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THE z-SCALING FROM TENS OF GeV TO TeV

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The concept of z-scaling that reflects the general features of internal particle substructure, constituent interaction, and mechanism of particle formation at high-p_T is reviewed. The experimental data on inclusive cross sections obtained at the U70, ISR, SpS, RHIC and Tevatron are analyzed. The properties of z-presentation of data such as the energy independence, power law, and A-dependence are discussed. The properties of z-scaling are argued to be connected with the fundamental symmetries such as self-similarity, locality and fractality. The use of z-scaling to search for new physics phenomena in collisions of hadrons and nuclei is suggested. RHIC data used in our new analysis confirm z-scaling in pp collisions. High-p_T particle spectra at LHC energies are predicted. Violation of z-scaling characterized by the change of the anomalous fractal dimension is considered as a new and complementary signature of new physics phenomena.

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1 Introduction

More prominent properties of particle production are observed at high energy and transverse momentum [1–4]. The kinematic regime is used to perform calculations of physical quantities in the framework of perturbative Quantum Chromodynamics (QCD). Deviations of the theoretical results obtained in the high-p_T region using available experimental data are often considered as manifestations of new physics phenomena. We should also note that the nonperturbative effects are not well controlled by the theory. Therefore search for new scaling features of particle interactions in the high-p_T region is of interest for development of the theory.

The fundamental problem of high energy physics is the origin of mass, spin, and charge of particles. The study of particle interactions over a wide kinematic range and especially at small scales is necessary to understand underlying physics phenomena. The Theory of Grand Unification (GUT)) assumes that all types of interactions are unified at small scales. New ideas such as extra dimensions, anisotropy and fractality of space-time, quark compositeness, and theories of Super Symmetry, and Super Gravity are intensively developed.

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Z-scaling as a new feature of high-p_T particle production in hadron-hadron and hadron-nucleus collisions at high energies was established in [5–11]. The scaling function ψ and scaling variable z are expressed via experimental quantities such as the inclusive cross section $Ed^3\sigma/dp^3$ and the multiplicity density of charged particles $dN/d\eta$. The z-presentation of data is found to reveal symmetry properties (energy independence, A-dependence, power law). The properties of ψ at high z are assumed to be relevant to the structure of space-time at small scales [12, 13]. The function $\psi(z)$ is interpreted as a probability density to produce a particle with the formation length z. The concept of z-scaling and the method of data analysis are developed for description of different particles (charged [6, 7, 14] and neutral [10, 11] hadrons, direct photons [8, 15], jets [9]) produced in high energy hadron-hadron and hadron-nucleus interactions. The proposed method is complementary to a method of direct calculations developed in the framework of QCD [16] and methods based on the Monte Carlo generators [17–24]. Therefore we consider that the use of the method of data analysis based on the concept of z-scaling allows us to reduce some theoretical uncertainties which are ambiguously estimated by theory.

In the report the general concept of z-scaling, the properties of z-presentation of data and some new results of data analysis are presented. The fundamental principles such as self-similarity, locality, fractality, and scale-relativity are formulated and discussed in the framework of the concept of z-scaling. Verification z-scaling validity at the RHIC and LHC is suggested. Violation z-scaling is considered to be indication of new physics phenomena.

2 Z-scaling

In the section we discuss basic ideas of z-scaling. A general scheme of z-presentation of data is described. The physical meaning of the introduced quantities is explained.

2.1 Locality

The idea of z-scaling is based on the assumptions [25] that the gross feature of inclusive particle distribution of the process (1) at high energies can be described in terms of the corresponding kinematic characteristics

$$M_1 + M_2 \rightarrow m_1 + X \tag{1}$$

of the constituent subprocess written in the symbolic form (2)

$$(x_1M_1) + (x_2M_2) \rightarrow m_1 + (x_1M_1 + x_2M_2 + m_2) \tag{2}$$

satisfying the condition

$$(x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{3}$$

The equation is the expression of locality of hadron interaction at constituent level. The x_1 and x_2 are the fractions of the incoming momenta P_1 and P_2 of the colliding objects with the masses M_1 and M_2. They determine the minimum energy, which is necessary for production of the secondary particle with the mass m_1 and the four-momentum p. The parameter m_2 is introduced to satisfy the internal conservation laws (for baryon number, isospin, strangeness, and so on).

Equation (3) reflects minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, the quantity Ω is introduced. It is chosen in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}, \tag{4}$$

where m is a mass constant and δ_1 and δ_2 are factors relating to the anomalous fractal dimensions of the colliding objects. The fractions x_1 and x_2 are determined to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling the condition (3)

$$d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \tag{5}$$

The fractions x_1 and x_2 are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

2.1.1 $\lambda - \chi$ decomposition of fraction x

Using equation (3) the relationship between the fractions x_1 and x_2 can be written in the form

$$x_1x_2 - x_1\lambda_2 - x_2\lambda_1 = \lambda_0, \tag{6}$$

where

$$\lambda_1 = \frac{(P_2p) + M_2m_2}{(P_1P_2) - M_1M_2}, \quad \lambda_2 = \frac{(P_1p) + M_1m_2}{(P_1P_2) - M_1M_2}, \quad \lambda_0 = \frac{0.5(m_2^2 - m_1^2)}{(P_1P_2) - M_1M_2}. \tag{7}$$

Using equation (5) it can be shown [6, 7] that the fractions x_1 and x_2 satisfy the $\lambda - \chi$ decomposition as follows

$$x_1 = \lambda_1 + \chi_1, \qquad x_2 = \lambda_2 + \chi_2, \tag{8}$$

with

$$\chi_1 = \sqrt{\mu_1^2 + \omega_1^2} + \omega_1, \qquad \chi_2 = \sqrt{\mu_2^2 + \omega_2^2} + \omega_2. \tag{9}$$

Here, the following notations are used

$$\mu_1^2 = (\lambda_1\lambda_2 + \lambda_0)\alpha\frac{(1 - \lambda_1)}{(1 - \lambda_2)}, \quad \mu_2^2 = (\lambda_1\lambda_2 + \lambda_0)\frac{1}{\alpha}\frac{(1 - \lambda_2)}{(1 - \lambda_1)}, \tag{10}$$

$$\omega_1 = (\lambda_1\lambda_2 + \lambda_0)\frac{(1 - \alpha)}{2(1 - \lambda_2)}, \quad \omega_2 = (\lambda_1\lambda_2 + \lambda_0)\frac{(\alpha - 1)}{2\alpha(1 - \lambda_1)}, \tag{11}$$

where $\alpha \equiv \delta_2/\delta_1$. In the case of proton-nucleus interactions, the nucleus is labeled by index 2. The value of α is chosen to be atomic weight A. The choice was justified by our analysis [7] of experimental data.

2.1.2 Scaling variable z

Determination of the self-similarity parameter z plays the crucial role in our approach. This should reflect general pattern of hadron production at high energies. Besides the scale invariance, it concerns the fractal character of the composite structures involved. Leading by the principles, we consider the variable

$$z = \frac{\hat{s}_{\perp}^{1/2}}{\Omega \cdot (dN/d\eta)}. \tag{12}$$

It includes according to the ansatz suggested in [6] the transverse kinetic energy of the elementary subprocess (2),

$$\hat{s}_{\perp}^{1/2} = \hat{s}_{\lambda}^{1/2} + \hat{s}_{\chi}^{1/2} - m_1 - (M_1 x_1 + M_2 x_2 + m_2), \tag{13}$$

the factor $\Omega(x_1, x_2)$ and the particle multiplicity density $dN/d\eta|_{\eta=0}(s)$. The transverse energy consists of two parts

$$\hat{s}_{\lambda}^{1/2} = \sqrt{(\lambda_1 P_1 + \lambda_2 P_2)^2}, \quad \hat{s}_{\chi}^{1/2} = \sqrt{(\chi_1 P_1 + \chi_2 P_2)^2}, \tag{14}$$

representing the transverse energy of the inclusive particle and its recoil, respectively. Note that the form of z, as defined by (12), determines its variation range. The boundaries of the range are 0 and ∞. These values are scale independent and kinematically accessible at any energy.

2.2 Self-similarity

Self-similarity is a scale-invariant property connected with dropping of certain dimensional quantities out of physical picture of the interactions. It means that dimensionless quantities for the description of physical processes are used. In accordance with the self-similarity principle, we search for the solution

$$\psi(z) \equiv \frac{1}{< N > \sigma_{inel}} \frac{d\sigma}{dz}, \tag{15}$$

which depends on a single scaling variable z. Here σ_{in} and $< N >$ are the inelastic cross section and the average multiplicity of charged particles, respectively. As shown in [6, 7] the scaling function $\psi(z)$ is expressed via the invariant cross section $Ed^3\sigma/dp^3$ as follows

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3} \tag{16}$$

Here, s is the center-of-mass collision energy squared, J is the corresponding Jacobian. The factor J is the known function of kinematic variables, momenta and masses of colliding and produced particles.

The function $\psi(z)$ is normalized as follows

$$\int_0^{\infty} \psi(z) dz = 1. \tag{17}$$

The relation allows us to interpret the function $\psi(z)$ as a probability density to produce a particle with the corresponding value of the variable z.

We would like to emphasize that existence of the function $\psi(z)$ depending on a single dimensionless variable z and revealing the scaling properties is not evident in advance. Therefore the proposed method to construct $\psi(z)$ and z could be only proved a posteriori.

2.3 Fractality

The principle of fractality states that the variables used in the description of the process diverge in terms of resolution. This property is a characteristic one of the scaling variable

$$z = z_0\Omega^{-1}, \tag{18}$$

where

$$z_0 = \sqrt{\hat{s}_\perp}/(dN/d\eta). \tag{19}$$

The variable z has a character of fractal measure, $z(\Omega) \to \infty$ at $\Omega \to 0$. For the given production process (1), its finite part z_0 is the ratio of the transverse energy released in the binary collision of constituents (2) and the average multiplicity density $dN/d\eta|_{\eta=0}$. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The quantity $\Omega(x_1, x_2)$ represents relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The parameters δ_1 and δ_2 are the anomalous fractal dimensions of the colliding objects (hadrons or nuclei). The momentum fractions x_1 and x_2 are determined in a way to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of the fractal measure z with respect to all possible sub-processes (2) subjected to the condition (3). The variable z was interpreted as a particle formation length.

As we will show later the scaling function of high-p_T particle production is described by the power law, $\psi(z) \sim z^{-\beta}$. Both quantities, ψ and z, are scale dependent. Therefore we consider that high energy interactions of hadrons and nuclei are interactions of fractals. In the asymptotic region the internal structure of particles, interactions of their constituents and mechanism of real particle formation should manifest self-similarity and fractality over a wide scale range.

2.4 Scale-relativity

The properties of particle interactions in space-time reflect symmetries of Nature. The principle of motion relativity has been used to formulate non-relativistic and relativistic theory. The special theory of relativity deals with only inertial coordinate systems while the expression of physical laws in the general theory of relativity should be written into any curvilinear coordinate system. The principle of general relativity states that "the laws of physics must be of such a nature that they apply to systems of references in any kind of motion". Application of the relativity principle can be extended to state of scale of reference system [13]. There are convincing evidence to consider that scale as well as other quantities characterizing a reference frame should be used to describe a particle state over a high-p_T range. In this range elementary probes such as direct photons, high-p_T hadrons and jets are not point-like objects. They have a complicated structure. The last is resulted from interactions of quarks, gluons and heavy bosons which are fundamental objects of the theory. Therefore experimentally measurable quantities should depend on a ratio between scales of the studied object and a probe. In other words the variables used in the description of the process depends on a resolution.

A generalization of the motion-relativity principle to the scale-relativity principle requires that "the laws of physics must be of such a nature that they apply to systems of references in any kind of motion and whatever its state of scale".

In the framework of the concept of z-scaling the mechanism of particle formation is described by the function $\psi(z)$. Both quantities ψ and z depend on the resolution Ω^{-1} while as found from our analysis of numerous experimental data the anomalous fractal dimension δ are to be resolution independent. We consider that the experimental verification and the study of z-scaling over a wide kinematic range of p_T and $\sqrt{s}$ and determination of δ could give new insight in the theory of scale-relativity [12, 13].

The various signatures are suggested to use for searching for new physics (quark compositeness, Higgs boson, new types of interactions, extra dimensions, black holes, phase transition, fractal structure of space-time etc.) at the collider of new generation, the Large Hardron Collider, at CERN. The regime of high-p_T particle production (or very small scales $\sim 10^{-4}$ [Fm]) will be accessible at the LHC. It is assumed that the coupling constants of electromagnetic, weak and strong interactions are to be the same order as the gravitational one at the scales. Therefore the structure of space-time itself should be reflected by the mechanism of particle formation. The z-scaling is suggested [5–11] as an effective tool to study features of particle formation at high-p_T. Violation of the scaling is considered to be a signature of new physics phenomena in collisions of hadrons and nuclei.

3 Properties of z-presentation of data

In the section we present and discuss some properties of p_T- and z-presentations of data of high-p_T particle production in pp, p̄p and pA collisions. We show that the scaling functions for different processes reveal the same properties. They are the energy independence of ψ, the power behavior of ψ at high-z and A-dependence.

3.1 Energy independence of $\psi(z)$

It is well known that numerous experimental data on high-p_T particle spectra manifest the strong dependence on collision energy $\sqrt{s}$. The effect enhances as the transverse momentum of produced particle increases.

3.1.1 π^+, π^-, K$^+$, K$^-$-mesons

Figures 1(a)-4(a) show the dependence of the inclusive cross sections of π^+, π^-, K$^+$ and K$^-$ mesons produced in pp collisions on the transverse momentum p_T at the incident proton momentum $p_L = 70, 200, 300, 400$ and 800 [GeV/c] and the angle $\theta_{cm} \simeq 90^o$. The data were obtained at Protvino [26] and Batavia [27, 28]. The transverse momenta of produced particles shown in Figs. 1(a)-4(a) change from 1 to 10 [GeV/c]. We would like to note that the data [27] and [28] corresponding to the momentum $p_L = 400$ [GeV/c] are complementary and are in a good agreement. As seen from Figs. 1(a)-4(a) that the particle spectra demonstrate a power behavior at $p_L = (200 - 800)$ [GeV/c]. The effect of kinematic boundary is visible at the end of spectrum at $p_L = 70$ [GeV/c].

The energy independence of z-presentation of data means that the scaling function $\psi(z)$ has the same shape for different $\sqrt{s}$ over a wide p_T range.

As seen from Figs. 1(b)-4(b) z-presentation of the same data sets demonstrates the energy independence of $\psi(z)$ over a wide collision energy and transverse momentum range. We would like to emphasize that the data [28] used in our new analysis confirm our earlier results [5, 29].

3.1.2 π^0-mesons

The PHENIX Collaboration published the new data [30] on the inclusive spectrum of π^0-mesons produced in pp collisions in the central rapidity range at RHIC energy $\sqrt{s} = 200$ [GeV]. The transverse momenta of π^0-mesons were measured up to 13 [GeV/c].

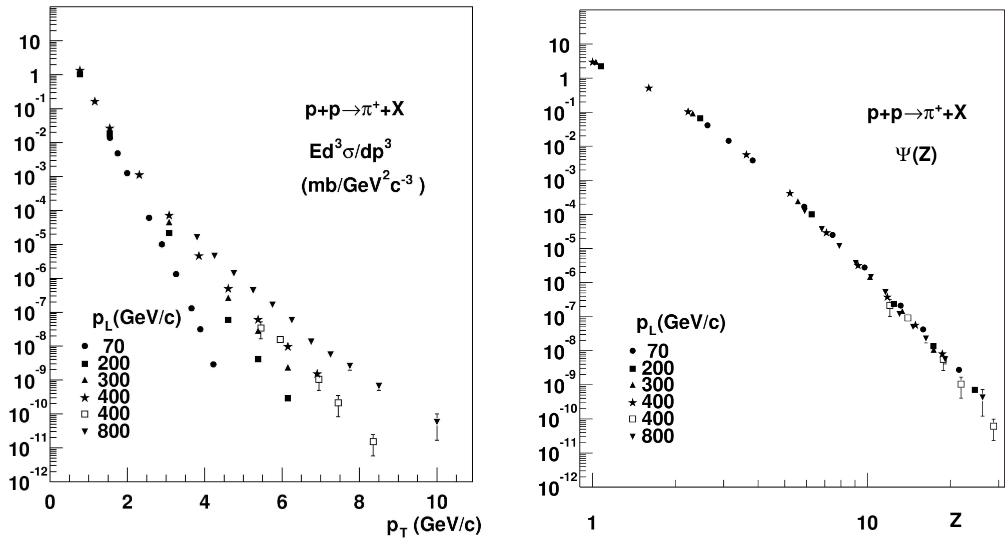
The p_T- and z-presentations of data for π^0-meson spectra obtained at ISR [31,32,34–36] and RHIC [30] are shown in Figs. 5(a) and 5(b). One can see that p_T-spectra of π^0-meson production reveal the properties similar to that found for charged hadrons. The new data [30] on π^0-meson inclusive cross sections obtained at the RHIC as seen from Fig. 5(b) are in a good agreement with our earlier results [10]. Thus we can conclude that the available experimental data on high-p_T π^0-meson production in pp collisions confirm the property of the energy independence of $\psi(z)$ in z-presentation.

3.1.3 Charged hadrons

The STAR Collaboration published the new data [37] on the inclusive cross sections of charged hadrons produced in pp collisions at RHIC energy $\sqrt{s} = 200$ [GeV]. The RHIC data and other ones obtained at the U70 [26], ISR [38] and Tevatron [27, 28] are shown in Fig. 6(a). The charged hadron spectra were measured over a wide kinematic range $\sqrt{s} = (11.5 - 200)$ [GeV] and $p_T = (0.5 - 9.5)$ [GeV/c]. The strong energy dependence and the power behavior of particle p_T-spectrum are found. The energy independence of z-presentation of data shown in Fig.6(b) is confirmed. It is of interest to verify the asymptotic behavior of ψ at $\sqrt{s} = 200$ [GeV] and reach value of z up to 30 and more.

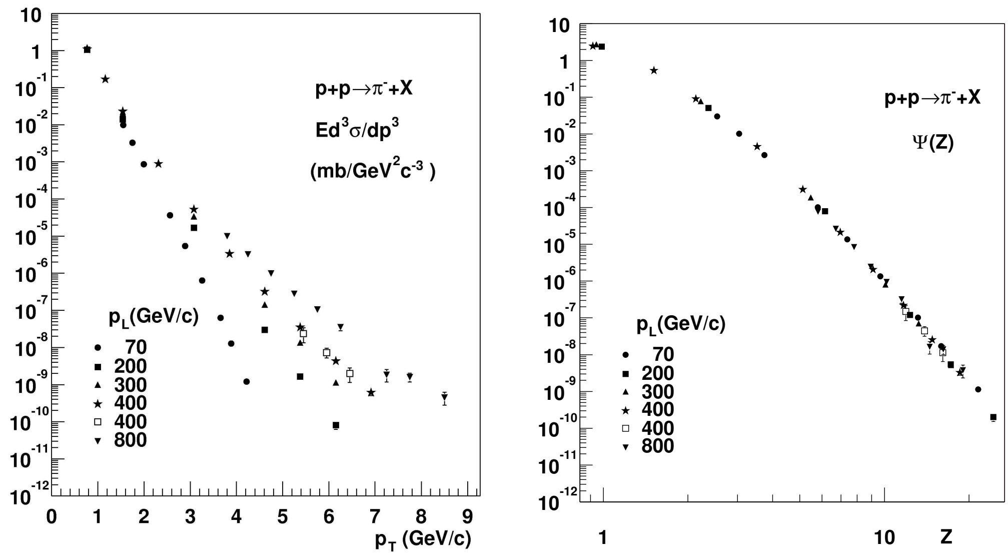
3.1.4 Direct photons

Direct photons are considered as the best probes of constituent interactions at high-p_T. The calculations of direct photon cross sections are developed in the next-to-next-to-leading order QCD [39]. The basic mechanisms of direct photon production in LO QCD are considered to be Compton scattering ($gq \to \gamma q$), and annihilation process ($\bar{q}q \to \gamma g$). These are direct mechanisms of photon production. In high-p_T range direct photons are also produced indirectly via bremsstrahlung of quarks ($qq \to qq\gamma,\ qg \to qg\gamma$). The contribution of indirect mechanisms of high-p_T photon production can be large enough. However there are significant theoretical uncertainties due to the choice of structure and fragmentation functions and renormalization, factorization and fragmentation scales for estimation of inclusive cross sections. Therefore any reliable estimates of direct photon cross sections allowing to reduce some of theoretical uncertainties are of interest. The results of data analysis for direct photon production in p̄p collisions are presented



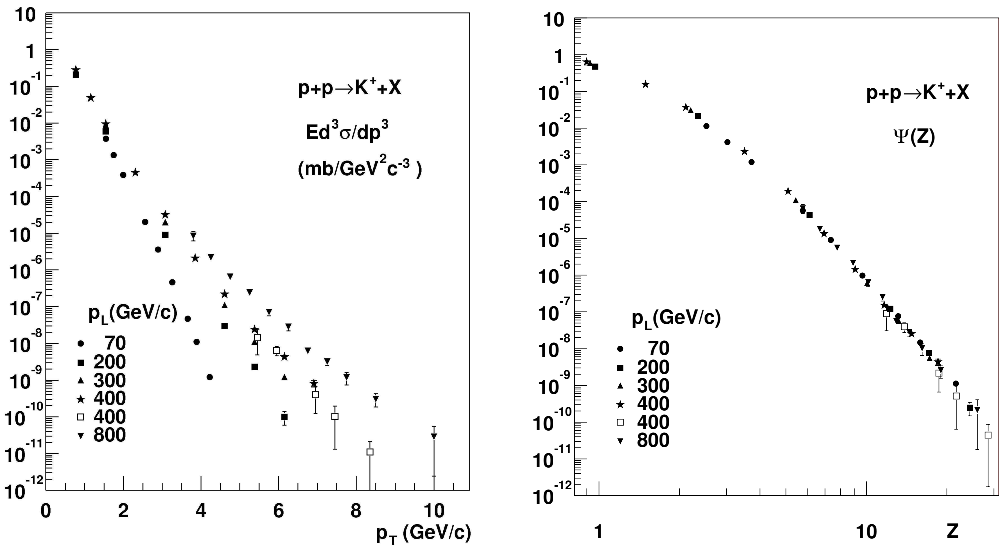
a) b)

Fig. 1. (a) The inclusive differential cross section for π^+-mesons produced in pp collisions at $p_L = 70, 200, 300, 400$ and 800 [GeV/c] and $\theta_{cm} \simeq 90^0$ as a function of the transverse momentum p_T. The experimental data are taken from [26–28]. (b) The corresponding scaling function $\psi(z)$.



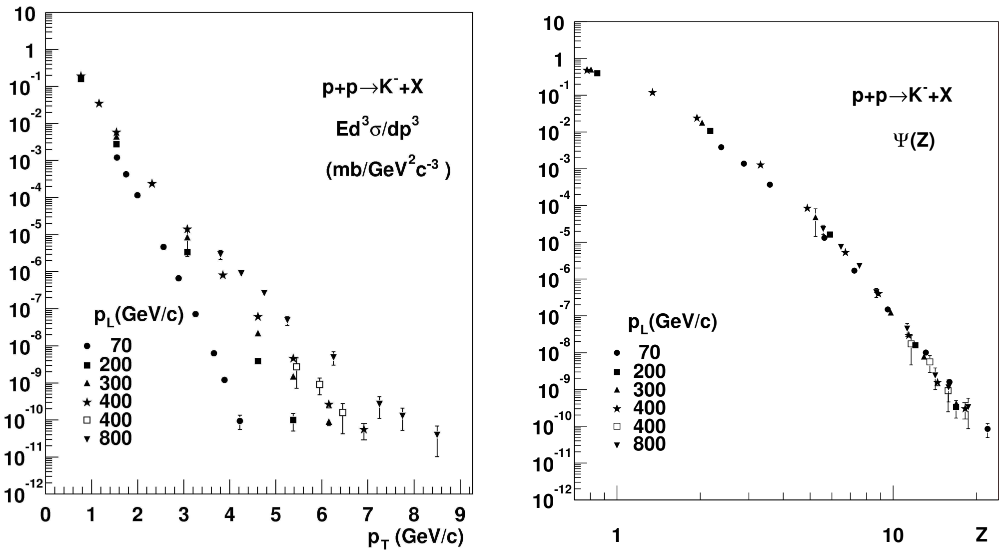
a) b)

Fig. 2. (a) The inclusive differential cross section for π^--mesons produced in pp collisions at $p_L = 70, 200, 300, 400$ and 800 [GeV/c] and $\theta_{cm} = 90^0$ as a function of the transverse momentum p_T. The experimental data are taken from [26–28]. (b) The corresponding scaling function $\psi(z)$.



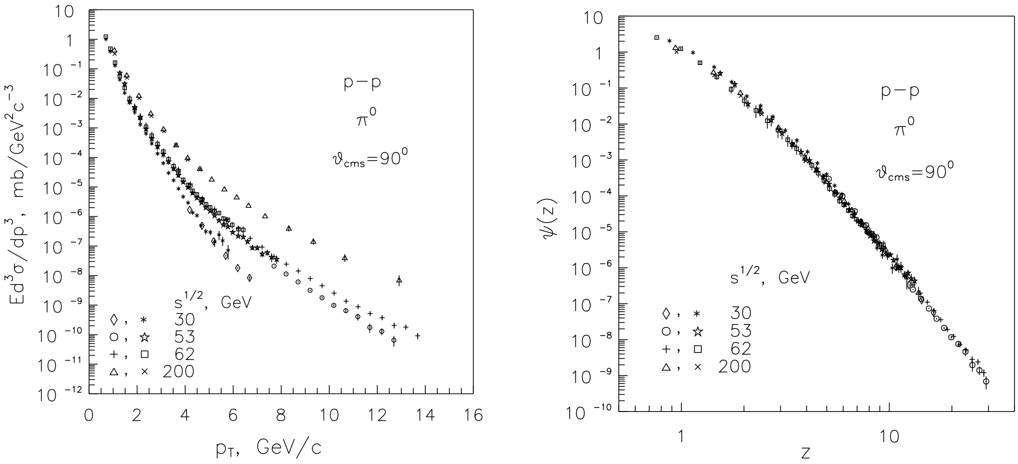
a) b)

Fig. 3. (a) The inclusive differential cross section for K^+-mesons produced in pp collisions at $p_L = 70, 200, 300, 400$ and 800 [GeV/c] and $\theta_{cm} \simeq 90^0$ as a function of the transverse momentum p_T. The experimental data are taken from [26–28]. (b) The corresponding scaling function $\psi(z)$.



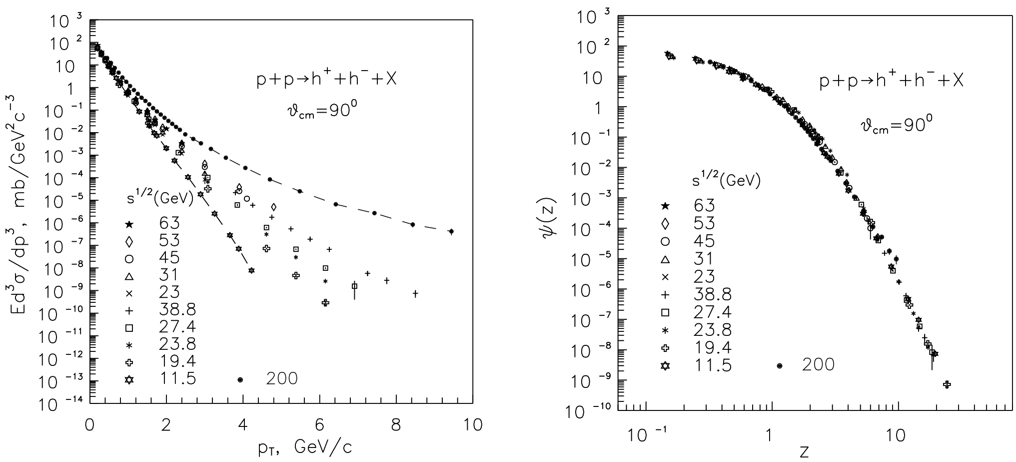
a) b)

Fig. 4. (a) The inclusive differential cross section for K^--mesons produced in pp collisions at $p_L = 70, 200, 300, 400$ and 800 [GeV/c] and $\theta_{cm} = 90^0$ as a function of the transverse momentum p_T. The experimental data are taken from [26–28]. (b) The corresponding scaling function $\psi(z)$.



a) b)

Fig. 5. (a) The dependence of the inclusive cross section of π^0-meson production on the transverse momentum p_T in pp collisions at $\sqrt{s} = 30, 53, 62$ and 200 [GeV] and the angle θ_{cm} of 90^0. The experimental data are taken from [30–32, 34–36]. (b) The corresponding scaling function $\psi(z)$.



a) b)

Fig. 6. (a) The inclusive cross section of charged hadron production in pp collisions versus transverse momentum at U70, ISR, Tevatron and RHIC energies $\sqrt{s} = (11.5 - 200)$ [GeV] and the angle θ_{cm} of 90^0. The experimental data are taken from [26–28, 37, 38]. (b) The corresponding scaling function $\psi(z)$.

in Fig. 7. The inclusive cross section versus the transverse momentum at $\sqrt{s} = (24 - 1800)$ [GeV] over the range $p_T = (4 - 110)$ [GeV/c] are shown in Fig. 7(a). Data used in the analysis are obtained by the UA1 [40], UA2 [41], UA6 [42], CDF [43] and D0 [44] Collaborations. The z-presentation of data (see Fig. 7(b)) demonstrates the energy independence of the scaling function ψ of high-p_T direct photon production over a wide kinematic range.

3.1.5 Jets

First observation of jets in p̄p collisions at the SpS was considered as compelling confirmation of parton structure of hadrons. In high energy collisions of hadrons copiously jet production due to hard parton scattering was observed. A jet represents a group of moving collimated particles. In the framework of QCD jets are distinguished to can be quark and gluon. Quark and gluon jets are initiated by fastest quark and gluon, respectively. It should be noted that a mechanism of jet formation is insufficient explored and not clearly understood till now. Therefore we hope that the scaling features of jet production could be useful to obtain additional constraints for models of jet formation.

In Figs. 8(a) we show the invariant cross sections of inclusive jet production in p̄p collisions at $\sqrt{s} = 630$ and 1800 [GeV]. These experimental data are obtained by the CDF [45] and D0 [46] Collaborations. A clear energy dependence of the cross section is observed to be. Difference between the cross sections at $\sqrt{s} = 630$ and 1800 [GeV] increases with a transverse energy of jet. The z-presentation of data shown in Fig. 8(b) demonstrates independence on the collision energy $\sqrt{s}$. The anomalous fractal dimension δ_{jet} for jet production in pp and p̄p collisions was found to be constant and equal to 1 [9].

3.2 A-dependence of $\psi(z)$

A comparison of particle yields in hadron-hadron and hadron-nucleus collisions is a basic method to study the nuclear matter influence on particle production. The elementary process is considered as probe of more complex system like nucleus. The difference between cross sections of particle production on free and bound nucleons was considered as an indication of unusual physics phenomena like EMC-effect, J/ψ-suppression, and Cronin effect.

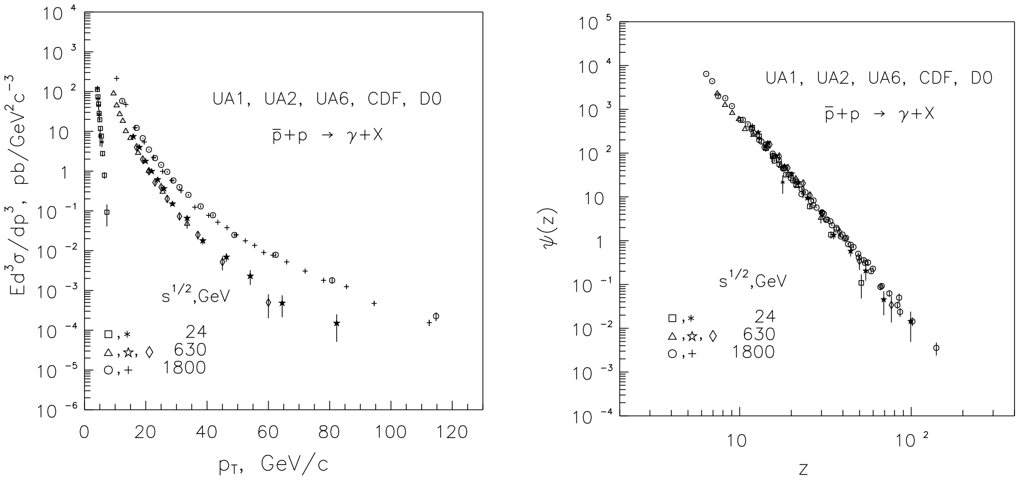
A change of the shape of p_T spectra is considered to be evidence that the mechanism of particle formation in nuclear matter is modified. Therefore it is convenient to compare scaling functions corresponding to different pA processes over a wide range of $\sqrt{s}$ and p_T.

The search for scaling features of particle formation in pA as well as in pp collisions and the study of their dependence on the atomic weight A are of interest for development of theory.

A-dependence of z-scaling of hadron production in pA collisions was studied in [7, 11]. It was established z-scaling for different nuclei (A=D-Pb) and types of produced particles ($\pi^{\pm,0}$, $K^{\pm}, \bar{p}$). To compare the scaling functions for different nuclei the symmetry transformation of z and $\psi(z)$

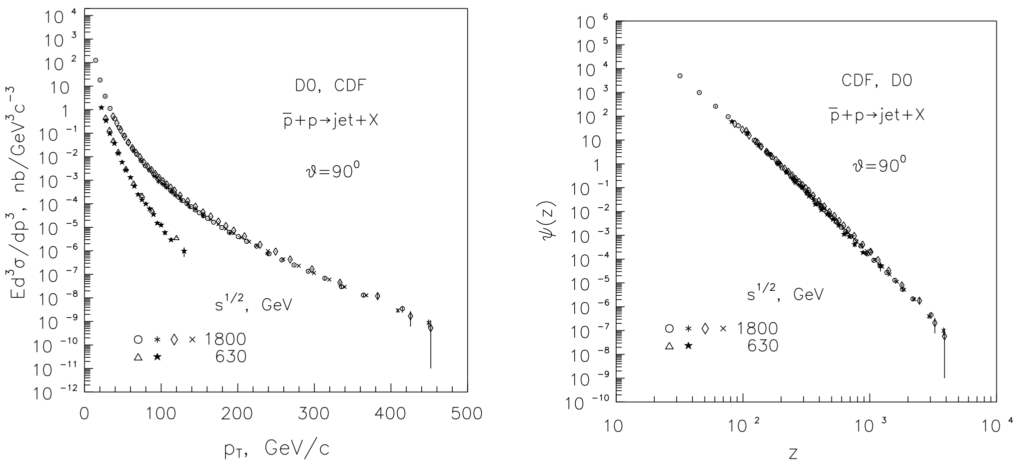
$$z \to \alpha_A z, \quad \psi \to \alpha_A^{-1} \psi \tag{20}$$

was used. The parameter α of the scale transformation (20) depends on the atomic weight A. It was parameterized by the formula $\alpha(A) = 0.9A^{0.15}$.



a) b)

Fig. 7. (a) The dependence of the inclusive cross section of direct photon production on the transverse momentum p_T in $\bar{\mathrm{p}}$p collisions at $\sqrt{s} = (24 - 1800)$ [GeV]. The experimental data obtained by the UA1, UA2, UA6, CDF and D0 Collaborations are taken from [40–44]. (b) The corresponding scaling functions.



a) b)

Fig. 8. (a) The inclusive cross section of jet production in $\bar{\mathrm{p}}$p collisions versus transverse momentum at Tevatron energies $\sqrt{s} = 630$ and 1800 [GeV] and $\theta_{cm} \simeq 90^0$ obtained by the CDF and D0 Collaborations. The experimental data are taken from [45, 46]. (b) The corresponding scaling function $\psi(z)$.

Figure 9(a) demonstrates the spectra of π^+-mesons produced in proton-nucleus collisions at $\sqrt{s} = 11.5$ and 27.4 [GeV] and $\theta_{cm}^{NN} \simeq 90^0$. Our new data analysis includes the experimental data obtained at Protvino [26] and Batavia [27, 28]. The data [28] extend transverse momentum range up to $p_T = 8.5$ [GeV/c]. A good compatibility of [27] and [28] data sets in the overlapping region was observed. The solid and dashed lines are obtained by fitting of the data for W, Pb and D, respectively. They demonstrate the strong dependence of p_T-spectra on the collision energy $\sqrt{s}$.

Figure 9(b) shows z-presentations of the same data. The obtained results are the new confirmation of z-scaling of high-p_T hadron production in pA collisions. The universality of the scaling function ψ for different nuclei means that mechanism of high-p_T particle formation in nuclear matter reveals property of self-similarity.

We use the parameterization $\alpha(A)$ to study A-dependence of π^0-meson and direct photon production in pA collisions. New data [47] obtained by the E706 Collaboration are used in the analysis. The experimental cross sections have been measured for pBe and pCu collisions at $\sqrt{s} = 31.6$ and 38.8 [GeV] and cover the p_T-range $(3 - 11)$ [GeV/c].

The p_T- and z-presentations of data for π^0-mesons produced in pA collisions are shown in Figs. 10(a) and 10(b), respectively.

Figure 11(a) demonstrates the spectra of photons produced in proton-nucleus collisions. As seen from Fig. 11(a) the p_T-spectra show the strong energy dependence. The difference between spectra at $\sqrt{s} = 31.6$ and 38.8 [GeV] increases with p_T. The z-presentation of the same data sets is shown in Fig.11(b). The scaling functions for both targets Be and Cu coincide each other. This is a direct confirmation that the nuclear effect for direct photon production can be described by the same function $\alpha(A)$ as for hadrons produced in proton-nucleus collisions [7]. The shape of the scaling functions is found to be a linear one on the log-log scale for both cases. The fit of the data is shown by the solid line in Fig. 11(b).

The value of the slope parameter β_{pA}^{γ} is constant over a wide p_T range and equal to 7.07. It means that the nuclear matter changes the probability of photon formation with different formation length z and does not change the fractal dimension of the mechanism of photon formation (photon "dressing").

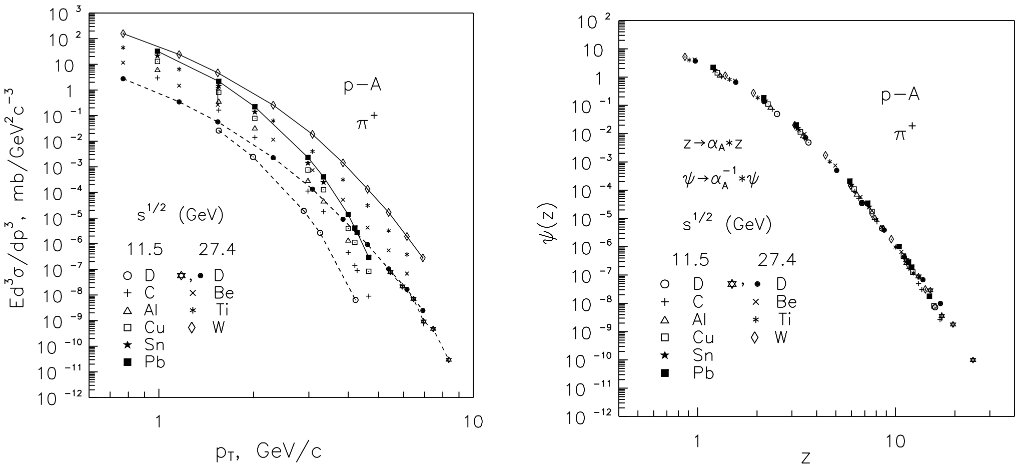
The obtained results show that the fractal dimension δ and the slope parameter β are independent of A. Therefore the experimental investigations of A-dependence of z-scaling for direct photons produced in hadron-nucleus collisions at RHIC and LHC energies are very important to obtain any indications on nuclear phase transition and formation of QGP.

3.3 Power law

One of the general properties of z-presentation of data is the power law of the scaling function

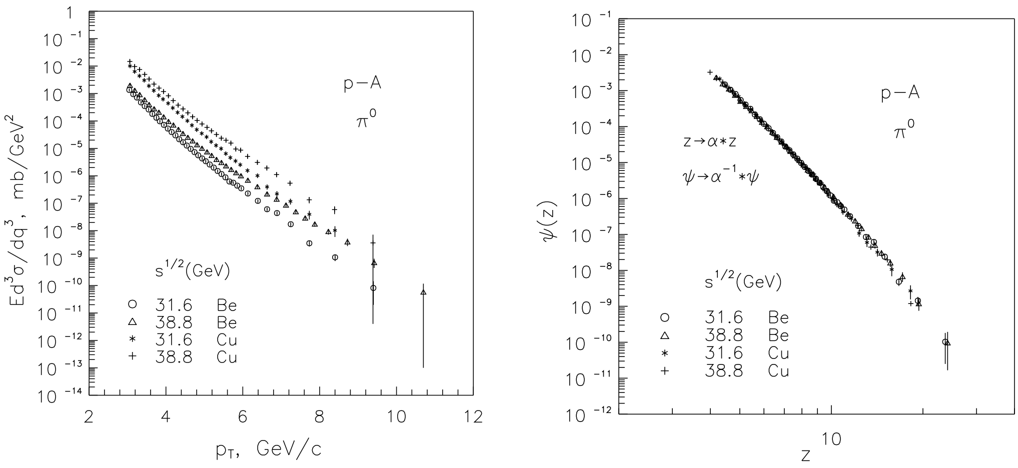
$$\psi(z) \sim z^{-\beta}. \tag{21}$$

Such behavior of ψ as seen from Figs. 1(b)-11(b) is observed for different particles (hadrons, direct photons) produced at $z > 4$. The data sets demonstrate a linear z-dependence of $\psi(z)$ on the log-log scale at high z. The quantity β is the slope parameter. The value of the slope parameter β is found to be constant with high accuracy. It is independent of energy $\sqrt{s}$ over a wide high transverse momentum range. Some indications (for π^0 mesons, charged hadrons,



a) b)

Fig. 9. (a) The inclusive differential cross section for π^+-mesons produced in pA collisions at $\sqrt{s} = 11.5$ and 27.4 [GeV] and $\theta_{cm}^{NN} \simeq 90^0$ as a function of the transverse momentum p_T. The solid and dashed lines are obtained by fitting of the data for W, Pb and D, respectively. The experimental data are taken from [26–28]. (b) The corresponding scaling function $\psi(z)$.



a) b)

Fig. 10. (a) The dependence of inclusive cross section of π^0-meson production on transverse momentum in pBe and pCu collisions at $\sqrt{s} = 31.6$ and 38.8 [GeV]. The experimental data are taken from [47]. The corresponding scaling functions $\psi(z)$.

direct photons, jets) are obtained that the value of slope parameter for pp is larger than the corresponding value for p̄p collisions, $\beta_{pp} > \beta_{\bar{p}p}$.

The existence of the power law means, from our point of view, that the mechanism of particle formation reveals fractal behavior.

4 Multiplicity density of charged particles $dN/d\eta$

The important ingredient of z-scaling concept is the multiplicity density of charged particles, $dN/d\eta(s, \eta)$. The scaling function ψ and the scaling variable z is proportional to $[dN/d\eta]^{-1}$. In the first case the quantity is included in the expression (5) to normalize the function ψ and to give the physical meaning for it as a probability density to produce a particle with the formation length z. In the second case (17) the multiplicity density is taken at $\eta = 0$. Therefore z is proportional to the energy of elementary subprocess per one particle produced in the initial hadron collision.

The energy dependence of the multiplicity density of charged particles for inelastic and non-single diffractive pp and p̄p collisions is shown in Fig. 12(a) [48]. The collision energy $\sqrt{s}$ changes from 14 to 1800 [GeV]. New data for $dN/d\eta(s, \eta)$ as well as for inclusive cross section $Ed^3\sigma/dp^3$ for pp collisions at RHIC energies are of interest for verification of z-scaling.

5 γ/π^0 ratio for pp and p̄p

The properties of the scaling function for direct γ and π^0-meson production can be used to estimate the dependence of the γ/π^0 ratio of inclusive cross sections on the transverse momentum p_T at LHC energies.

The asymptotic behavior of $\psi(z)$ was found to be described by the power law for π^0-mesons and direct photons produced in pp and $\bar{p}p$ collisions. The slope parameters are satisfied to the relations $\beta^{\gamma}_{pp} > \beta^{\gamma}_{\bar{p}p}$, $\beta^{\pi^0}_{pp} > \beta^{\pi^0}_{\bar{p}p}$, $\beta^{\pi^0}_{pp} > \beta^{\gamma}_{pp}$ and $\beta^{\pi^0}_{\bar{p}p} > \beta^{\gamma}_{\bar{p}p}$.

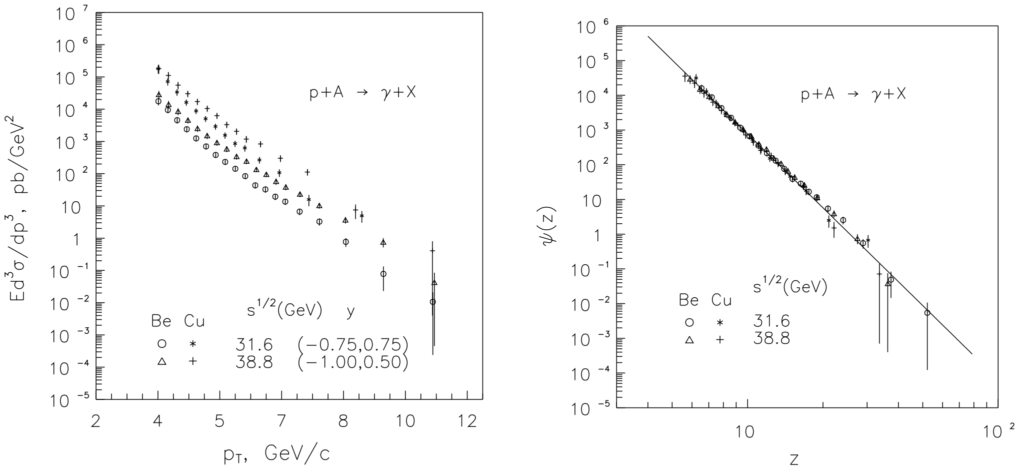
As seen from Fig. 5(b) the cross section data [30] of π^0-mesons produced in pp collisions obtained by the PHENIX Collaboration at RHIC are in a good agreement with the asymptotic behavior of $\psi(z)$.

Figure 12(b) shows the γ/π^0 ratio of inclusive cross sections as a function of the transverse momentum p_T at $\sqrt{s} = 5.5$ and 14. [TeV]. The ratio was found to be different for pp and p̄p collisions. It increases with p_T. The ratio has the cross-over point at $p_T \simeq (60 - 70)$ [GeV/c] and $p_T \simeq (110 - 130)$ [GeV/c] for pp and p̄p collisions, respectively.

6 $z - p_T$ plot

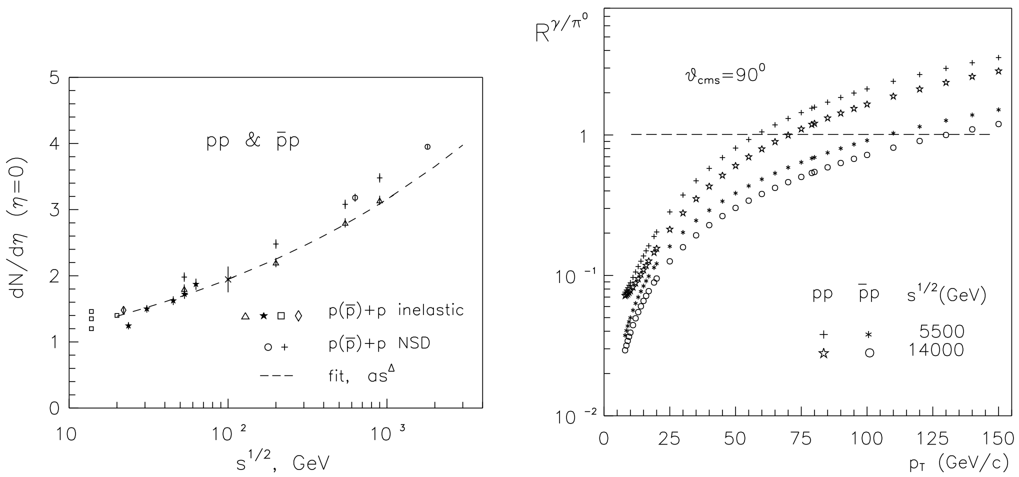
The $z - p_T$ plot is the dependence of the variable z on the transverse momentum p_T of produced particle for a given process. The plot allows us to determine a high transverse momentum range that is experimentally inaccessible till now, interesting for verification of z-scaling and searching for the scaling violation.

Figure 13(a) shows the $z - p_T$ plot for the pp→ π^+X process at $\sqrt{s} = (24 - 14000)$ [GeV]. As seen from Fig. 1(b) the scaling function $\psi(z)$ was measured up to $z \simeq 30$. The function $\psi(z)$ demonstrates the power behavior at $z > 4$. Therefore the kinematic range $z > 30$ is of more preferable for experimental investigations of z-scaling violation. The boundary $z = 30$



a) b)

Fig. 11. (a) The dependence of inclusive cross section of direct photon production on transverse momentum in pBe and pCu collisions at $\sqrt{s} = 31.6$ and 38.8 [GeV]. The experimental data are taken from [47]. (b) The corresponding scaling functions $\psi(z)$.



a) b)

Fig. 12. (a) The multiplicity density of charged particles $dN/d\eta$ as a function of the energy $\sqrt{s}$ at $\eta = 0$ for pp and p̄p collisions. The experimental data are taken from [48]. (b) The γ/π^0 ratio of inclusive cross sections versus the transverse momentum p_T in pp and p̄p collisions at $\sqrt{s} = 5.5$ and 14. [TeV].

corresponds to the different values of the transfers momentum p_T depending on the collision energy $\sqrt{s}$.

Figure 13 (b) shows our predictions of the dependence of the inclusive cross section $Ed^3\sigma/dp^3$ on the transverse momentum p_T for π^+-mesons produced in pp collisions at the ISR, RHIC and LHC energies and the angle θ_{cm} of 90^0. The verification of the predictions is of interest to determine the region of the scaling validity and search for new physics phenomena.

7 "δ-jump"

Mechanism of particle formation at high transverse momenta in z-presentation is described by the power law (21). Such behavior of $\psi(z)$ depends on the values of the anomalous fractal dimension of colliding particles, δ_1 and δ_2. The dimensions for hadrons, direct photons and jets produced in pp collisions were found to satisfy the relation $\delta_h < \delta_\gamma < \delta_{jet}$ and to be independent of $\sqrt{s}$ and p_T. The anomalous fractal dimension for nucleus δ_A is expressed via the dimension for nucleon δ_N as follows $\delta_A = A \cdot \delta_N$.

Figure 14(a) shows the dependence of the anomalous fractal dimension δ for π^0-meson production in pp and p̄p [49] collisions on the energy $\sqrt{s}$. The value of $\delta_h = 0.5$ used in our previous data analysis is confirmed by the new data [30] on inclusive cross section (see Fig. 5(b)) obtained by the PHENIX Collaboration at RHIC. Figure 14(b) gives evidence that the values of the slope parameter β of the scaling function for pp and p̄p collisions differ each other at $z > 6$.

The change of the fractal dimension δ or "δ-jump" is considered as an indication on new mechanism of particle formation. It is assumed that the energy dependence of the quantity is especially sensitive in the high-p_T range. Therefore the study of z-scaling at higher $\sqrt{s}$ and p_T is of interest for search for new physics phenomena.

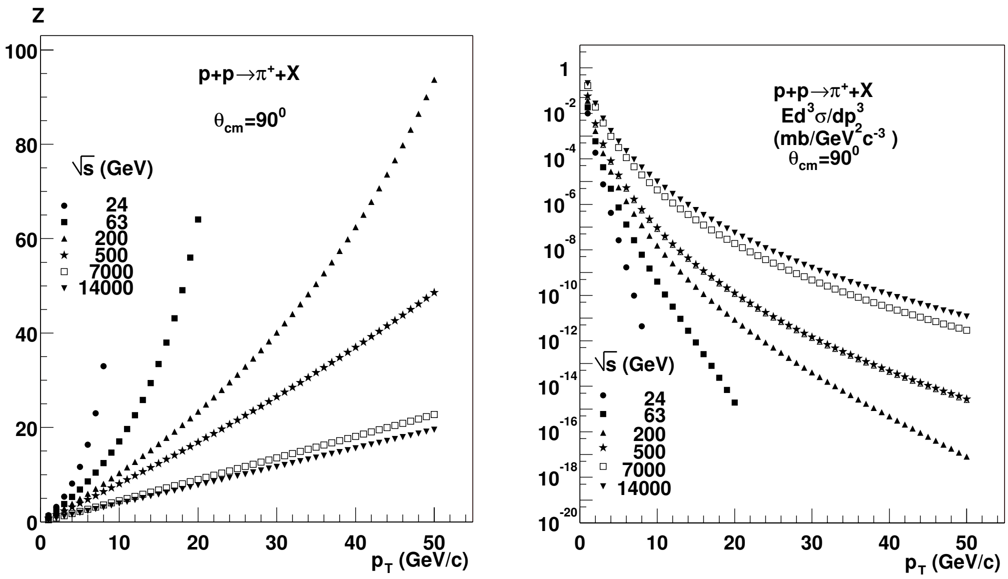
8 Direct γ and η^0-meson yields in pp and pPb collisions at RHIC and LHC

The scaling properties of z-presentation of data for pp and pA collisions can be used to estimate particle yields in the kinematic region experimentally inaccessible at present time and to compare with other model predictions.

Figures 15 and 16 show our predictions of the dependence of the inclusive cross section $Ed^3\sigma/dp^3$ on the transverse momentum p_T for direct photons (a) and η^0-mesons (b) produced in pp and pPb collisions at RHIC and LHC energies and the angle θ_{cm}^{NN} of 90^0. The data on the cross sections [33, 42, 50] obtained at ISR energy $\sqrt{s} = (24 - 63)$ [GeV] are also shown for comparison.

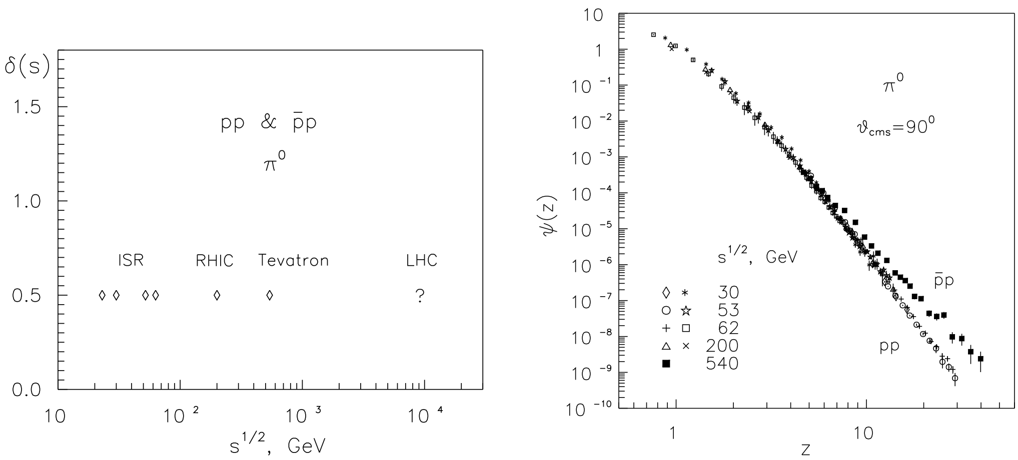
9 Conclusions

The general concept of z-scaling for particle production in hadron-hadron and hadron-nucleus collisions with high transverse momenta was reviewed. The development of the new method of data analysis based on z-presentation of data was presented. The scaling function $\psi(z)$ and scaling variable z were shown to be expressed via the experimental quantities, the invariant inclusive cross section $Ed^3\sigma/dp^3$ and the multiplicity density of charged particles $dN/d\eta(s,\eta)$.



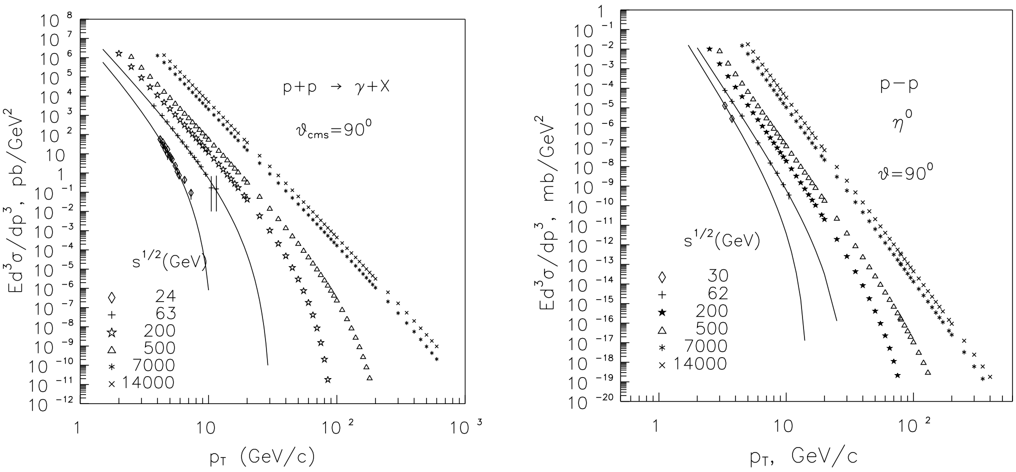
a) b)

Fig. 13. (a) The $z - p_T$ plot and (b) the dependence of the inclusive cross section of π^+-meson production in pp collisions on the transverse momentum p_T at $\sqrt{s} = (24 - 14000)$ [GeV] and the angle θ_{cm} of 90^0.



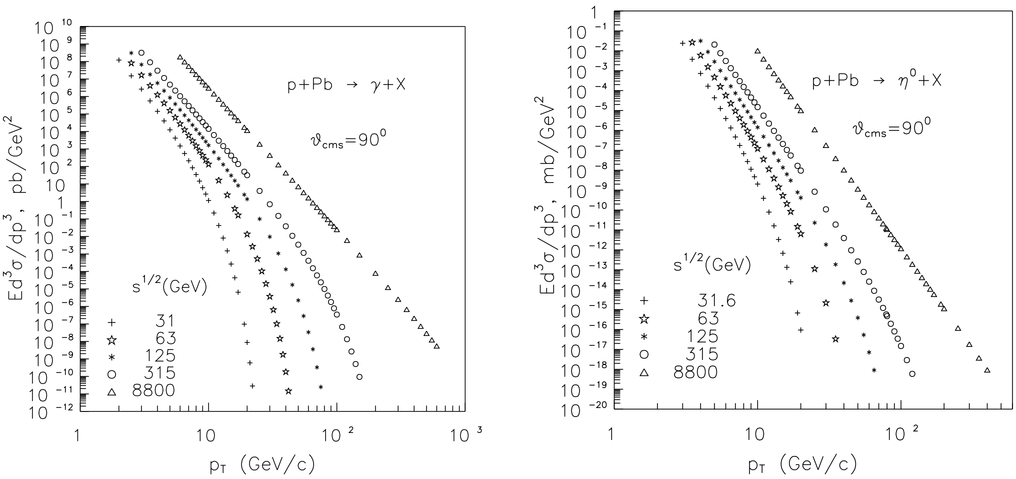
a) b)

Fig. 14. (a) The dependence of the anomalous fractal dimension $\delta(s)$ on the collision energy $\sqrt{s}$. (b) The scaling function $\psi(z)$ of π^0-meson production in pp and $\bar{\text{p}}$p collisions on the transverse momentum p_T at the energy $\sqrt{s} = 30 - 200$ and 540 [GeV] and the angle θ_{cm} of 90^0, respectively. The experimental data are taken from [30–32, 34–36] and [49].



a) b)

Fig. 15. The dependence of the inclusive cross section of direct photon (a) and η^0-meson (b) production on the transverse momentum p_T in pp collisions at $\sqrt{s} = (24 - 14000)$ [GeV]. The experimental data are taken from [33,42,50]. The solid lines and points ($\star$, $\triangle$, $*$, $\times$) are the calculated results.



a) b)

Fig. 16. The dependence of the inclusive cross section of direct photon (a) and η^0-meson (b) production on the transverse momentum p_T in pPb collisions at $\sqrt{s} = (31 - 8800)$ [GeV]. The points ($\triangle, \circ, *, \star, +$) are the calculated results.

Physical interpretation of the scaling function $\psi(z)$ and variable z as a probability density to produce a particle with the formation length z was argued. The quantity z was shown to reveal the property of the fractal measure and $\delta_{1,2}$ are the anomalous fractal dimensions of colliding particles. It was argued that z-scaling reflects the fundamental symmetries such as locality, self-similarity, fractality, and scale relativity.

Results of new analysis of the experimental data on the inclusive cross sections obtained at the U70, ISR, SpS, Tevatron and RHIC were presented. The scaling properties of z-presentation of data such as the energy independence, A-dependence, and the power law were discussed. A complementary confirmation of z-scaling for π^0-meson and charged hadron production in pp collisions at the RHIC was obtained.

New measurements of the multiplicity density of charged particles, and the inclusive cross sections of particle production in the experimentally inaccessible kinematic region for the study of z-scaling were suggested. The change of the anomalous fractal dimension ("δ-jump") was suggested to consider as a new and complementary signature of new physics phenomena of high-p_T particle production. The $z - p_T$ plot was used to determine the regions that are of more preferable for experimental search for z-scaling violation. The properties of z-presentation of data were used to predict high-p_T particle spectra at RHIC and LHC energies.

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