

**SOPHISTICATED VISUALIZATION ALGORITHMS FOR ANALYSIS OF
MULTIDIMENSIONAL EXPERIMENTAL NUCLEAR SPECTRA****M. Morhác^{1*‡}, J. Kliman^{*‡}, V. Matoušek^{2*}, I. Turzo^{*}**^{*}*Institute of Physics, Slovak Academy of Sciences, Bratislava, Slovakia*[‡]*Flerov Laboratory of Nuclear Reactions, JINR, Dubna, Russia*

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This paper describes graphical models of visualization of 2-, 3-, 4-dimensional scalar data used in nuclear data acquisition, processing and visualization system developed at the Institute of Physics, Slovak Academy of Sciences. It focuses on presentation of nuclear spectra (histograms). However it can be successfully applied for visualization of arrays of other data types. In the paper we present conventional as well as new developed surface and volume rendering visualization techniques used.

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1 Introduction

Scientists, engineers and others often need to analyze a large amount of experimental information. Scanning these large sets of numbers to determine trends and relationships is a tedious and ineffective process. If the data are converted to a visual form the trends are often immediately apparent.

Nuclear physics is one of the scientific fields where the requirements for using higher dimensions and higher resolutions are continuously increasing. In today's nuclear and high energy physics experiments, the number of detectors being included in the measurements is going up to one hundred or more. This number of detectors determines the dimensionality of histograms produced by multidimensional amplitude analysis. As a result the analysis of coincidence data produces 2-, 3-, 4- or more dimensional histograms or nuclear spectra.

The objective of the paper is to present visualization techniques and graphical models to display 2-, 3-, 4-dimensional nuclear spectra (histograms), which were implemented in the nuclear data acquisition, processing and visualization system [1, 2]. Though the software package is designed mainly for use with nuclear data, any kind of data can be processed as well. Other systems for nuclear spectra graphical representation were presented in [3, 4].

¹E-mail address: Miroslav.Morhac@savba.sk²E-mail address: Vladislav.Matousek@savba.sk, Corresponding author. Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 845 11 Bratislava, Slovakia

2 Visualization of 2-parameter nuclear spectra

Two-parameter coincidence nuclear spectrum (histogram) is represented by a matrix of data with two independent variables (parameters) and one dependent variable (counts), i.e., $c = f(x, y)$. The perspective transformation is employed to project the 3-dimensional scene onto a 2-dimensional screen. We have employed the following model to display 3-dimensional data on screen.

$$\begin{aligned} x' &= t_{xx} \cdot i + t_{xy} \cdot j + v_x \\ y' &= t_{yx} \cdot i + t_{yy} \cdot j + t_{yz} \cdot c + v_y \end{aligned} \quad (1)$$

where $t_{xx}, t_{xy}, t_{yx}, t_{yy}, t_{yz}, v_x, v_y$ are the transform coefficients reflecting translation in both original 2-dimensional scalar field (in x, y dimensions as well as in counts) and in the position on screen, scaling, rotation around z - axis and elevation of the view. The position of a point on the screen is x', y' and

$$\begin{aligned} x &= x_{\min} + k_x \cdot i; \\ y &= y_{\min} + k_y \cdot j; \\ i &\in \langle 0, n_x \rangle; \quad j \in \langle 0, n_y \rangle, \end{aligned}$$

where

$$k_x = \frac{x_{\max} - x_{\min}}{n_x}; \quad k_y = \frac{y_{\max} - y_{\min}}{n_y} \quad (2)$$

and n_x, n_y are numbers of nodes of a regular grid in variables x and y . The model proposed in such a way allows:

- to choose and display any part of the spectrum by setting $x_{\min}, x_{\max}, y_{\min}, y_{\max}$ to appropriate values
- to set any range of displayed counts — c_{\min}, c_{\max}
- to place the display of spectrum anywhere on the screen
- to rotate and elevate the view of the spectrum
- to change the density of display nodes. This is important when displaying accumulated spectra in on-line mode, i.e., during the acquisition of spectra.

During the measurement the experimenter needs to have at his disposal an information about the overall shape of the spectrum without being interested in details. The display of 2-parameter spectrum with a high density of displayed points (nodes) is very frequently a time-consuming operation and it may increase inadequately the dead time of the measuring system. Therefore sometimes the only remedy to cope with this situation is the reduction of the number of displayed nodes during spectra collection. An example of the display of 2-parameter spectrum is shown in Fig. 1.

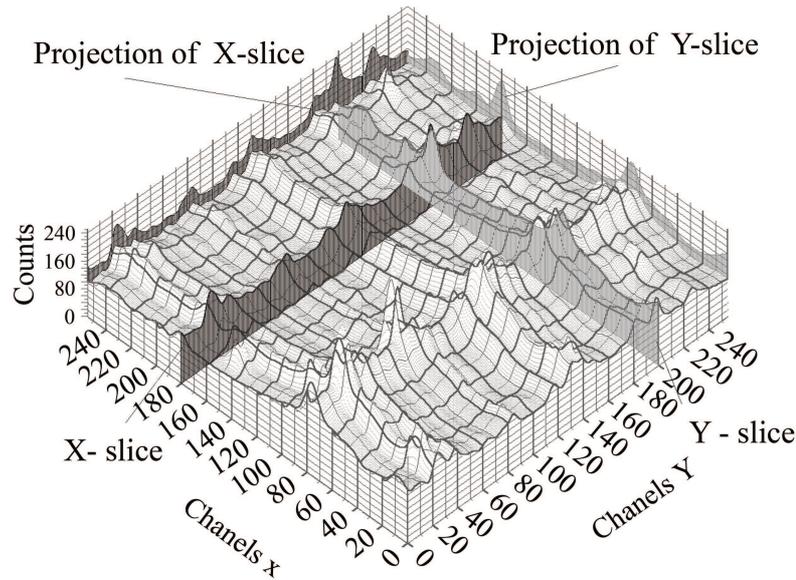


Fig. 1. An example of the display of 2-parameter spectrum.

One can put a grid on the spectrum to identify the position of its physically interesting parts (peaks). The displayed spectrum can be slid in both directions. Projections of the slices can be shown on backplanes of the coordinate system. This makes it possible to read-out the count values in the slices in individual channels.

The contents of spectra can be shown in line-drawing mode (grid, bars, slices, contours, etc.) or in shading surface mode. One can change the basic attributes of pen (color, size, style, pattern, background pattern, etc.). The surface shading (using Gouraud shading algorithm [5]) can be realized in dependence on the position of the fictive light source, that can be changed in 3-D space. The nuclear spectrum can be shown without shadows or with shadows. An example of display of nuclear spectrum is presented in Fig. 2.

Another way of surface shading is shown in Fig. 3. The shading in the channel x, y is proportional to the contents in the given channel, i.e., to the counts $c(x, y)$. Then the entry in the palette for the point x, y is

$$pal(x, y) = step \cdot (c(x, y) - c_{min}), \quad (3)$$

where

$$step = k \cdot \frac{palette_size}{c_{max} - c_{min}} \quad (4)$$

with $k = 1, 2, 3, \dots$. For $k = 1$ the range of the palette can be scaled automatically to the range of the displayed counts (Fig. 3).

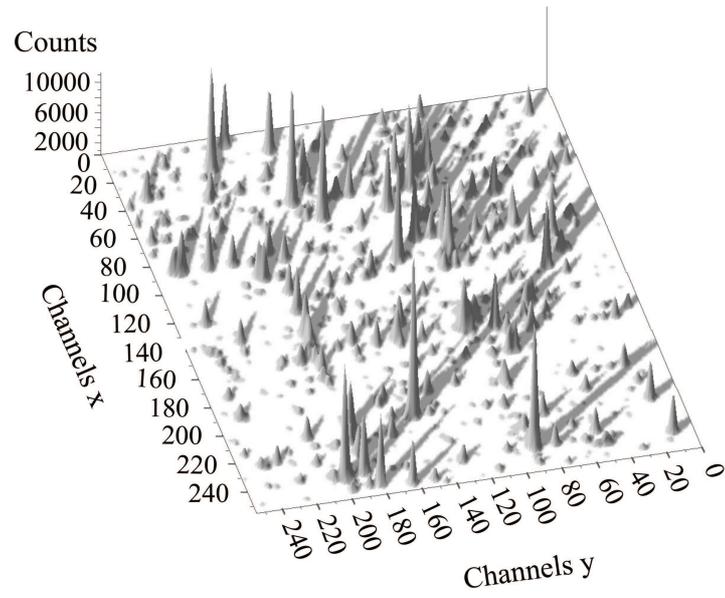


Fig. 2. An example of 2-parameter spectrum in the shaded-surface display mode with shadows.

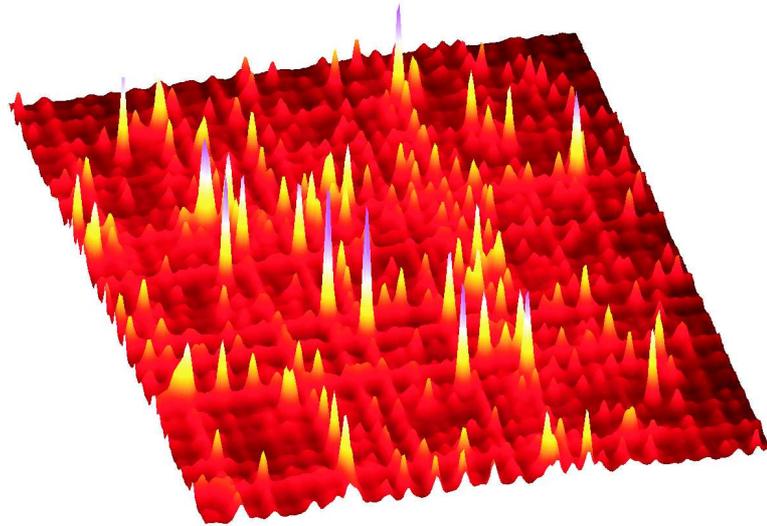


Fig. 3. An example of 2-parameter spectrum in the shaded-surface color display mode.

Sometimes it is necessary to emphasize the interesting parts of a spectrum by contours. By choosing higher value of the coefficient k , we obtain shaded contours display mode. An example of this display mode in rectangular view for $k = 10$ can be seen in Fig. 4. This way of presentation allows one to observe the positions of both big and small peaks in the spectrum in detail.

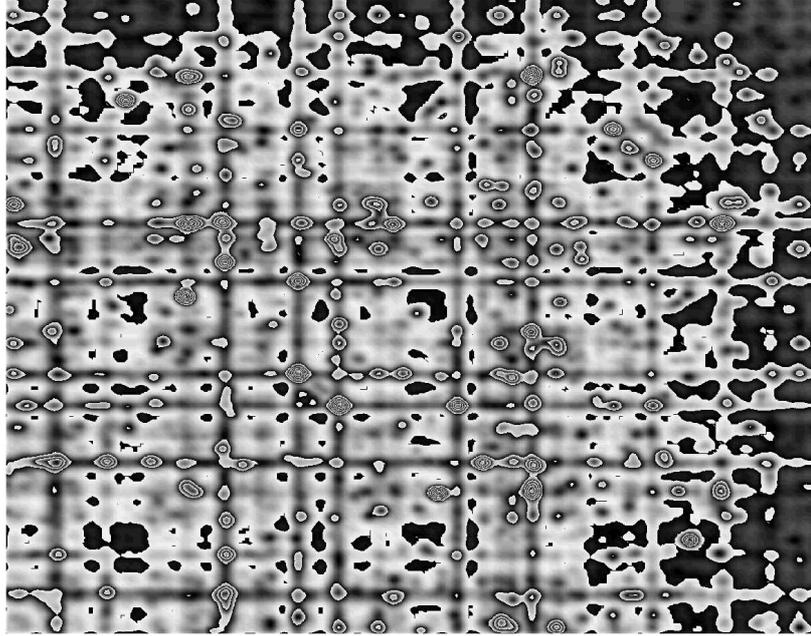


Fig. 4. An example of the display of 2-parameter spectrum using shaded-contour display mode in rectangular view.

In the above given display modes, one can choose among different display palettes (transition between black and white, combinations of transitions among basic colors, etc.).

In any of the above mentioned models one can change the density of grid nodes. To interpolate the surface shape among chosen control grid nodes one can use a spline method – in our case the Bezier surfaces and B-spline surfaces [5, 6], where one can change degree of spline and density of the interpolation. This makes it possible to generate continuous surfaces even for small number of measured points and thus to estimate the trends in surface shapes. In Fig. 5 we can see an example of 2-parameter spectrum interpolated by the Bezier surface in the shaded surface method (degree 3).

3 Graphical models of 3-parameter spectra display

Analogously to the previous section 3-parameter γ -ray coincidence nuclear spectrum is 3-dimensional scalar field with three independent parameters x, y, z (particle energies) and one dependent

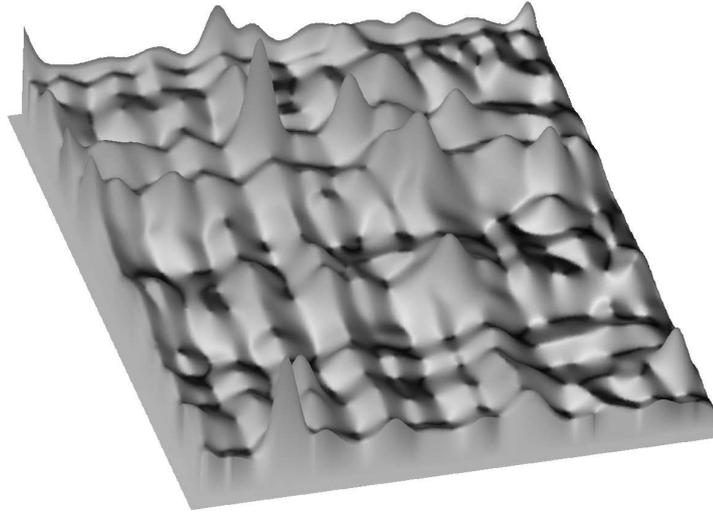


Fig. 5. An example of the display of 2-parameter spectrum with shaded surface interpolated by B – splines method (degree 3).

variable — counts $c = f(x, y, z)$. As with 2-parameter scalar fields one can idealize the display of 3-parameter scalar field using discrete symbols at specific locations in space, or use techniques that show the variations in the 3-parameter space. Hence each channel is defined by three parameters – coordinates x, y, z in the original space, which determine the position of a channel. We have employed a model where the channel is shown as a sphere (other marks as square, triangle, star etc. also can be used) with the size proportional to the event counts it contains. The position of the channel x, y, z on the screen is given by the following relations

$$\begin{aligned} x' &= t_{xx} \cdot i + t_{xy} \cdot j + t_{xz} \cdot k + v_x; \\ y' &= t_{yx} \cdot i + t_{yy} \cdot j + t_{yz} \cdot k + v_y \end{aligned} \quad (5)$$

where $t_{xx}, t_{xy}, t_{xz}, t_{yx}, t_{yy}, t_{yz}, v_x, v_y$ are the display transform coefficients reflecting translations in both original 3-dimensional scalar field (in x, y, z dimensions as well as in counts) and in the position on screen, scaling, rotation around axes x, y, z and

$$x = x_{\min} + k_x \cdot i; \quad y = y_{\min} + k_y \cdot j; \quad z = z_{\min} + k_z \cdot k,$$

where

$$k_x = \frac{x_{\max} - x_{\min}}{n_x}; \quad k_y = \frac{y_{\max} - y_{\min}}{n_y}; \quad k_z = \frac{z_{\max} - z_{\min}}{n_z}, \quad (6)$$

and where n_x, n_y, n_z are the numbers of nodes of regular grid in variables x, y, z . The model proposed in such a way allows:

- to choose and display any part of the 3-dimensional array — $x_{min}, x_{max}, y_{min}, y_{max}, z_{min}, z_{max}$
- to choose a range of displayed counts — c_{min}, c_{max}
- to place the picture anywhere on the screen
- to rotate with the displayed spectrum around any of the axes x, y, z
- to change the density of the displayed nodes.

Like as in 2-parameter spectrum, the low density of displayed nodes allows one to observe the collection of the displayed spectrum in real time and thus to obtain an idea about its shape during the measurement.

An example of the display of 3-parameter spectrum is given in Fig. 6. Sometimes this algorithm of visualization is called the particle gradient display in literature [7]. The spectrum can be slid by planes parallel to the planes of a coordinate system. The projections of these slices can be shown in the contour display mode in the planes of the coordinate systems (Fig. 6). It enables us to obtain information about the distribution of events in 3-dimensional space and to determine the positions of interesting parts in spectrum.

However, one needs to see details in the slices sometimes. Then the package allows to slice the spectrum for one fixed value (x, y, z – dimension) and to analyze 2-dimensional distribution in this slice (Fig. 7). By moving the position of the slice along the whole scale one can analyze entirely the 3-parameter spectrum.

One can continue in the analysis of 3-parameter spectrum by investigating 1-dimensional slices (Fig. 8). Again the slices can be moved in the interactive way and thus allowing the experimenter to find interesting parts in the spectrum.

The channels can be shown in line-drawing mode as spheres having the possibility to change the pen attributes or as shaded balls (Fig. 9). The shading depends on the position of the fictive light source, which can be changed.

Another technique to display 3-dimensional arrays is the creation of an isosurface in 3-dimensional space using the marching cube algorithm [5]. The isosurface is the 3-D surface representing locations of constant counts within a volume. Every point on the surface has the same constant count value of the scalar variable within the field. Isosurfaces form the 3-dimensional analogy to the isolines that form a contour display on a surface [7]. In our case the isosurface divides 3-D space to two subspaces, a subspace of events with the counts higher and a subspace of events with the counts lower than a given value.

Again the contents of spectra can be shown in line-drawing mode (combinations of slices in all 3 dimensions) or in shading surface mode. To interpolate the shape of the isosurface among the chosen control grid nodes one can again use the method of B-splines. Their degree and the density of the interpolation can be set as desired. An example of visualization algorithm for spectra interpolated by B-splines of third degree are shown in Fig. 10.

The above presented technique shows only the exterior visible surface. The isosurface can hide other surfaces. Important tool in the visualization of 3-parameter scalar fields is the volume

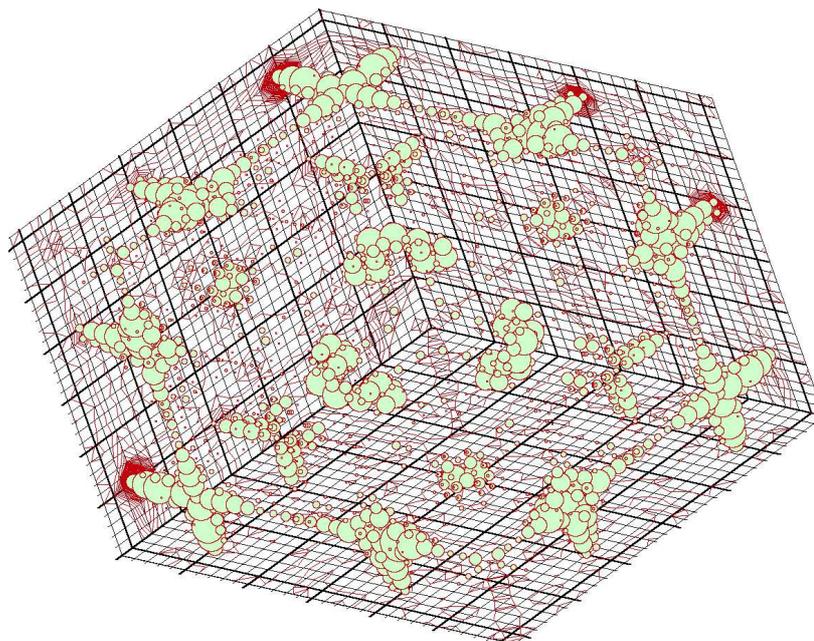


Fig. 6. An example of the display of 3-parameter spectrum with projections of slices shown in the contours display mode in the planes of the coordinate system.

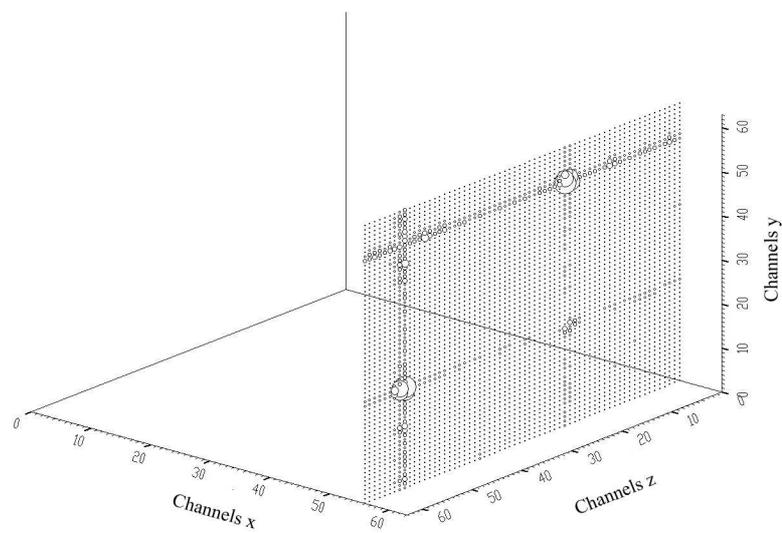


Fig. 7. An example of 2-dimensional slice in 3-parameter spectrum.

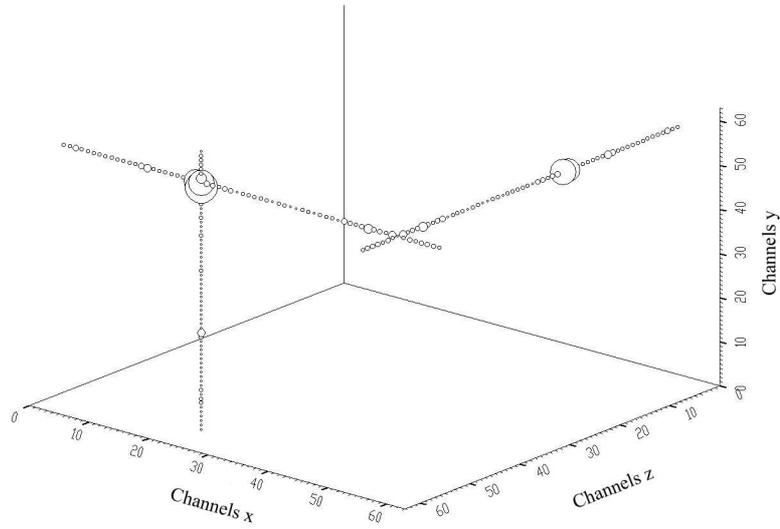


Fig. 8. An example of three one-dimensional slices in 3-parameter spectrum.

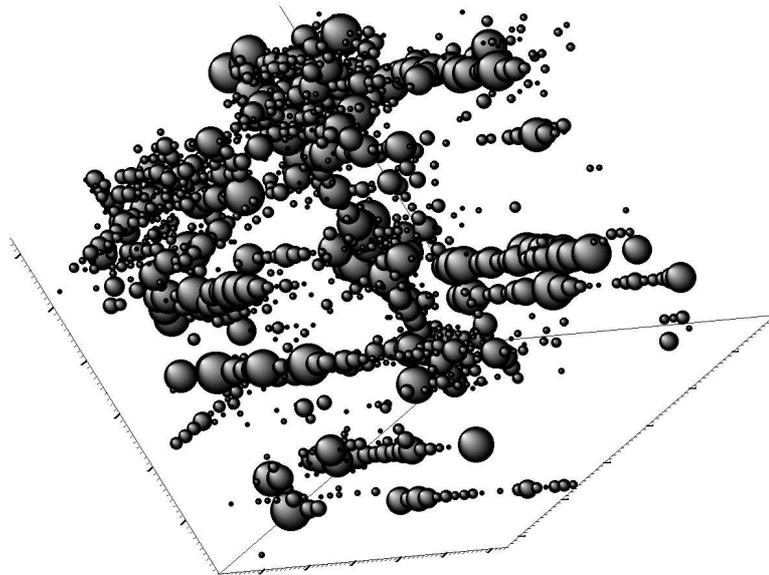


Fig. 9. An example of the display of 3-parameter spectrum with channels shown as shaded balls.

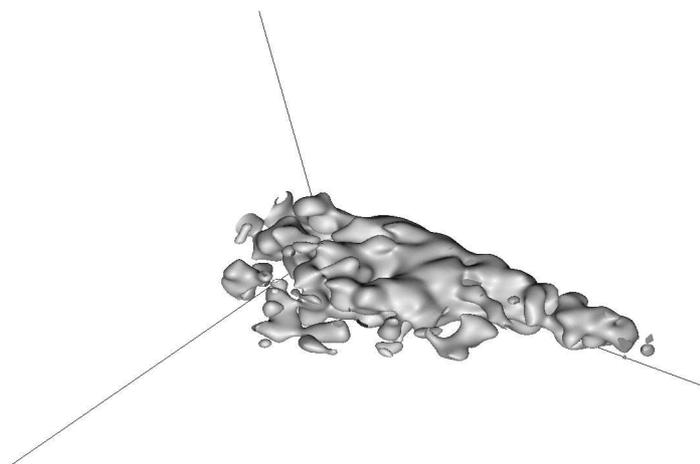


Fig. 10. An example of shaded surface visualization algorithm for spectra interpolated by B-splines of the third degree.

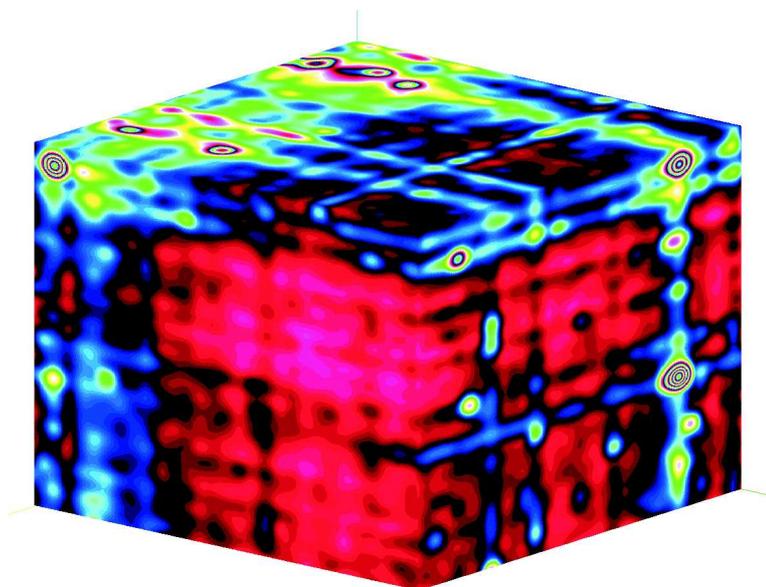


Fig. 11. The example of the display of 3-parameter spectrum using the volume slicing technique.

slicing technique with the possibility to cut away part of a volume to see what is behind. An example of the spectrum displayed through the use of this algorithm is shown in Fig. 11.

The palette of the display can be changed in the same way as in the 2-parameter spectra visualization according to relations (3) and (4). By increasing k , one can increase the number of contours and to see some parts of the spectrum in more detailed way. Like in the case of 2-parameter spectra one can choose from a set of different color palettes.

4 Display of 4-parameter spectra

In the last years there is a tendency to create 4-parameter histograms in some laboratories (e.g., from experiments with GAMMASPHERE [8]). Therein the counts are a function of four parameters (particle energies), i.e., $c = f(x, y, z, v)$. Let us imagine that now this point represents a slice in the fourth parameter instead of one channel belonging to one point of 3-D space in 3-parameter nuclear spectrum visualization, i.e.,

$$c_{i,j,k}(v) = f(x_i, y_j, z_k, v). \quad (7)$$

We depict each slice as a closed polygon with the center positioned in analogy with (5) at the location

$$\begin{aligned} x'_{i,j,k} &= t_{xx} \cdot i + t_{xy} \cdot j + t_{xz} \cdot k + v_x \\ y'_{i,j,k} &= t_{yx} \cdot i + t_{yy} \cdot j + t_{yz} \cdot k + v_y, \end{aligned} \quad (8)$$

where the meaning of the coefficients and parameters is in accordance with (5). Then for the positions of points of the polygon i, j, k on screen we have

$$x''_{i,j,k}(v) = x'_{i,j,k} - r_{\max} \cdot \frac{c_{i,j,k}(v) - c_{\min}}{c_{\max} - c_{\min}} \cdot \cos \left(\frac{2\pi(v - v_{\min})}{v_{\max} - v_{\min} + 1} + \phi_0 \right) \quad (9)$$

$$y''_{i,j,k}(v) = y'_{i,j,k} + r_{\max} \cdot \frac{c_{i,j,k}(v) - c_{\min}}{c_{\max} - c_{\min}} \cdot \sin \left(\frac{2\pi(v - v_{\min})}{v_{\max} - v_{\min} + 1} + \phi_0 \right), \quad (10)$$

where the fourth parameter $v \in \langle v_{\min}, v_{\max} \rangle$, r_{\max} (constant value) is the maximum distance of a polygon point from its center and ϕ_0 is starting angle of the display of the first point of polygon. Changing the parameter r_{\max} one can change the sizes of polygons on screen. The distance of each point of the polygon from its center carries the information about the counts that the corresponding channel contains. Parameters c_{\min} and c_{\max} determine the range of displayed counts. The channels with the counts outside this range are ignored. An example of the display of 4-parameter spectrum employing this technique is presented in Fig. 12. It allows one to observe correlation among spectrum counts in all four parameters, i.e., by changing coordinates x, y, z of the 3-D space and by changing the angle in individual polygons. For $\phi_0 = 0$ the channels with $v = v_{\min}$ are displayed at the “9 o'clock” positions. Changing the angle ϕ_0 one can turn with the

polygons clockwise or counterclockwise. All other display parameters, attributes, palettes have the same meaning and can be changed in the same way as in the case of 2- and 3-dimensional arrays.

When required one can display also the positions of the channels in the fourth dimension. This makes it possible to get an idea about counts values in individual channels (see Fig. 12).

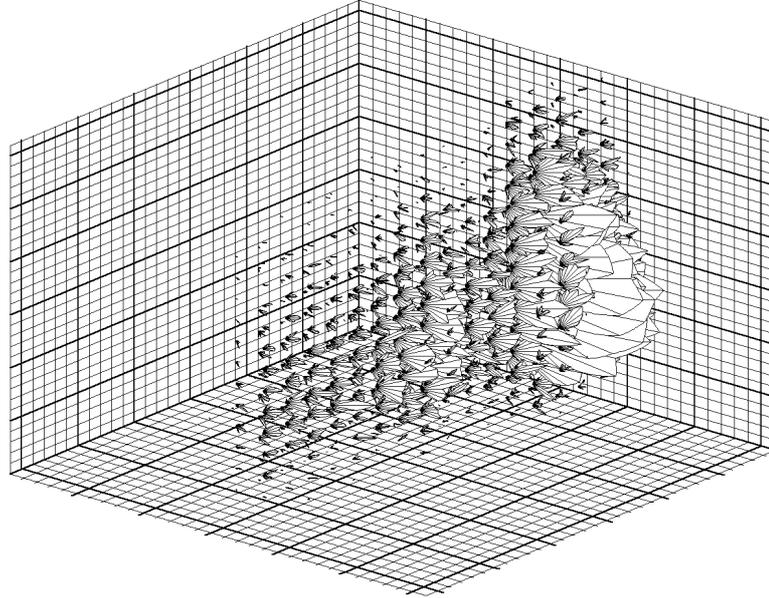


Fig. 12. An example of the display of 4-parameter spectrum with displayed positions of the channels in the fourth dimension.

The lines connecting the points of polygons can be interpolated using the Bezier curves (Fig. 13).

However, one can change also the color (level of shading) with the angle while keeping the radius of circle constant. According to the resolution in the fourth independent variable, the circle is divided to the pies. The size of the circle is proportional to the sum of counts in the fourth dimension,

$$\sum_{v=v_{\min}}^{v_{\max}} f(x_i, y_j, z_k, v).$$

The color of a pie is proportional to the counts in appropriate channel and predefined color palette. An example of such a display mode is shown in Fig. 14.

One can also make 3-parameter slices from which it is possible to study the distribution of events – Fig. 15.

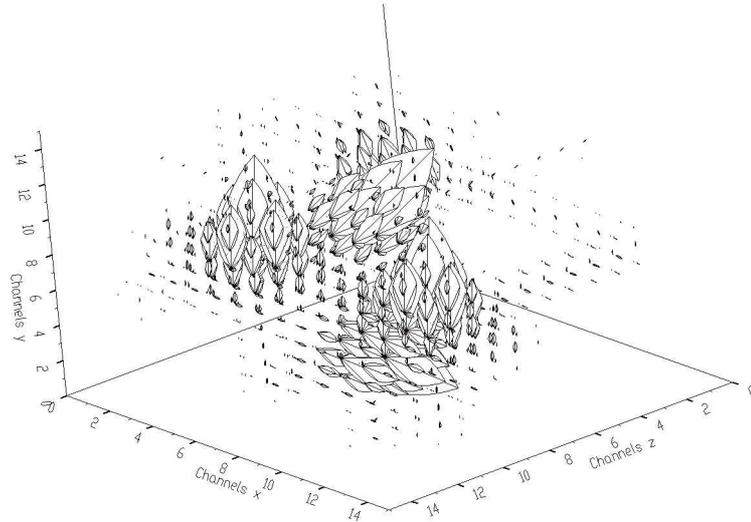


Fig. 13. An example of the display of 4-parameter spectrum with interpolation of the points of polygons using Bezier curves.

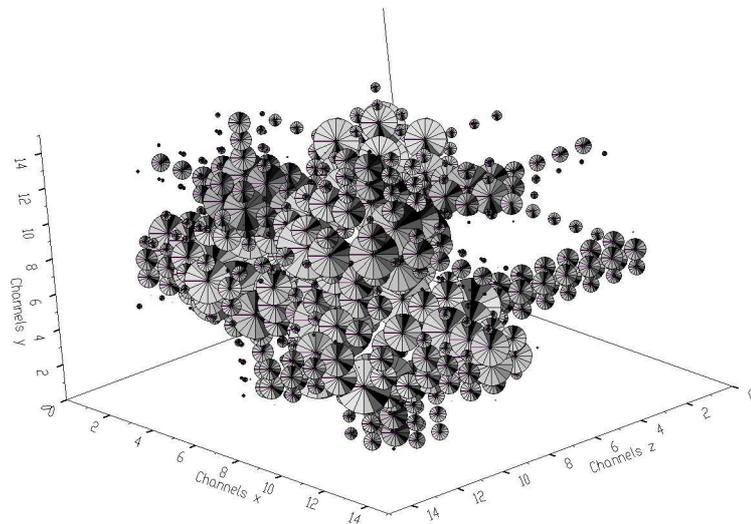


Fig. 14. An example of the display of 4-parameter spectrum using pie display mode.

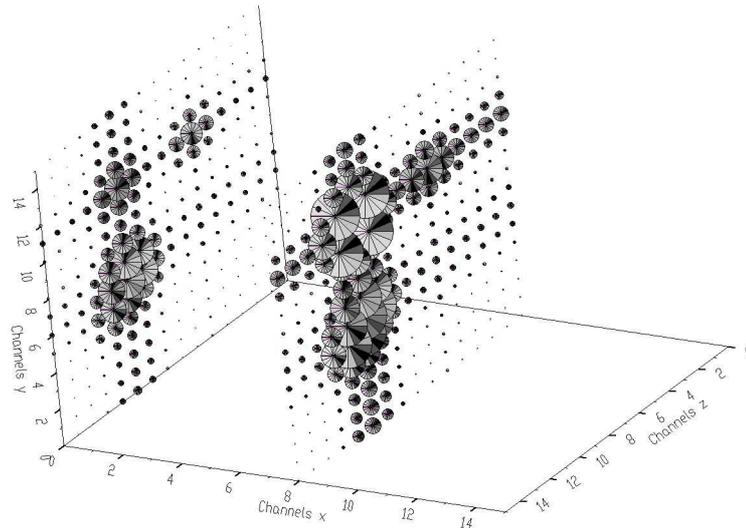


Fig. 15. An example of the display of 3-parameter slices in 4-parameter spectrum space.

Another technique to display 4-dimensional arrays is based on generation of isovolume in 3-D space. All points of displayed volume have the same counts. In each point of 3-D space defined by 3 independent variables x, y, z , we make slice in the direction of the fourth variable. If all counts in the slice are less than given isovolume counts, the point x, y, z is not displayed. Otherwise the value v of the first occurrence of a given isovolume counts determines the shade (color) of the corresponding x, y, z grid point. By interpolation of shades (colors) of all points belonging to the isovolume we get continual surface. Isovolume is the 4-dimensional analogy to the isolines and isosurfaces described in the two previous sections. An example of the display employing this algorithm of visualization is shown in Fig. 16.

Analogously to two previous sections one can change palettes and number of surface contours in the same way.

5 Conclusions

We have presented conventional 2-dimensional as well as newly developed 3- and 4-dimensional visualization algorithms. We have proposed the algorithm to display the 3-dimensional array in the form of spheres. One value (scalar), which determines the size of a sphere, is assigned to one point of 3-dimensional space. This way of presentation allows experimenter to locate positions of interesting peaks of spectra (peaks) in 3-dimensional space (see e.g. Fig. 6). If necessary more precise positions can be obtained through the use of moving slices (see Figs. 7 and 8).

We have extended this algorithm to the 4-dimensional array. In this case a set of values (vector – a slice in the fourth variable) is assigned to one point of the 3-dimensional space. The vector can be displayed either as closed polygon, where the distance of each polygon point from

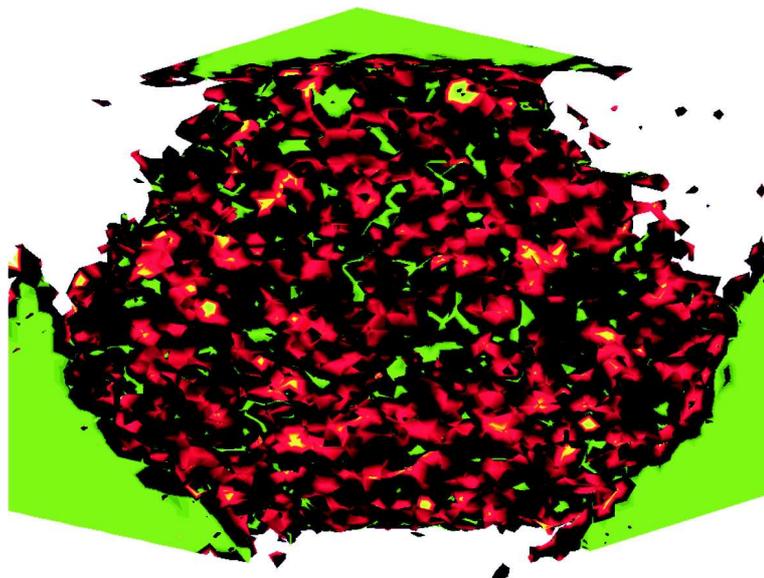


Fig. 16. An example of the display of 4-parameter spectrum employing the isovolume algorithm of visualization.

the center of the polygon carries the information about the counts in the corresponding channel, or as circle divided to pies, where their shades (colors) are proportional to corresponding counts. Again, this technique provides experimenter with information about correlations among the peaks in 4-dimensional spectra. One can observe the distribution of counts even in the 4th dimension as well as in other dimensions (see e.g. Fig. 14). This provides experimenter with more complex view of 4-dimensional data. Analogously to 3-dimensional case, the technique of lower dimensional slices is also applicable (see Fig. 15). Successive decreasing the dimensionality makes it possible to determine exactly the position of appropriate 4-dimensional peaks.

Theoretically we could continue in these considerations for the next dimension, i.e., for the display of 5-dimensional array. Then we should have to assign a matrix of two variables (2-D array) to each point of 3-D space. Each of these matrices could be displayed by employing one of the known algorithms for 2-D arrays, e.g., polygon surfaces [5]. In principle one can continue in these thoughts even for higher dimensions. However we are able to visualize data only in 3-D space. Using the above suggested technique one can insert the subspaces of the remaining variables into points of 3-D space.

Color versions of the Figs. 3, 6, 11, and 16 are available on <http://www.fu.sav.sk/aps/>.

References

- [1] M. Morháč, I. Turzo, J. Krištiak: *IEEE Trans. on Nuclear Science* **42** (1995) 1
- [2] M. Morháč, J. Kliman, V. Matoušek, I. Turzo: *Nuclear Instruments & Methods in Physics Research A* **389** (1997) 89

- [3] D. C. Radford: *Nuclear Instruments & Methods in Physics Research A* **361** (1995) 297
- [4] J. Baczyński: *Nuclear Instruments & Methods in Physics Research A* **325** (1993) 523
- [5] D. Hearn, M. P. Baker: *Computer Graphics*: Prentice Hall International, Inc. (1994)
- [6] W. M. Newman, R. F. Sproull: *Principles of Interactive Computer Graphics*: McGraw-Hill, Inc. (1979)
- [7] R. S. Gallagher: *Computer Visualization Graphics Techniques for Scientific and Engineering Analysis*: CRC Press, Boca Raton, Ann Arbor, London, Tokyo (1995)
- [8] M. A. Delaplanque, R. M. Diamond: *GAMMASPHERE A national γ -ray facility*: Tech. Rep. Pub-5202, Lawrence Berkeley Laboratory, Berkeley (1988)