8\pi^2 f_\pi^2} \left(1 + \frac{1}{10} \frac{m_\pi^2}{\hat{m}^2}\right) \approx 2.946\%. \quad (8)$$

The first term on the rhs is independent of  $\hat{m}$ , while in the small second term we take  $\hat{m} = m_N/3$ . This then leads to a net 2.946% correction in Eq. (8).

Since the physical GTr becomes exact ( $f_\pi g_{\pi NN} = m_N g_A$ ) when  $m_\pi \rightarrow 0$  for a conserved axial current, it should not be surprising that the measured GTd in Eq. (4) of  $(2.259 \pm 0.591)\%$  is within 1.16 standard deviations from the dispersion-theoretical GTd  $\bar{\Delta} = 2.946\%$  in Eq. (8). Appreciate that  $g_A$  is measured at  $q^2 = 0$  while  $f_\pi$  is measured at  $q^2 = m_\pi^2$  but  $f_\pi(0)$  is inferred at  $q^2 = 0$  via Eq. (8).

Just as the chiral-breaking  $SU(2)$  GTd is 2–3%, the  $SU(2) \times SU(2)$   $\pi N$   $\sigma$  term of 63 MeV corresponds to a dimensionless ratio of about 3%:

$$\frac{\sigma_N}{2m_N} = \frac{63 \text{ MeV}}{2 \times 938.9 \text{ MeV}} \approx 3.35\%. \quad (9)$$

Alternatively the chiral-limiting (CL) nucleon mass is related to the  $\pi N$   $\sigma$  term as [5]

$$m_N^2 = (m_N^{CL})^2 + m_N \sigma_N, \quad \text{or with } \sigma_N = 63 \text{ MeV}, \quad (10)$$

$$\frac{m_N}{m_N^{CL}} - 1 = 3.53\%, \quad \text{with } m_N^{CL} = 906.85 \text{ MeV}. \quad (11)$$

Note the many 3% CL relations in Eqs. (4), (8), (9), (11) above. Now we justify the  $\sigma$  term  $\sigma_N = 63 \text{ MeV}$ .

The explicit  $SU(2) \times SU(2)$  chiral-breaking  $\sigma$  term is the sum of the perturbative GMOR [6] or quenched APE [7] part

$$\sigma_N^{\text{GMOR}} = (m_\Xi + m_\Sigma - 2m_N) \frac{m_\pi^2}{m_K^2 - m_\pi^2} = 26 \text{ MeV}, \quad (12)$$

$$\sigma_N^{APE} = (24.5 \pm 2) \text{ MeV}, \quad (13)$$

plus the nonperturbative linear  $\sigma$  model (L $\sigma$ M) nonquenched part [8] due to  $\sigma$  tadpoles for the chiral-broken  $m_\pi^2$  and  $\sigma_N$ , with ratio predicting

$$\sigma_N^{L\sigma M} = \left(\frac{m_\pi}{m_\sigma}\right)^2 m_N \approx 40 \text{ MeV} \quad (14)$$

for  $m_\sigma \approx 665 \text{ MeV}$  [9], a model-independent coupled channel dispersion theory and parameter-free relation. Specifically, Eq. (14) stems from semi-strong L $\sigma$ M tadpole graphs generating  $\sigma_N$  and  $m_\pi^2$ . Their ratio cancels out the  $\langle \sigma | H_{ss} | 0 \rangle$  factor. The L $\sigma$ M couplings  $2g_{\sigma\pi\pi} = m_\sigma^2/f_\pi$  and  $f_\pi g_{\sigma NN} = m_N$  then give  $\sigma_N^{L\sigma M} = (m_\pi/m_\sigma)^2 m_N$  as found in Eq. (14). Since the  $\sigma(600)$  has been observed [2], with a broad width, but the central model-independent value [9] is known to be 665 MeV, the chiral L $\sigma$ M mass ratio in Eq. (14) is expected to be quite accurate - while being free of model-dependent parameters. The authors of [10] find the  $\sigma$  meson between 400 MeV and 900 MeV, with the average mass 650 MeV near 665 MeV from [9]. Then the sum of (12,13) plus (14) is

$$\sigma_N = \sigma_N^{GMOR,APE} + \sigma_N^{L\sigma M} \approx (25 + 40) \text{ MeV} = 65 \text{ MeV}. \quad (15)$$

Rather than add the perturbative plus nonperturbative parts as in Eq.(15), one can instead work in the infinite momentum frame (IMF) requiring squared masses [11] and only one term (tadpole terms  $\rightarrow 0$  in the IMF) [12]

$$\sigma_N^{IMF} = \frac{m_\Xi^2 + m_\Sigma^2 - 2m_N^2}{2m_N} \left( \frac{m_\pi^2}{m_K^2 - m_\pi^2} \right) = 63 \text{ MeV}. \quad (16)$$

Note that Eqs. (15) and (16) are both very near the observed value [13]  $(65 \pm 5) \text{ MeV}$ .

With hindsight, we can also deduce the  $\pi N \sigma$  term via PCAC (partially conserved axial current) at the Cheng-Dashen (CD) point [14] with background isospin-even  $\pi N$  amplitude

$$\bar{F}^+(\nu = 0, t = 2m_\pi^2) = \sigma_N/f_\pi^2 + O(m_\pi^4). \quad (17)$$

At this CD point, a recent Karlsruhe data analysis by G. Höhler [13] finds

$$\bar{F}^+(0, 2m_\pi^2) = \sigma_N/f_\pi^2 + 0.002m_\pi^{-1} = 1.02m_\pi^{-1}, \quad (18)$$

implying  $\sigma_N = 63 \text{ MeV}$  for  $f_\pi = 93 \text{ MeV}$ ,  $m_\pi = 139.57 \text{ MeV}$ .

We can unify the earlier parts of this paper by first inferring from Eq. (8) the chiral limit (CL) pion decay constant

$$f_\pi^{CL} = f_\pi/1.02946 \approx 89.775 \text{ MeV} \quad (19)$$

using Eq. (8) and the observed [2]  $f_\pi = (92.42 \pm 0.26) \text{ MeV}$ . Then the quark-level GTr using the meson-quark coupling  $g = 2\pi/\sqrt{3}$  [15] predicts the nonstrange quark mass in the CL as <sup>4</sup>

$$\hat{m}^{CL} = f_\pi^{CL} g = 325.67 \text{ MeV}, \quad (20)$$

<sup>4</sup>Also see L $\sigma$ M ref. [8] and the  $Z = 0$  compositness condition [16].

close to the expected  $\hat{m}^{CL} = m_N/3 \approx 313$  MeV. This in turn predicts the scalar  $\sigma$  mass in the CL as [8, 17]

$$m_\sigma^{CL} = 2\hat{m}^{CL} = 651.34 \text{ MeV} \quad (21)$$

and then the on-shell L $\sigma$ M  $\sigma$  mass obeys

$$m_\sigma^2 - m_\pi^2 = (m_\sigma^{CL})^2 \approx (651.34 \text{ MeV})^2 \quad \text{or} \quad m_\sigma \approx 665.76 \text{ MeV}, \quad (22)$$

almost exactly the model-independent  $\sigma$  mass found in ref. [9], also predicting  $\sigma_N^{L\sigma M}$  in Eq. (14).

In this letter we have linked the GT discrepancy Eqs. (4), (8) and the  $\pi N \sigma$  term Eqs. (15), (16) with the L $\sigma$ M values Eqs. (19)-(22). The predicted L $\sigma$ M value of  $g_{\pi NN}$  is

$$g_{\pi NN} = N_c g g_A = 3(2\pi/\sqrt{3})1.267 \approx 13.79, \quad (23)$$

near the observed value in Eq. (1) with meson-quark coupling  $g$ . Substituting Eq. (23) into the GTd (Eq. (4)) in turn predicts in the quark model

$$\Delta = 1 - \frac{m_N}{3m_q} \rightarrow 0 \quad \text{as} \quad m_q \rightarrow m_N/3. \quad (24)$$

However meson-baryon couplings for pseudoscalars (P), axial-vectors (A) and  $SU(6)$ -symmetric states are known [18] to obey

$$(d/f)_P \approx 2.0, \quad (d/f)_A \approx 1.74, \quad (d/f)_{SU(6)} = 1.50, \quad (25)$$

where the scales of  $d, f$  characterize the symmetric, antisymmetric  $SU(3)$  structure constants. Note that the ratio remains the same:

$$\frac{(d/f)_A}{(d/f)_P} = \frac{1.74}{2.0} = 0.87, \quad \frac{(d/f)_{SU(6)}}{(d/f)_A} = \frac{1.50}{1.74} \approx 0.86. \quad (26)$$

Thus to predict the quark-based  $\pi NN$  coupling constant we weight Eq. (23) by the scale factor of Eq. (26) in order to account for the  $SU(6)$  quark content of  $g_A$ :

$$g_{\pi NN} = 3 \times 2\pi/\sqrt{3} \times 1.267 \times 0.87 \approx 12.00 \quad (27)$$

and this predicted coupling constant is near 13.169 from ref. [1], or 13.145 from ref. [19], or nearer still to 13.054 from ref. [20]. One could alter this 0.87 reduction of  $g_A$  in Eq. (27) by using the quark-based factor  $3/5=0.6$ , where the  $SU(6)$  factor for  $g_A$  of  $5/3$  becomes inverted for quarks as suggested in [21]. In any case the predicted  $\pi NN$  coupling lies between 12.00 and 13.79 in Eqs. (27), (23), midway near the recent data in Eq. (1).

In passing, we note that the large model-independent [9] scalar  $\sigma$  mass of  $m_\sigma \approx 665$  MeV is recovered via the L $\sigma$ M combined with the CL quark-level GTR Eqs. (19)–(22). Also the large almost model-independent interior dispersion relation version of the  $\pi N \sigma$  term [22,23] is between 65-80 MeV. While this  $\sigma$  term follows from the two GMOR + L $\sigma$ M terms in Eq. (15) or from the IMF term in Eq. (16), original chiral perturbation theory (ChPT) of the 1970s suggested [24]  $\sigma_N \approx 25$  MeV near the GMOR value.

Modern ChPT now predicts [25] a  $\sigma_N$  of 45 MeV at  $t = 0$  extended up to the above presumably measured value of 60 MeV according to [25–27]<sup>5</sup>

$$\begin{aligned} 60 \text{ MeV} &= \sigma_N^{\text{GMOR}}(25 \text{ MeV}) + \sigma_N^{\text{higher order ChPT}}(10 \text{ MeV}) \\ &+ \sigma_N^{\text{t-dep.}}(15 \text{ MeV}) + \sigma_N^{\bar{s}s}(10 \text{ MeV}) \end{aligned} \quad (28)$$

and the latter "three pieces happen to have the same sign as  $\sigma_N^{\text{GMOR}}$ " [27]. The 15 MeV in Eq.(28) was found not to occur in our  $c$ -number  $\sigma_N$  analysis, since the  $t$ -dependence is minimal in Eqs. (17), (18). The latter  $\sigma_N^{\bar{s}s}$  of 10 MeV in (28) is 8 MeV higher than the  $y \leq 6\%$  value in refs. [28] with  $\sigma_N^{\bar{s}s} = (1 - y)^{-1} \sigma_N^{\text{GMOR}} - \sigma_N^{\text{GMOR}} \leq 2 \text{ MeV}$ . Lastly the  $\sigma_N^{\text{ChPT}}$  correction of 10 MeV in (28) does not apply to our  $L\sigma M$   $\sigma_N$  addition to  $\sigma_N^{\text{GMOR}}$ .

In summary, as  $m_\pi \rightarrow 0$ ,  $\partial A_\pi \rightarrow 0$ , the quark-level GT relation requires the observed 2 – 3% GTd and 3%  $\sigma$  term ratio to predict  $g_{\pi NN}$ , with  $\Delta \rightarrow 0$  as  $m_q \rightarrow m_N/3$  or  $\bar{\Delta} \rightarrow 0$  when  $m_\pi^2 \rightarrow 0$ . We have computed the  $\pi N \sigma$  term in many different ways to find approximately  $\sigma_N = 63 \text{ MeV}$ .

**Acknowledgement:** One of us (MDS) appreciates D.V. Bugg's help in first obtaining Eq. (8) via a computer. Nonetheless the reader may be more convinced with Eq. (8) using the Dwight integral solution combined with the Taylor series in footnote 4. This work is in part supported by the Slovak Agency for Science, Grant 2/3105/23.

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<sup>5</sup>However the ChPT paper [26] attempts to rule out the  $L\sigma M$  in App. B.

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