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QUANTUM INTERACTION OF FERMIONS WITH THE "FAR FIELD" OF A KERR BLACK-HOLE

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Using the S-matrix formalism and Feynman's diagram technique, the gravitational scattering of the zero-rest-mass and massive 1/2-spin and 3/2-spin particles on the "far field" of a Kerr black-hole is studied. The calculations have been made in the first order of the Born approximation, but taking into account the contribution brought by the covariant derivative, according to the principle of minimal coupling in Quantum Gravity.

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1 Introduction

The problem of the interaction of particles of various spin values by a gravitating body which generates an axisymmetric gravitational field has been considered in many studies (see for example [1–5]). In the present paper we develop an unified approach for the description of scattering processes for the massless and massive half-integer spin particles (more precisely, for the massless and massive 1/2-spin (Dirac) and 3/2-spin (Rarita-Schwinger) particles), in the frame of Quantum Gravity (QG). The concrete results (scattering cross-sections and the basic characteristics of the process of interaction between the half-integer spin particles and the static gravitational field described by Kerr geometry) generalize all the results previously obtained in Refs. [4, 6, 7], in the long-wavelength, weak-field limit (wavelength corresponding to the incident particles "size of body/radius of scatterer" gravitational radius/mass of scatterer).

As a difference from the method of other authors, in order to determine the interaction Lagrangians between the gravitational field and the considered fields we will use the principle of minimal coupling in QG, that is equivalent with considering the Lagrangians of the studied fields on a curved space-time. In fact, this implies replacing the usual derivatives in the expressions of the Lagrangians with the corresponding covariant derivatives. As we will see, this fact determines the appearance of some correction terms in the expressions of the interaction Lagrangians even in the first-order of approximation.

We have to underline that unlike the big majority of the authors that usually use Gupta's interaction Lagrangian (in the so-called "Gupta coupling formalism"; see for example [8]):

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$L_{int}^{Gupta} = -\frac{1}{2}\kappa h_{\mu\nu}T^{\mu\nu}$, we obtained the interaction Lagrangians using the above-mentioned method in the form: $L_{int} = L_{int}^{Gupta} + L_{int}^{corr}(\kappa)$, where $L_{int}^{corr}(\kappa)$ is the first-order correction in the coupling constant κ.

2 General conventions and notations

By $g^{\mu\nu}$, $\eta^{\mu\nu}$ and $y^{\mu\nu}$ we denoted the metric tensor, the Minkowski tensor - diag (+1, -1, -1, -1) - and the tensor of the weak gravitational field, respectively. Following Feynman [9] and since we require that $|y^{\mu\nu}| \ll 1$ everywhere, we expand the gravitational field as follows: $\sqrt{-g}g^{\mu\nu} = \eta^{\mu\nu} - \kappa y^{\mu\nu}$, where $g = \det(g_{\mu\nu})$ and $\kappa = \sqrt{16\pi G}$ (in natural unit system, G being the Newton constant).

In the Boyer-Lindquist coordinates, the geometry of a Kerr black-hole is described by the line element:

$$\begin{array}{l} ds^2 = \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \left(r^2 + a^2\right)\sin^2\theta d\varphi^2 - c^2dt^2 + \frac{2MGr}{\Sigma}\left(a\sin^2\theta d\varphi - cdt\right)^2; \\ \Sigma \equiv r^2 + a^2\cos^2\theta, \Delta \equiv r^2 - 2MGr + a^2. \end{array} \tag{1}$$

For large and very large r it goes over to the line element of a flat space-time. Thus, if in the "far field" one passes in the metric (1) to Cartesian coordinates: $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, then one obtains:

$$\begin{array}{l} ds^2 = \left[1 - \frac{2MG}{r} + \mathcal{O}\left(\frac{1}{r^3}\right)\right]dt^2 + \left[4\varepsilon_{jkl}G\frac{S^k x^l}{r^3} + \mathcal{O}\left(\frac{1}{r^3}\right)\right]dtdx^j - \\ - \left[\left(1 + \frac{2MG}{r}\right)\delta_{jk} + \left(\begin{array}{c}\text{gravitational radiation terms} \\ \text{that die out as } \mathcal{O}\left(\frac{1}{r}\right)\end{array}\right)\right] \times \\ \times dx^j dx^k,\ \left(j,k,l = \overline{1,3}\right). \end{array} \tag{2}$$

Here $\overrightarrow{S} = M\overrightarrow{a}$ is the angular momentum and M is the mass of the body that creates the gravitational field, so that

$$\begin{array}{l} g_{00} = 1 - \frac{2MG}{r}, g_{0j} = g_{j0} = \frac{2MG}{r^3}\varepsilon_{jkl}a_k x_l, \\ g_{jk} = -\left(1 + \frac{2MG}{r}\right)\delta_{jk},\ \left(j,k,l = \overline{1,3}\right). \end{array} \tag{3}$$

Taking into account the relations: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $h_{\mu\nu} = y_{\mu\nu} - \frac{1}{2}y\eta_{\mu\nu}$, $y = y_\alpha^\alpha$, we find

$$h_{00} = -\frac{2MG}{r}, h_{0j} = h_{j0} = 2\varepsilon_{jkl}\frac{MG}{r^3}a_k x_l, h_{jk} = -\frac{2MG}{r}\delta_{jk},\ \left(j,k,l = \overline{1,3}\right), \tag{4}$$

and the Fourier transform for the $y_{\mu\nu}$ is given by

$$\begin{array}{l} y_{0j}\left(\overrightarrow{q}\right) = y_{j0}\left(\overrightarrow{q}\right) = -iy_{4j}\left(\overrightarrow{q}\right) = -iy_{j4}\left(\overrightarrow{q}\right) = \frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{2|\overrightarrow{q}|^2}\varepsilon_{jkl}a_k q_l, \\ y_{00}\left(\overrightarrow{q}\right) = -y_{44}\left(\overrightarrow{q}\right) = \frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}, y_{jk}\left(\overrightarrow{q}\right) = 0,\ \left(j,k,l = \overline{1,3}\right). \end{array} \tag{5}$$

If the angular momentum per unit mass $\overrightarrow{a}$ vanishes in the above expressions, we recover the linearized Schwarzschild geometry [10]. In the following the components of 3-vectors are labeled by Latin indices, while components of 4-vectors carry Greek indices. Also, comma denotes the usual derivative, whereas semicolon denotes the covariant derivative with respect to the space-time coordinates.

3 Gravitational scattering of the spinor and spin-vector particles

The problem of the scattering of particles of various spins "propagating" in a slightly curved space-time may be treated by quantizing both the gravitational background and the scattered field. In such scenario the two fields couple according to the Feynman vertex rules. However, since our interest is restricted to a gravitational background geometry generated by energy-momentum distributions which are not appreciably affected by the scattering process, we may replace the virtual graviton by an external gravitational field, *i.e.*, we may use the external field approximation for stationary gravitational field described by Kerr geometry.

In this paper we shall limit ourselves to interactions proportional to κ^2 (single graviton exchange), calculating the scattering cross-sections in the first Born approximation. As one knows, the matter field theories in a fixed curved background are in general non-renormalizable. It is not even clear, so far, up to which extent the semiclassical approximation, *i.e.*, quantized matter in a classical background geometry, can provide reliable results. However, the processes studied in this paper prove to be finite, that is, the concrete calculus can be done to the end without obtaining divergent terms, at least in the first order of the Born approximation.

3.1 The case of zero-rest-mass particles

In order to obtain the first-order Lagrangians expressing the interaction between the gravitational field and the massless spinor and spin-vector fields, we use the principle of minimal coupling in QG. According to this principle, for massless spinor field we must add to the expression of the gravitational field Lagrangian, the massless spinor field Lagrangian written in the curved space [11]:

$$L_{spinor} \equiv L_{0,1/2} = \frac{i}{4}\sqrt{-g}\left[\overline{\psi}\gamma^{\mu}\left(1+\gamma^{5}\right)\psi_{;\mu} - \overline{\psi}_{;\mu}\gamma^{\mu}\left(1+\gamma^{5}\right)\psi\right]. \tag{6}$$

We have preferred to use the two-component massless neutrino theory, instead of four-component one, *i.e.*, for this field we have used the classical expression of Dirac Lagrangian in which (taking into account the Landau & Lifschitz's ideas [12]) there were made the following substitutions: $\psi \to \frac{1}{2}\left(1+\gamma^{5}\right)\psi$, $\overline{\psi} \to \frac{1}{2}\overline{\psi}\left(1-\gamma^{5}\right)$, where $\gamma^{5} = -i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}(= \gamma_{5} = \gamma_{1}\gamma_{2}\gamma_{3}\gamma_{4}$, when we pass to the flat space).

For the zero-rest-mass spin-3/2 field, by analogy with the spin-1/2 case, we considered the classical expression of the massless Rarita-Schwinger Lagrangian in which the analogous substitutions have been performed: $\psi_{\mu} \to \frac{1}{2}\left(1+\widetilde{\gamma}^{5}\right)\psi_{\mu}$, $\overline{\psi}_{\mu} \to \frac{1}{2}\overline{\psi}_{\mu}\left(1-\widetilde{\gamma}^{5}\right)$. For this field, besides the principle of minimal coupling we also used the vierbein formalism, so that the Lagrangian of this system can be written as follows (see also [13]):

$$\begin{aligned}&L_{spin-vector} \equiv L_{0,3/2} = \tfrac{i}{4}\sqrt{-g}g^{\mu\nu}\times\\ &\times\left[\overline{\psi}_{\mu}\overleftarrow{D}_{\lambda}\widetilde{\gamma}^{\lambda}\left(1+\widetilde{\gamma}^{5}\right)\psi_{\nu} - \overline{\psi}_{\mu}\widetilde{\gamma}^{\lambda}\left(1+\widetilde{\gamma}^{5}\right)\overrightarrow{D}_{\lambda}\psi_{\nu}\right],\end{aligned} \tag{7}$$

where $\widetilde{\gamma}^{\mu}$ are the generalized Dirac matrices: $\widetilde{\gamma}^{\mu} = L^{\mu}(\alpha)\widehat{\gamma}(\alpha)$, $\widetilde{\gamma}_{\mu} = L_{\mu}(\alpha)\widehat{\gamma}(\alpha)$; $L^{\mu}(\alpha)$ are the vierbein coefficients, $\widehat{\gamma}(\alpha)$ are the usual Dirac matrices in (-2) hyperbolic representation of Minkowski space-time and $D_{\mu}\psi_{\nu}$ is the covariant derivative of the spin-vector ψ_{ν}. It's worth mentioning that exactly the same results are obtained if, in order to describe the zero-rest-mass spin-3/2 particles, we use the appropriately spin-3/2 massless part of the supergravity Lagrangian:

$$L^{SG}_{0,3/2} = \frac{i}{2}\sqrt{-g}\varepsilon^{\alpha\beta\mu\nu}\left(\overline{\psi}_\alpha \overleftarrow{D}_\nu\, \widetilde{\gamma}_5\widetilde{\gamma}_\mu\psi_\beta - \overline{\psi}_\alpha\widetilde{\gamma}_5\widetilde{\gamma}_\mu \overrightarrow{D}_\nu\, \psi_\beta\right).$$

This fact was already proved by us [3, 7] for the Schwarzschild geometry, the choice used here being justified by reasons of calculus simplicity. in addition, we notice that the gravitational scattering of Rarita-Schwinger particles is rarely studied in concrete applications, even it exist an outstanding rigorous general approach by use of extended supergravities (see for example [14]).

Taking into account the de Donder-Fock gauge: $\left(\sqrt{-g}g^{\mu\nu}\right)_{,\nu} = 0$, developing all quantities in series in terms of κ and passing to the flat complex Minkowski space-time, the first-order interaction Lagrangians are:

$$L^{(int)(1)}_{0,1/2}(\kappa) = -\frac{1}{8}\kappa\left[\overline{\psi}\gamma_\mu(1+\gamma_5)\psi_{,\nu} - \overline{\psi}_{,\nu}\gamma_\mu(1+\gamma_5)\psi\right]s_{\mu\nu}, \tag{8}$$

and

$$\begin{aligned} L^{(int)(1)}_{0,3/2}(\kappa) = {} & \tfrac{1}{8}\kappa\left[\overline{\psi}_{\mu,\lambda}\gamma_\nu(1+\gamma_5)\psi_\mu - \overline{\psi}_\mu\gamma_\nu(1+\gamma_5)\psi_{\mu,\lambda}\right]y_{\lambda\nu} - \\ & -\tfrac{1}{32}\kappa\overline{\psi}_\mu\left[\gamma_\rho\gamma_\nu\gamma_\lambda(1+\gamma_5) + \gamma_\lambda(1+\gamma_5)\gamma_\rho\gamma_\nu\right]\psi_\mu\left(h_{\nu\lambda,\rho} - h_{\rho\lambda,\nu}\right), \end{aligned} \tag{9}$$

where: $h^{\mu\nu} = y^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}y$, $s_{\mu\nu} = y_{\mu\nu} + \frac{1}{2}\delta_{\mu\nu}y$, $y = y^\alpha_\alpha$. We also have taken advantage of the massless Rarita-Schwinger field equations: $i\widehat{\gamma}^\lambda\left(1+\widehat{\gamma}^5\right)\psi_{\nu,\lambda} = 0$, $i\overline{\psi}_{\nu,\lambda}\widehat{\gamma}^\lambda\left(1+\widehat{\gamma}^5\right) = 0$ and the well known anticommutation relations:

$$\{\widehat{\gamma}^\mu, \widehat{\gamma}^\nu\} = \widehat{\gamma}^\mu\widehat{\gamma}^\nu + \widehat{\gamma}^\nu\widehat{\gamma}^\mu = 2\eta^{\mu\nu}.$$

The processes are described by the Feynman's type diagram in Fig. 1, where p and (r) and also p' and (s) are the four-momenta and polarizations of the initial and final particles, respectively, and q is the four-momentum of the virtual graviton. The spatial orientation of the angular momentum $\overrightarrow{a}$ and the scattered direction $\overrightarrow{p}'$ relative to the incident direction $\overrightarrow{p}$ are shown in Fig. 2. Using the S-matrix formalism we deduced the Feynman rules for diagrams in the external gravitational field described by Kerr geometry, which allowed us to calculate the matrix element $\langle p'|S|p\rangle$ in the mentioned approximation. Thus we find that

$$\begin{cases} y^{ext}_{jk}\left(\overrightarrow{q}\right) = 0, \\ y^{ext}_{j4}\left(\overrightarrow{q}\right) = y^{ext}_{4j}\left(\overrightarrow{q}\right) = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}\left(\overrightarrow{a}\times\overrightarrow{q}\right)_j, \\ y^{ext}_{44}\left(\overrightarrow{q}\right) = y^{ext}\left(\overrightarrow{q}\right) = \frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}. \end{cases}$$

$$\begin{cases} s^{ext}_{jk}\left(\overrightarrow{q}\right) = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}\delta_{jk}, \\ s^{ext}_{j4}\left(\overrightarrow{q}\right) = s^{ext}_{4j}\left(\overrightarrow{q}\right) = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}\left(\overrightarrow{a}\times\overrightarrow{q}\right)_j, \\ s^{ext}_{44}\left(\overrightarrow{q}\right) = \frac{3}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\overrightarrow{q}|^2}. \end{cases}$$

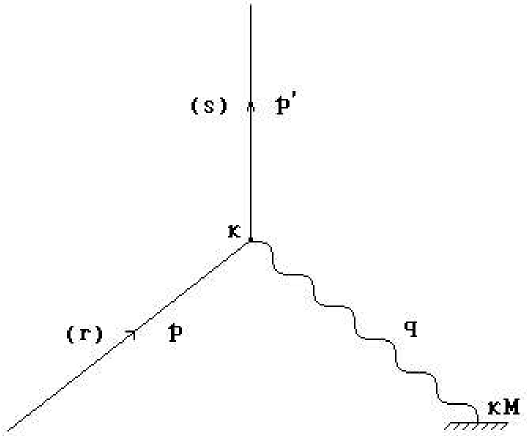

Fig. 1. The (fermionic field) - graviton - (fermionic field) vertex. The wavy line represents a graviton. The solid lines represent either spinor or spin-vector quanta.

$$\begin{cases} h_{jk}^{ext}\left(\vec{q}\right) = -\frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\vec{q}|^2}\delta_{jk}, \\ h_{j4}^{ext}\left(\vec{q}\right) = h_{4j}^{ext}\left(\vec{q}\right) = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\vec{q}|^2}\left(\vec{a}\times\vec{q}\right)_j, \\ s_{44}^{ext}\left(\vec{q}\right) = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\frac{\kappa M}{|\vec{q}|^2}. \end{cases}$$

We specify that any derivation with respect to the x_α is equivalent with the appearance of a supplementary factor iq_α. Taking into account the above relations, the matrix elements - in the external field approximation - corresponding to the diagram in Fig. 1 can be expressed as follows:

$$S_{p'p}^{(*)} \equiv \left\langle p' \left| S^{(*)} \right| p \right\rangle = F^{(*)}\left(p', p\right)\delta\left(q_0\right), \tag{10}$$

where the superscript symbol (*) takes place for each the 1/2 and 3/2, depending on which field is considered (*i.e.*, the spinor or spin-vector field respectively). In relation (10), $q_0 = p_0^{'} - p_0 = 0$ states for the energy conservation law. The differential cross-section is given by the well-known expression:

$$d\sigma = (2\pi)^2 \left\langle \sum_{f.sp.} \left|F\left(p', p\right)\right|^2 \right\rangle_{i.sp.} p_0^2 d\Omega, \tag{11}$$

where $d\Omega = \sin\theta\ d\theta\ d\varphi$, θ being the scattering angle. After a lengthy but straightforward calculation, for differential cross-sections of the two processes we obtained:

$$\begin{gathered} d\sigma_0^{(1/2)} = (GM)^2 \frac{d\Omega}{\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2} - (GM)^2 a p_0 \left(2\mathrm{ctg}^2\frac{\theta}{2}\cos\alpha + \sin\theta\csc^2\frac{\theta}{2}\times\right. \\ \left.\times\sin\alpha\cos\varphi\right) + (GM)^2 a^2 p_0^2 \left[\cos^2\frac{\theta}{2} + \frac{1}{2}\csc^2\frac{\theta}{2}\left(4 + 3\cos\theta + \cos^2\theta\right)\times\right. \\ \left.\times\sin^2\alpha + \frac{1}{2}\sin\theta\sin 2\alpha\cos\varphi - 4\mathrm{ctg}^2\frac{\theta}{2}\sin^2\alpha\cos^2\varphi\right] d\Omega, \end{gathered} \tag{12}$$

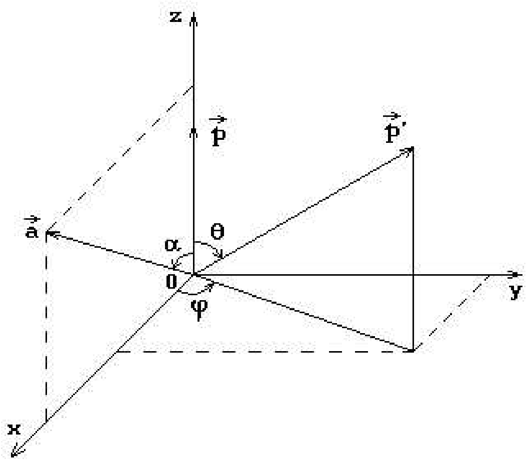

Fig. 2. The spatial orientation of the angular momentum $\vec{a}$ and the scattered direction $\vec{p}\,'$ relative to the incident direction $\vec{p}$.

$$\begin{aligned} d\sigma_0^{(3/2)} &= (GM)^2 \tfrac{d\Omega}{\sin^4 \frac{\theta}{2}} \cos^2 \tfrac{\theta}{2} + 2\,(GM)^2\, a p_0 \mathrm{ctg}^2 \tfrac{\theta}{2} \left(\sin\alpha \tan\tfrac{\theta}{2} \cos\varphi + \right. \\ &\left. + \cos\alpha\right) + (GM)^2\, a^2 p_0^2 \left[\cos^2 \tfrac{\theta}{2} + \tfrac{1}{2} \csc^2 \tfrac{\theta}{2} \left(4 + 3\cos\theta + \cos^2\theta\right) \sin^2\alpha + \right. \\ &\left. + \tfrac{1}{2} \sin\theta \sin 2\alpha \cos\varphi - 4\mathrm{ctg}^2 \tfrac{\theta}{2} \sin^2\alpha \cos^2\varphi\right] d\Omega, \end{aligned} \tag{13}$$

3.2 The case of massive particles

In this case we have started with the following actions describing the massive Dirac and Rarita-Schwinger fields:

$$L_{spinor} \equiv L_{m,1/2} = \frac{i}{2}\sqrt{-g}\left[\overline{\psi}\widetilde{\gamma}^{\mu}\psi_{;\mu} - \overline{\psi}_{;\mu}\widetilde{\gamma}^{\mu}\psi\right] - \sqrt{-g}m\overline{\psi}\psi, \tag{14}$$

and

$$\begin{aligned} L_{spin-vector} &\equiv L_{m,3/2} = \sqrt{-g} g^{\mu\nu}\left[\tfrac{i}{2}\left(\overline{\psi}_{\mu} \overleftarrow{D}_{\lambda}\, \widetilde{\gamma}^{\lambda}\psi_{\nu} - \overline{\psi}_{\mu}\widetilde{\gamma}^{\lambda} \overrightarrow{D}_{\lambda}\, \psi_{\nu}\right) + \right. \\ &\left. + m\overline{\psi}_{\mu}\psi_{\nu}\right] = {}^1 L_{m,3/2} + {}^2 L_{m,3/2}, \end{aligned} \tag{15}$$

where

$${}^1L_{m,3/2} = \sqrt{-g}g^{\mu\nu}\left[\frac{i}{2}\left(\overline{\psi}_{\mu,\lambda}\widetilde{\gamma}^{\lambda}\psi_{\nu} - \overline{\psi}_{\mu}\widetilde{\gamma}^{\lambda}\psi_{\nu,\lambda}\right) + m\overline{\psi}_{\mu}\psi_{\nu}\right], \tag{16}$$

and

$${}^2L_{m,3/2} = \frac{i}{8}\sqrt{-g}g^{\mu\nu}\overline{\psi}_{\mu}\left(\widetilde{\gamma}_{\rho;\lambda}\widetilde{\gamma}^{\rho}\widetilde{\gamma}^{\lambda} + \widetilde{\gamma}^{\lambda}\widetilde{\gamma}_{\rho;\lambda}\widetilde{\gamma}^{\rho}\right)\psi_{\nu}, \tag{17}$$

$\psi_{\nu,\lambda}$ being the usual derivative of the Rarita-Schwinger field function. Obviously there is a big difference in studying the interaction of massless and massive particles, that means of a system described by a Lagrangian containing no mass-term in comparison to one that contains such a term [15]. The presence of a mass term in the Lagrangian seriously affects the physical content of the problem. Even if gravitino is considered at the moment to have no mass, it can have a non-zero rest mass due to the cosmological constant, as shown by Deser and Zumino [16–18]. Concerning the Rarita-Schwinger field we used the mass term proposed by Deser, Kay and Stelle in [19], that means: $im\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu$, a term that for reasons of simplicity has been written under the more convenient form: $mg^{\mu\nu}\overline{\psi}_\mu\psi_\nu$, this fact being perfectly possible because of the following constraints: $\widetilde{\gamma}^\mu\psi_\mu = 0$ and $\partial_\mu\psi^\mu = 0$. The above expressions for ${}^1L_{m,3/2}$ and ${}^2L_{m,3/2}$ have been obtained inserting the expression of the covariant derivative of the spin-vectors: $\overrightarrow{D}_\mu\,\psi_\nu = \psi_{\nu,\mu} - \Gamma_\mu\psi_\nu$, $\overline{\psi}_\nu\,\overleftarrow{D}_\mu = \overline{\psi}_{\nu,\mu} - \overline{\psi}_\nu\Gamma_\mu$, where Γ_μ are the Fock-Ivanenko spin coefficients of the affine connection. It is very simple to show that the ${}^2L_{m,3/2}$ term in (15) gives no contribution in the first-order approximation, so that for the first-order interaction Lagrangian between the weak gravitational and the massive Rarita-Schwinger fields reads

$$\begin{aligned} &L^{(int)(1)}_{m,3/2}(\kappa) = -\tfrac{i}{4}\kappa\left(\overline{\psi}_{\lambda,\mu}\widehat{\gamma}_\nu\psi^\lambda - \overline{\psi}_\lambda\widehat{\gamma}_\nu\psi^\lambda_{,\mu}\right)h^{\mu\nu} - \tfrac{i}{2}\kappa\times \\ &\times\left(\overline{\psi}_{\mu,\lambda}\widehat{\gamma}^\lambda\psi_\nu - \overline{\psi}_\mu\widehat{\gamma}^\lambda\psi_{\nu,\lambda}\right)y^{\mu\nu} - \kappa m\overline{\psi}_\mu\psi_\nu y^{\mu\nu}. \end{aligned} \tag{18}$$

Taking into account the previous considerations and passing to the flat Minkowski space-time, the first-order interaction Lagrangians between the weak gravitational and the massive 1/2-spin and 3/2-spin fields are

$$L^{(int)(1)}_{m,1/2}(\kappa) = -\frac{1}{4}\kappa\left[\overline{\psi}\gamma_\mu\psi_{,\nu} - \overline{\psi}_{,\nu}\gamma_\mu\psi\right]s_{\mu\nu} - \frac{1}{2}\kappa m\overline{\psi}\psi y, \tag{19}$$

and

$$L^{(int)(1)}_{m,3/2}(\kappa) = \frac{1}{4}\kappa\left(\overline{\psi}_{\alpha,\nu}\gamma_\mu\psi_\alpha - \overline{\psi}_\alpha\gamma_\mu\psi_{\alpha,\nu}\right)y_{\mu\nu} - \frac{1}{4}\kappa m\overline{\psi}_\alpha\psi_\alpha y. \tag{20}$$

We also have taken advantage of the massive Rarita-Schwinger field equation: $\gamma_\mu\psi_{\alpha,\mu} = -m\psi_\alpha$ and its adjoint. After the same steps as in the case of zero-rest-mass particles, for the differential cross-sections of the massive 1/2-spin and 3/2-spin particles we obtained:

$$\begin{aligned} &d\sigma^{(1/2)}_m = (GM)^2\frac{d\Omega}{\sin^4\frac{\theta}{2}}\left[\left(\frac{1+v^2}{2v^2}\right)^2 - \frac{3+v^2}{4v^2}\sin^2\frac{\theta}{2}\right] + (GM)^2a^2p_0^2\times \\ &\times\left[f_1(\theta) + f_2(v,\theta)\sin^2\alpha + f_3(\theta)\sin 2\alpha\cos\varphi + \right. \\ &\left. + f_4(\theta)\cos^2\varphi + f_5(v,\theta)\sin^2\alpha\cos^2\varphi\right]d\Omega, \end{aligned} \tag{21}$$

$$\begin{aligned} &d\sigma^{(3/2)}_m = (GM)^2\frac{d\Omega}{\sin^4\frac{\theta}{2}}\left\{\left(\frac{1+v^2}{2v^2}\right)^2 - \frac{1}{36v^4(1-v^2)^2}\left[\left(15 - 41v^2 + \right.\right.\right. \\ &\left. +5v^4 + 21v^6\right)v^2\sin^2\frac{\theta}{2} + 4\left(3 - 6v^2 - 5v^4\right)v^4\sin^4\frac{\theta}{2} + 8\left(3 + v^2\right)\times \\ &\left.\left.\times\, v^6\sin^6\frac{\theta}{2}\right]\right\} + (GM)^2a^2p_0^2\left[f_1(v,\theta) + f_2(v,\theta)\sin^2\alpha + f_3(v,\theta)\times\right. \\ &\left.\times\sin 2\alpha\cos\varphi + f_4(v,\theta)\cos^2\frac{\alpha}{2}\cos^2\varphi\right]d\Omega, \end{aligned} \tag{22}$$

where we denoted by v the $\frac{|\overrightarrow{p}|}{p_0}$ ratio. The corresponding expressions for $f_i(v,\theta)$, $i = \overline{1,5}$ appearing in the above relations are given in Appendix A.

4 Special limit cases and integral cross-sections

Allowing $\vec{a}$ to vanish (linearized Schwarzschild geometry) for the massless fields we arrive at

$$d\sigma_{Schw}^{(1/2)massless} = d\sigma_{Schw}^{(3/2)massless} = (GM)^2 \frac{d\Omega}{\sin^4 \frac{\theta}{2}} \cos^2 \frac{\theta}{2}, \tag{23}$$

i.e., both massless neutrino and gravitino are scattered in the same manner by a slightly curved Schwarzschild background, while for the massive fields we get

$$d\sigma_{Schw}^{(1/2)massive} = (GM)^2 \frac{d\Omega}{\sin^4 \frac{\theta}{2}} \left[\left(\frac{1+v^2}{2v^2} \right)^2 - \frac{3+v^2}{4v^2} \sin^2 \frac{\theta}{2} \right], \tag{24}$$

and

$$\begin{aligned} d\sigma_{Schw}^{(3/2)massive} &= (GM)^2 \tfrac{d\Omega}{\sin^4 \frac{\theta}{2}} \left\{ \left(\tfrac{1+v^2}{2v^2} \right)^2 - \tfrac{1}{36v^4(1-v^2)^2} \left[\left(15 - 41v^2 + 5v^4 + \right.\right.\right. \\ &\left.\left.\left. +21v^6 \right) v^2 \sin^2 \tfrac{\theta}{2} + 4 \left(3 - 6v^2 - 5v^4 \right) v^4 \sin^4 \tfrac{\theta}{2} + 8 \left(3 + v^2 \right) v^6 \sin^6 \tfrac{\theta}{2} \right] \right\}. \end{aligned} \tag{25}$$

If, supplementary, we ask for the angle θ to take small and/or very small values, then, for the massless fields both the differential cross-sections are of this form:

$$d\sigma_{Schw}^{massless}\big|_{small\theta} = (GM)^2 \frac{d\Omega}{\sin^4 \frac{\theta}{2}} \equiv d\sigma_{Ruth}^{massless} \tag{26}$$

and for the massive fields we have

$$d\sigma_{Schw}^{massive}\big|_{small\theta} = (GM)^2 \frac{d\Omega}{\sin^4 \frac{\theta}{2}} \left(\frac{1+v^2}{2v^2} \right)^2 \equiv d\sigma_{Ruth}^{massive}, \tag{27}$$

result which in the ultrarelativistic limit $(v \to 1)$ coincides with that obtained for massless particles. As one can see from relations (26) and (27), for $\vec{a} = 0$ the differential cross-sections are all of Rutherford type. This result coincides with that obtained for the massless 0-, 1- and 2-spin particles in the same approximation [1] and also for the massive integer spin particles (when supplementary, the ultrarelativistic limit is supposed) [10, 20], proving that in this special limit case the gravitational particle scattering is spin independent.

The method we have used in order to determine the differential cross-sections allows us to study the backward scattering limit case, since the obtained results (concerning $\frac{d\sigma}{d\Omega}$) are valid for any value of the scattering angle. Thus, the massless 1/2-spin and 3/2-spin particles have finite values for the differential cross-sections that depend on the squared angular momentum per unit mass $\vec{a}^2$ and also on the angle α :

$$\left.\frac{d\sigma}{d\Omega}^{(1/2)}\right|_{\theta=\pi} = \left.\frac{d\sigma}{d\Omega}^{(3/2)}\right|_{\theta=\pi} = (GM)^2 \vec{a}^2 p_0^2 \sin^2 \alpha. \tag{28}$$

The massive 1/2-spin particles have also finite value for the differential backward cross-section that depends on the squared angular momentum per unit mass $\vec{a}^2$ and also on the angles

α and φ. It's interesting to point out that the angular dependence of this differential backward cross-section still appear and for the UR limit case, when

$$\left.\frac{d\sigma}{d\Omega}^{(1/2)}\right|_{\theta=\pi, v=1} = (GM)^2 \vec{a}^{\,2} p_0^2 \left(\frac{9}{2}\cos^2\varphi + \sin^2\alpha\right). \tag{29}$$

Concerning the massive 3/2-spin field, as it can be easily to check up, it has α - depending differential backward cross-section, and in the UR limit case this scattering cross-section has diverging expression. This fact can be better seen from Fig. 7, that represents the integral scattering cross-section as function of the both ratio $\frac{|\vec{p}|}{p_0} \equiv v$ and angle α. As this figure shows, for the UR case the graph tends asymptotically to the straight-line $v = 1$.

Because of the fact that in the $\theta \to 0$ limit the differential cross-sections are all of Rutherford type, there appear to be problems in computing the integral scattering cross-sections. To be more precisely, in the $\theta \to 0$ limit there are obtained diverging terms due to a dependence in $\frac{1}{\sin^4\frac{\theta}{2}}$ of the differential cross-sections on the diffusion angle, so the integral scattering cross-sections tend to infinity due to the infinite action range of the gravitational interaction. We overpassed this difficulty using a cut-off procedure based on the Leibniz-Newton formula. The computations give the following expressions for the integral cross-sections:

$$\begin{aligned}\sigma^{(1/2)}_{massless}(\alpha,\theta) = 4\pi\Big\{&\tfrac{1}{4}\Big[\vec{a}^{\,2} p_0^2(1-2\cos 2\alpha) - 4ap_0\cos\alpha\Big]\cos\theta - \tfrac{1}{16}\vec{a}^{\,2} p_0^2\times\\ &\times\cos 2\alpha\cos 2\theta - \csc^2\tfrac{\theta}{2} + 2\Big(2\vec{a}^{\,2} p_0^2\sin^2\alpha - 2ap_0\cos\alpha - 1\Big)\ln\left|\sin\tfrac{\theta}{2}\right|\Big\},\end{aligned} \tag{30}$$

$$\begin{aligned}\sigma^{(3/2)}_{massless}(\alpha,\theta) = 4\pi\Big\{&\tfrac{1}{4}\Big[\vec{a}^{\,2} p_0^2(1-2\cos 2\alpha) + 4ap_0\cos\alpha\Big]\cos\theta - \tfrac{1}{16}\vec{a}^{\,2} p_0^2\times\\ &\times\cos 2\alpha\cos 2\theta - \csc^2\tfrac{\theta}{2} - 2\Big(1 - 2ap_0\cos\alpha - 2\vec{a}^{\,2} p_0^2\sin^2\alpha\Big)\ln\left|\sin\tfrac{\theta}{2}\right|\Big\},\end{aligned} \tag{31}$$

$$\begin{aligned}\sigma^{(1/2)}_{massive}(v,\alpha,\theta) = &\tfrac{\pi}{4}\vec{a}^{\,2} p_0^2\left(3 + 2v^2\sin^2\alpha - 7\cos 2\alpha\right)\cos\theta - \tfrac{\pi}{16}\vec{a}^{\,2} p_0^2\times\\ &\times\left(1 + 2v^2\sin^2\alpha - 3\cos 2\alpha\right)\cos 2\theta - \tfrac{\pi}{v^4}\left(1+v^2\right)^2\csc^2\tfrac{\theta}{2} -\\ &-\tfrac{2\pi}{v^2}\Big[3 + v^2\Big(1 - 13\vec{a}^{\,2} p_0^2 + 4\vec{a}^{\,2} p_0^2\cos 2\alpha\Big)\Big]\ln\left|\sin\tfrac{\theta}{2}\right|,\end{aligned} \tag{32}$$

$$\begin{aligned}\sigma^{(3/2)}_{massive}(v,\alpha,\theta) = &\tfrac{\pi}{432(1-v^2)^2}\Big[6f_1(v,\alpha)\cos\theta + 6f_2(v,\alpha)\cos 2\theta + \vec{a}^{\,2} p_0^2\times\\ &\times\left(f_3(v,\alpha)\cos 3\theta + \tfrac{3}{2}f_4(v,\alpha)\cos 4\theta\right)\Big] - \tfrac{\pi v^4}{360(1-v^2)}\vec{a}^{\,2} p_0^2\cos^2\tfrac{\alpha}{2}\times\\ &\times\cos 5\theta - \tfrac{\pi\left(1-v^2\right)^2}{v^4}\csc^2\tfrac{\theta}{2} - \tfrac{2\pi}{9v^2(1-v^2)}f_5(v,\alpha)\ln\left|\sin\tfrac{\theta}{2}\right|,\end{aligned} \tag{33}$$

where the corresponding functions $f_i(v,\alpha)$, $i = \overline{1,5}$ are given in Appendix A, and the graphs for these integral cross-sections are given in different cases in Figs. 3÷8. We have to say that in relations (30)÷(33) we disconsidered the factor $(GM)^2$ which, as a constant, does not affects the shape of the surfaces and curves, and for convenience, in numerical calculations we considered $\vec{a}^{\,2} p_0^2 = 1$. Therefore the numerical values appearing in all the figures do not have quantitative significance, but only qualitative one.

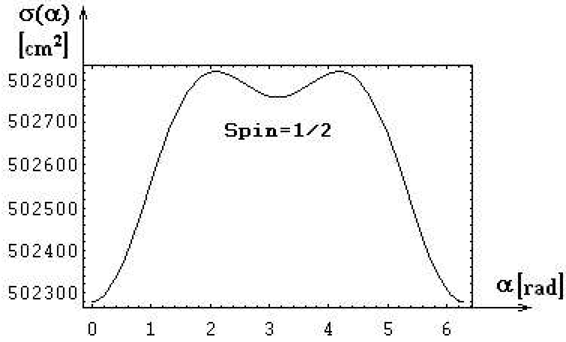

Fig. 3. Variation of integral cross-section $\sigma\,(\alpha)$ with respect to the angle α, for massless spinor particles.

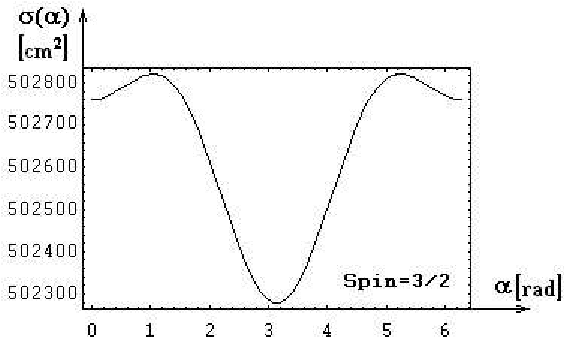

Fig. 4. Variation of integral cross-section $\sigma\,(\alpha)$ with respect to the angle α, for massless spin-vector particles.

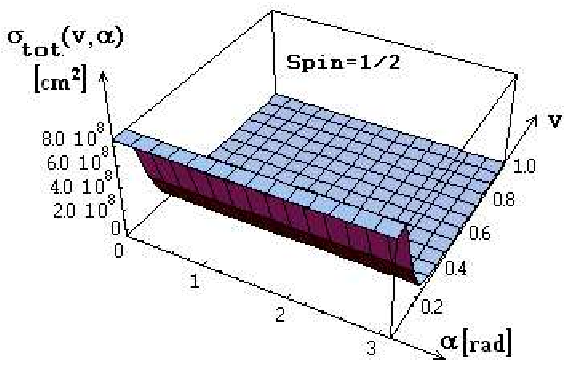

Fig. 5. Variation of integral (total) cross-section $\sigma_{tot.}\,(v,\alpha)$ with respect to the both ratio $v = \frac{|\vec{p}|}{p_0}$ and angle α, for massive spinor particles.

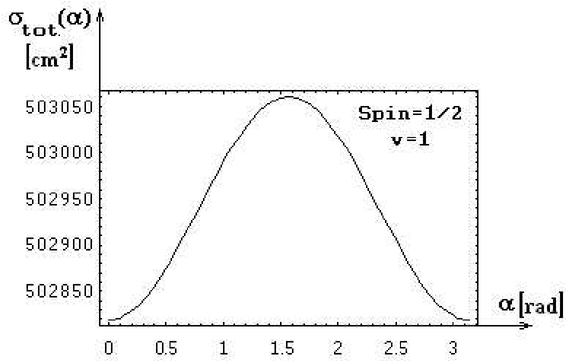

Fig. 6. Variation of integral (total) cross-section $\sigma_{tot.}(\alpha)$ with respect to the angle α, for massive spinor particles in the UR case ($v = 1$).

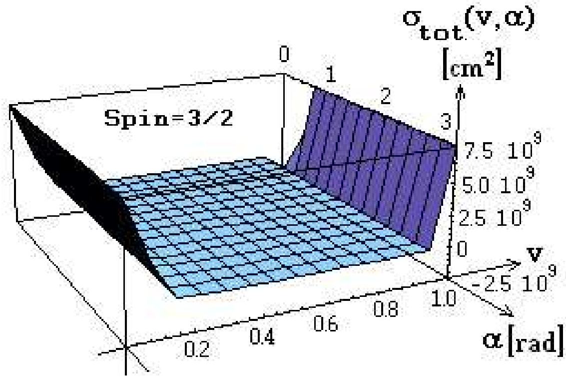

Fig. 7. Variation of integral (total) cross-section $\sigma_{tot.}(v, \alpha)$ with respect to the both ratio $v = \frac{|\vec{p}|}{p_0}$ and angle α, for massive spin-vector particles.

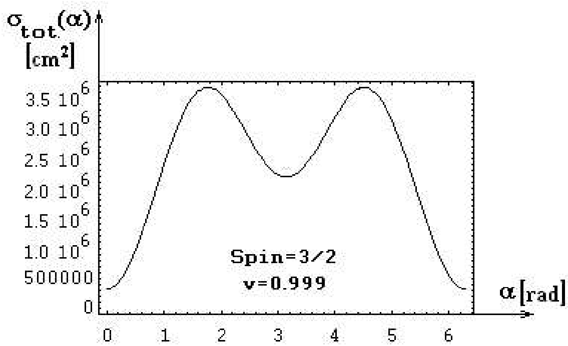

Fig. 8. Variation of integral (total) cross-section $\sigma_{tot.}(\alpha)$ with respect to the angle α, for massive spin-vector particles in the quasi-UR case ($v = 0.999$).

5 Summary and Conclusions

We develop an unified approach to the description of scattering processes for the massless and massive spinor and spin-vector (Rarita-Schwinger) particles in the frame of QG. The basic method we used here is that offered by the principle of minimal coupling in QG. The calculations have been made in the first order of approximation but also taking into account the contribution given by the covariant derivative, according to the principle of minimal coupling in QG. Consequently comes a greater generality of the results, referring to the values scale of the scattering angle, which thus is not forced to take only small enough values. This fact allows us to approach the special backward scattering limit case. The differential cross-sections for the weak-field gravitational scattering of spinor and spin-vector particles have been calculated using Feynman perturbative methods. The expressions giving the differential cross-sections for massive half-integer spin particles are of the form: $\left(\frac{d\sigma}{d\Omega}\right)_{Kerr} = \left(\frac{d\sigma}{d\Omega}\right)_{Schw} + terms\ which\ are\ proportional\ to\ \vec{a}^{\,2}\ p_0^2$, whereas for the massless particles the contribution of the angular momentum to the differential cross-section includes a term proportional to ap_0. For $\vec{a} = 0$ and $\theta \ll 1$ they exhibit a Rutherford-type dependence. We have studied the backward scattering limit case and we also determined the integral scattering cross-sections as $\theta-$ indefinite integrals of the differential cross-sections. The graphical representations of these integral cross-sections were done.

Finally it's worthwhile to point out that for $\vec{a} = 0$ and $\theta \ll 1$ the scattering cross-sections of massless half-integer spin particles and also of corresponding massive particles have the same form, and in UR case they coincide with those corresponding to the massless scalar particles, photons and gravitons, *i.e.*, the gravitational particle scattering in this limit case is spin-independent in agreement with other authors' results (*e.g.* [21,22]) obtained by different means in few particular cases.

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Appendix A

Below we enlist the explicit expressions appearing in relations (21), (22) and (33).

1. In relation (21):

$f_1(\theta) = \frac{1}{4}\sin^2\theta \csc^2\frac{\theta}{2};$

$f_2(v,\theta) = \frac{1}{4}\csc^2\frac{\theta}{2}\left(9 + 6cos\theta + \cos^2\theta - v^2\sin^2\theta\right);$

$f_3(\theta) = \frac{1}{2}\sin\theta;$

$f_4(\theta) = \frac{9}{2}\csc^2\frac{\theta}{2};$

$f_5(v,\theta) = \frac{1}{4}\csc^2\frac{\theta}{2}\left(-9 - 8\cos\theta + \cos^2\theta + v^2\sin^2\theta\right).$

2. In relation (22):

$f_1(v,\theta) = \frac{1}{3}\csc^2\frac{\theta}{2}\{82 - v^2\left(129 - 68v^2 + 3v^4\right) + 2\cos\theta\left(31 - 70v^2 + \right.$
$\left. + 27v^4\right) - v^2\cos^2\theta\left(19 - 14v^2 - v^4\right) + 2v^4\cos^3\theta\left(4 + v^2\cos\theta\right) - [77 -$
$-v^2\left(123 - 65v^2 + 3v^4\right) + 2\cos\theta\left(31 - 68v^2 + 27v^4\right) + \cos^2\theta\left(5 - 25v^2 + \right.$
$\left. + 15v^4 + v^6\right) - 4v^2\cos^3\theta\left(1 - 2v^2\right) + 2v^4\cos^4\theta\left(1 + v^2\right)]\}\left(1 - v^2\right)^{-2};$

$f_2(v,\theta) = \frac{1}{36}\csc^2\frac{\theta}{2}\{82 - v^2\left(129 - 68v^2 + 3v^4\right) + 2\cos\theta\left(31 - 70v^2 + \right.$
$\left. + 27v^4\right) - v^2\cos^2\theta\left(19 - 14v^2 - v^4\right) + 2v^4\cos^3\theta\left(4 + v^2\cos\theta\right)] \times$
$\times\left(1 - v^2\right)^{-2} - f_1(v,\theta);$

$f_3(v,\theta) = \frac{1}{18}\sin\theta\left(5 - 6v^2 + 3v^4 - 4v^2\cos\theta + 2v^4\cos^2\theta\right)\left(1 - v^2\right)^{-2};$

$f_4(v,\theta) = -\frac{2}{9}\cos^2\frac{\theta}{2}\left(1 - v^2\right)^{-2}\left[77 - v^2\left(123 - 65v^2 + 3v^4\right) - \cos\theta\left(5 + 25v^2 + \right.\right.$
$\left.\left. + 5v^4 - 3v^6\right) + 2v^2\cos^2\theta\left(2 + 7v^2 - v^4\right) - 2v^4\cos^3\theta\left(1 - v^2\right)\right]\left(1 - v^2\right)^{-2}.$

3. In relation (33):

$f_1(v,\alpha) = 48\left(1 - v^2\right) - 64v^4 + \vec{a}^2 p_0^2\left[\left(531 - 1121v^2 + 566v^4 + 4v^6\right) + \right.$
$+ \left(303 - 513v^2 + 268v^4 - 10v^6\right)\cos\alpha - \left(268 - 664v^2 + 326v^4 + 14v^6\right) \times$
$\left. \times \cos 2\alpha\right];$

$f_2(v,\alpha) = -4v^2\left(3 + v^2\right) + \vec{a}^2 p_0^2\left[\left(67 - 155v^2 + 84v^4 + 4v^6\right) + \right.$
$\left. + \left(72 - 146v^2 + 66v^4\right)\cos\alpha - \left(5 - 29v^2 + 26v^4 + 4v^6\right)\cos 2\alpha\right];$

$f_3(v,\alpha) = -10 - 34v^2 + 27v^4 + 9v^6 - \left(10 + 42v^2 - 15v^4 - 5v^6\right)\cos\alpha +$
$+ \left(8v^2 - 20v^4 - 4v^6\right)\cos 2\alpha;$

$f_4(v,\alpha) = 2 + 5v^2 + v^4 + \left(2 + 6v^2\right)\cos\alpha - v^2\left(1 + v^2\right)\cos 2\alpha;$

$f_5(v,\alpha) = 15 - 26v^2 - 21v^4 - 144v^2\,\vec{a}^2 p_0^2\left(1 - v^2\right)\sin^2\alpha.$