QUANTUM TELEPORTATION OF QUDITS BETWEEN MANY PARTIES

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The paper presents general protocols for quantum teleportation between many parties. It is shown how N parties can teleport unknown quantum states to M other parties with the use of $N + M$ qudits in the maximally entangled state.

PACS: 03.67.-a

1   Introduction

Quantum teleportation lies at the heart of quantum information. In the original scheme of Bennett et al. [1] two parties (Alice and Bob) share a pair of qubits in maximally entangled state. In addition one party, e.g. Alice, has a qubit in an unknown quantum state. The two parties can perform a very simple protocol which enables a teleportation of the qubit in an unknown quantum state from Alice to Bob. In the same paper these results were generalized from qubits to qudits (d-dimensional quantum particles). On the other hand, Karlsson and Bourennane [2] and Hillery, Buzek and Barthiaume [3] proposed a different scheme where three parties (Alice, Bob 1, and Bob 2) share three qubits in the entangled state of the Greenberger-Horne-Zeilinger (GHZ) class. In their scheme one party (Alice) teleports an unknown quantum state of a qubit. At the end of the protocol the state of the original qubit is encoded in a two-qubit entangled state shared by Bob 1 and Bob 2 in such a way that none of them can reconstruct the original qubit by himself. However, when they cooperate and use only local quantum operations and classical communication, one of them can reconstruct the original qubit. This protocol can be easily generalized to qudits [4]. Quantum teleportation between many parties was also considered in [5–10].

Let us now suppose that three parties (Bob 1, Bob 2, and Alice) share three qudits of dimensions d in entangled state of the GHZ class. In addition let two of them have qudits of dimensions d_1 and d_2 ($d_1 d_2 = d$) in unknown quantum states. We will present a protocol which enables them to teleport the states of their qudits to Alice with the use of the above mentioned qudits in maximally entangled state. At the end of the protocol Alice will possess one qudit of dimension d, whose state encodes the states of the teleported qudits. The reverse problem will also be considered. Now let Alice has a qudit whose state encodes the states of two qudits of lower dimensions. It will be shown how Alice can teleport the first encoded qudit to Bob 1 and

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the second one to Bob 2. Both protocols will be generalized to the case of more than three parties. Finally, we will consider quantum teleportation between many senders and many receivers of quantum information. As a special case we will consider teleportation of two qudits in two opposite directions with the use of one pair of qudits in the maximally entangled state. Namely, if Alice and Bob share such a pair then Alice can teleport a qudit of dimension d_1 to Bob and Bob can teleport a qudit of dimension d_2 to Alice.

The paper is organized as follows. Part 2 describes quantum teleportation from one party to many parties while part 3 deals with quantum teleportation from many parties to one party. Finally in part 4 quantum teleportation from many senders to many receivers is described. In the sake of clarity simple examples are presented first and then the results are generalized.

2   Quantum teleportation: many to one

Let three parties (Bob 1, Bob 2, and Alice) share three qudits in entangled state of the GHZ-class:

$$|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle_1 |k\rangle_2 |k\rangle_3, \tag{1}$$

where d is a dimension of each qudit. We will assume that d can be written as a product of two integers d_1 and d_2 ($d_1 d_2 = d$) where both d_1 and d_2 are greater than 1. Let in addition Bob 1 and Bob 2 have qudits of dimension d_1 and d_2, respectively, in unknown quantum states. Because both d_1 and d_2 are smaller than d each of these qudits can be encoded in the qudit of dimension d. We are thus free to assume that Bob 1 has a qudit of dimension d in the state

$$|\alpha^1\rangle = \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 |k_1\rangle, \tag{2}$$

while Bob 2 has a qudit of dimension d in the state

$$|\alpha^2\rangle = \sum_{k_2=0}^{d_2-1} \alpha_{k_2}^2 |k_2 d_1\rangle. \tag{3}$$

Below we present the protocol which enables Bob 1 and Bob 2 to teleport their qudits "α^1" and "α^2" to Alice with the use of three qudits in the state "ψ".

Bob 1 performs the operations given below. (B1.2) He makes the unitary operation on his qudit "α^1"

$$|k_1\rangle \rightarrow \frac{1}{\sqrt{d_2}} \sum_{k_2=0}^{d_2-1} |k_2 d_1 + k_1\rangle. \tag{4}$$

The new state of his qudit is

$$|\alpha^1\rangle^{(1)} = \frac{1}{\sqrt{d_2}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 |k_2 d_1 + k_1\rangle. \tag{5}$$

(B1.3) He measures the qudits "α^1" and "1" from "ψ" in the generalized Bell basis

$$|\psi_{mn}\rangle = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \omega^{nl} |l\rangle |l \oplus m\rangle, \tag{6}$$

where $\omega = \exp(i2\pi/d)$ and $\oplus$ denotes addition modulo d. If the result of his measurement is ψ_{mn} then Bob 2 and Alice share two qudits in the state

$$\left|\Psi^{23}\right\rangle^{(1)} = \frac{1}{\sqrt{d_2}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \omega^{-n(k_2 d_1 + k_1)} |(k_2 d_1 + k_1) \oplus m\rangle_2 |(k_2 d_1 + k_1) \oplus m\rangle_3. \tag{7}$$

(B1.4) He sends the result of his measurement to Alice and Bob 2. (A.1) Alice performs the unitary operation

$$\omega^{-n(k_2 d_1 + k_1)} |(k_2 d_1 + k_1) \oplus m\rangle_3 \rightarrow |k_2 d_1 + k_1\rangle_3. \tag{8}$$

(B2.1) Bob 2 performs the unitary operation

$$|(k_2 d_1 + k_1) \oplus m\rangle_2 \rightarrow |k_2 d_1 + k_1\rangle_2. \tag{9}$$

Now Bob 2 and Alice share two qudits in the state

$$\left|\Psi^{23}\right\rangle^{(2)} = \frac{1}{\sqrt{d_2}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 |k_2 d_1 + k_1\rangle_2 |k_2 d_1 + k_1\rangle_3. \tag{10}$$

Bob 2 performs the operations given below. (B2.2) He makes the unitary operation on his qudit "α^2"

$$|k_2 d_1\rangle \rightarrow \frac{1}{\sqrt{d_1}} \sum_{k_1=0}^{d_1-1} |k_2 d_1 + k_1\rangle. \tag{11}$$

The new state of his qudit is

$$\left|\alpha^2\right\rangle^{(1)} = \frac{1}{\sqrt{d_1}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_2}^2 |k_2 d_1 + k_1\rangle. \tag{12}$$

(B2.3) He measures the qudits "α^2" and "2" from "ψ" in the generalized Bell basis. If the result of his measurement is ψ_{mn} then Alice has the qudit in the state

$$\left|\Psi^3\right\rangle^{(1)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \alpha_{k_2}^2 \omega^{-n(k_2 d_1 + k_1)} |(k_2 d_1 + k_1) \oplus m\rangle_3. \tag{13}$$

(B2.4) He sends the result of his measurement to Alice. (A.2) Alice performs the unitary operation (8)

Now Alice has the qudit in the state

$$\left|\Psi^3\right\rangle^{(2)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \alpha_{k_2}^2 |k_2 d_1 + k_1\rangle_3. \tag{14}$$

This state contains the same information as the states of qudits "α^1" and "α^2", so the teleportation has been performed successfully.

These results can be generalized to the case of more than three parties (Bob 1, Bob 2, ..., Bob N and Alice). $(N + 1)$ parties share $(N + 1)$ qudits in entangled state of the GHZ-class

$$|\Psi\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle_1 |k\rangle_2 \dots |k\rangle_N |k\rangle_{N+1}. \tag{15}$$

Bob i has a qudit of dimension d_i encoded in the qudit of dimension d $(d = d_1 d_2 \dots d_N)$ in the following way

$$|\alpha^i\rangle = \sum_{k_i=0}^{d_i-1} \alpha_{k_i}^i |k_i p_i\rangle, \tag{16}$$

where $p_1 = 1$ and $p_i = d_1 \dots d_{i-1}$ $(i \neq 1)$. Below we present the protocol which enables all Bobs to teleport their qudits to Alice.

Bob i has to perform the following operations. (Bi.1) He makes the unitary operation

$$|(k_n p_n + \dots + k_2 p_2 + k_1) \oplus m\rangle_i \rightarrow |k_n p_n + \dots + k_2 p_2 + k_1\rangle_i \tag{17}$$

for all results of measurements he receives from Bob 1, Bob 2,..., Bob $i - 1$. Bob 1 does not perform this step. (Bi.2) He makes the unitary operation

$$|k_i p_i\rangle_i \rightarrow \sqrt{\frac{d_i}{d}} \sum_{m_1}^{d_1-1} \dots \sum_{m_{i-1}}^{d_{i-1}-1} \sum_{m_{i+1}}^{d_{i+1}-1} \dots \sum_{m_N}^{d_N-1} |m_N p_N$$
$$+ \dots m_{i+1} p_{i+1} + k_i p_i + m_{i-1} p_{i-1} + \dots m_1\rangle_i. \tag{18}$$

(Bi.3) He measures the qudits "α^2" and "i" from "ψ" in the generalized Bell basis. (Bi.4) He sends the result of his measurement to Alice and Bob $i + 1$, Bob $i + 2$,..., Bob N.

Alice has to perform the unitary operation

$$\omega^{-n(k_N p_N + \dots + k_2 p_2 + k_1)} |(k_N p_N + \dots + k_2 p_2 + k_1) \oplus m\rangle_{N+1}$$
$$\rightarrow |k_N p_N + \dots + k_2 p_2 + k_1\rangle_{N+1} \tag{19}$$

for all results of measurements she receives from Bob 1, Bob 2,...,Bob N. At the end of this protocol Alice has the qudit in the state

$$|\Psi^{N+1}\rangle = \sum_{k_1=0}^{d_1-1} \sum_{k_2=0}^{d_2-1} \dots \sum_{k_N=0}^{d_N-1} \alpha_{k_1}^1 \alpha_{k_2}^2 \dots \alpha_{k_N}^N |k_N p_N + \dots + k_2 p_2 + k_1 p_1\rangle_{N+1}. \tag{20}$$

3   Quantum teleportation: one to many

Now we will consider the reverse problem. Let three parties (Alice 1, Alice 2, and Bob) share three qudits in entangled state (1). Let Bob have two qudits "α^1" and "α^2" of dimensions d_1 and d_2 encoded in one qudit of dimension d $(d = d_1 d_2)$ in the following way

$$|\alpha^{1,\,2}\rangle = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \alpha_{k_2}^2 |k_2 d_1 + k_1\rangle. \tag{21}$$

Although we assume that the state of two qudits is separable it does not have to be the case. If the state of two qudits is entangled then Alice 1 and Alice 2 at the end of the protocol will have also two qudits in entagled state. Below we present the protocol which enables Bob to teleport his qudits "α^1" and "α^2" to Alice 1 and Alice 2 respectively with the use of three qudits in the state (1).

Bob performs the operations given below (B.1) He measures the qudits "$\alpha^{1,\,2}$" and "3" from "ψ" in the generalized Bell basis. If the result of his measurement is ψ_{mn} then Alice 1 and Alice 2 share two qudits in the state

$$\left| \Psi^{1,\,2} \right\rangle^{(1)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha^1_{k_1} \alpha^2_{k_2} \omega^{-n(k_2 d_1 + k_1)} \left| (k_2 d_1 + k_1) \oplus m \right\rangle_1 \left| (k_2 d_1 + k_1) \oplus m \right\rangle_2. \tag{22}$$

(B.2) He sends the result of his measurement to Alice 1 and Alice 2. (A1.1) Alice 1 performs the unitary operation

$$\omega^{-n(k_2 d_1 + k_1)} \left| (k_2 d_1 + k_1) \oplus m \right\rangle_1 \rightarrow \left| k_2 d_1 + k_1 \right\rangle_1. \tag{23}$$

(A2.1) Alice 2 performs the unitary operation

$$\left| (k_2 d_1 + k_1) \oplus m \right\rangle_2 \rightarrow \left| k_2 d_1 + k_1 \right\rangle_2. \tag{24}$$

Now Alice 1 and Alice 2 share two qudits in the state

$$\left| \Psi^{1,\,2} \right\rangle^{(2)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha^1_{k_1} \alpha^2_{k_2} \left| k_2 d_1 + k_1 \right\rangle_1 \left| k_2 d_1 + k_1 \right\rangle_2. \tag{25}$$

Alice 1 performs the operations given below (A1.2) She makes the unitary operation

$$\left| k_2 d_1 + k_1 \right\rangle_1 \rightarrow \frac{1}{\sqrt{d_2}} \sum_{m=0}^{d_2-1} \omega^{k_2 d_1 m} \left| m d_1 + k_1 \right\rangle_1. \tag{26}$$

The new state is

$$\left| \Psi^{1,\,2} \right\rangle^{(3)} = \frac{1}{\sqrt{d_2}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \sum_{m=0}^{d_2-1} \omega^{k_2 d_1 m} \alpha^1_{k_1} \alpha^2_{k_2} \left| m d_1 + k_1 \right\rangle_1 \left| k_2 d_1 + k_1 \right\rangle_2. \tag{27}$$

(A1.3) She carries out a von Neumann measurement described by the projectors

$$P_m = \sum_{k_1=0}^{d_1-1} \left| m d_1 + k_1 \right\rangle_{1\,1} \left\langle m d_1 + k_1 \right|. \tag{28}$$

If the result of her measurement is P_m then Alice 1 and Alice 2 share two qudits in the state

$$\left| \Psi^{1,\,2} \right\rangle^{(4)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \omega^{k_2 d_1 m} \alpha^1_{k_1} \alpha^2_{k_2} \left| m d_1 + k_1 \right\rangle_1 \left| k_2 d_1 + k_1 \right\rangle_2. \tag{29}$$

(A1.4) She sends the result of her measurement to Alice 2. (A1.5) She makes the unitary operation

$$|md_1 + k_1\rangle_1 \rightarrow |k_1\rangle_1. \tag{30}$$

Alice 2 performs the operations given below. (A2.6) She makes the unitary operation (This step can be performed after step (A2.5), however for clarity I have put it here.)

$$\omega^{k_2 d_1 m} |k_2 d_1 + k_1\rangle_2 \rightarrow |k_2 d_1 + k_1\rangle_2. \tag{31}$$

The new state is

$$\left| \Psi^{1,\,2} \right\rangle^{(5)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \alpha_{k_2}^2 |k_1\rangle_1 |k_2 d_1 + k_1\rangle_2. \tag{32}$$

(A2.2) She makes the unitary operation

$$|k_2 d_1 + k_1\rangle_2 \rightarrow \frac{1}{\sqrt{d_1}} \sum_{m=0}^{d_1-1} \omega^{k_1 d_2 m} |k_2 d_1 + m\rangle_2. \tag{33}$$

The new state is

$$\left| \Psi^{1,\,2} \right\rangle^{(6)} = \frac{1}{\sqrt{d_1}} \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \sum_{m=0}^{d_2-1} \omega^{k_1 d_2 m} \alpha_{k_1}^1 \alpha_{k_2}^2 |k_1\rangle_1 |k_2 d_1 + m\rangle_2. \tag{34}$$

(A2.3) She carries out a von Neumann measurement described by the projectors

$$P_m = \sum_{k_2=0}^{d_2-1} |k_2 d_1 + m\rangle_{2\,2} \langle k_2 d_1 + m|. \tag{35}$$

If the result of his measurement is P_m then Alice 1 and Alice 2 share two qudits in the state

$$\left| \Psi^{1,\,2} \right\rangle^{(7)} = \sum_{k_2=0}^{d_2-1} \sum_{k_1=0}^{d_1-1} \omega^{k_1 d_2 m} \alpha_{k_1}^1 \alpha_{k_2}^2 |k_1\rangle_1 |k_2 d_1 + m\rangle_2. \tag{36}$$

(A2.4) She sends the result of her measurement to Alice 1. (A2.5) She performs the unitary operation

$$|k_2 d_1 + m\rangle_2 \rightarrow |k_2 d_1\rangle_2. \tag{37}$$

(A1.6) Alice 1 performs the unitary operation

$$\omega^{k_1 d_2 m} |k_2 d_1 + k_1\rangle_1 \rightarrow |k_2 d_1 + k_1\rangle_1. \tag{38}$$

Now Alice 1 and Alice 2 have two qudits in the state

$$\left| \Psi^{1,\,2} \right\rangle^{(8)} = \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 |k_1\rangle_1 \sum_{k_2=0}^{d_2-1} \alpha_{k_2}^2 |k_2 d_1\rangle_2. \tag{39}$$

It should be noted that the corresponding steps for Alice 1 and Alice 2 (e.g. A1.3 and A2.3) can be performed in parallel.

These results can also be generalized to the case of $(N + 1)$ parties (Alice 1, Alice 2, ..., Alice N, and Bob). $(N + 1)$ parties share $(N + 1)$ qudits in entangled state (15). Bob has N qudits "α^1", "α^2",..., "α^N" of dimensions $d_1, d_2, \ldots, d_N$ encoded in one qudit of dimension d in the following way

$$\left| \alpha^{1,\,2,\ldots,N} \right\rangle = \sum_{k_1=0}^{d_1-1} \sum_{k_2=0}^{d_2-1} \ldots \sum_{k_N=0}^{d_N-1} \alpha_{k_1}^1 \alpha_{k_2}^2 \ldots \alpha_{k_N}^N \left| k_N p_N + \ldots + k_2 p_2 + k_1 p_1 \right\rangle, \tag{40}$$

where $p_1 = 1$ and $p_i = d_1 \ldots d_{i-1}$ $(i \neq 1)$. Below we present the protocol which enables Bob to teleport N qudits to N remaining parties (one qudit to one party). Because the generalization of steps performed by Bob and the first step performed by each Alice is straightforward we consider only the generalization of all the next steps performed by each Alice. (Ai.2) Alice i makes the unitary operations

$$\left| k_N p_N + \ldots k_{i+1} p_{i+1} + k_i p_i + k_{i-1} p_{i-1} + \ldots k_1 \right\rangle_i$$
$$\to \sqrt{\frac{d_i}{d}} \sum_{m_1^i=0}^{d_1-1} \ldots \sum_{m_{i-1}^i=0}^{d_{i-1}-1} \sum_{m_{i+1}^i=0}^{d_{i+1}-1} \ldots \sum_{m_N^i=0}^{d_N-1} \omega_N^{k_N m_N^i} \ldots \omega_{i+1}^{k_{i+1} m_{i+1}^i}$$
$$\times \omega_{i-1}^{k_{i-1} m_{i-1}^i} \ldots \omega_1^{k_1 m_1^i} \left| m_N^i p_N + \ldots m_{i+1}^i p_{i+1} + k_i p_i + m_{i-1}^i p_{i-1} + \ldots m_1^i \right\rangle_i.$$

where $\omega_i = \omega^{d/d_i}$. (A$i$.3) Alice i carries out a von Neumann measurement described by the projectors

$$P_{m_N^i \ldots m_{i+1}^i m_{i-1}^i \ldots m_1^i}^i = \sum_{k_i=0}^{d_i-1} \left| m_N^i p_N + \ldots m_{i+1}^i p_{i+1} + k_i p_i \right.$$
$$\left. + m_{i-1}^i p_{i-1} + \ldots m_1^i \right\rangle_i {}_i\left\langle m_N^i p_N + \ldots m_{i+1}^i p_{i+1} + k_i p_i + m_{i-1}^i p_{i-1} + \ldots m_1^i \right| \tag{41}$$

(Ai.4) Alice i sends the result m_j^i of her measurement to Alice j. (Ai.5) Alice i makes the unitary operation

$$\left| m_N^i p_N + \ldots + m_{i+1}^i p_{i+1} + k_i p_i + m_{i-1}^i p_{i-1} + \ldots + m_1^i \right\rangle_i \to \left| k_i p_i \right\rangle_i. \tag{42}$$

(Ai.6) Alice i performs the unitary operation

$$\omega_i^{k_i m_i^j} \left| k_N p_N + \ldots + k_2 p_2 + k_1 \right\rangle_i \to \left| k_N p_N + \ldots + k_2 p_2 + k_1 \right\rangle_i \tag{43}$$

for all results of measurements $j \neq i$ she receives from Alice 1, Alice 2,...,Alice N. At the end of this protocol Alice 1, Alice 2,...,Alice N have N qudits in the state

$$\left| \Psi^{1,\,2,\ldots,N} \right\rangle = \sum_{k_1=0}^{d_1-1} \alpha_{k_1}^1 \left| k_1 p_1 \right\rangle_1 \sum_{k_2=0}^{d_2-1} \alpha_{k_2}^2 \left| k_2 p_2 \right\rangle_2 \ldots \sum_{k_N=0}^{d_N-1} \alpha_{k_N}^N \left| k_N p_N \right\rangle_N. \tag{44}$$

4 Quantum teleportation: many to many

Let us now assume that $2N$ parties (N Bobs, who are senders of quantum information and N Alices, who are receivers) share $2N$ qudits of dimension d in entangled state of the GHZ-class. Let N of them (each Bob) have qudits of dimension $d_1, d_2, \ldots, d_N$ and $N+1$ parties (each Bob and Alice 1) perform the steps described in part 2. In addition let each Bob send the result of his measurement in step (Bi.2) not only to Alice 1 but also to Alice 2, Alice 3,..., Alice N. If Alice 2, Alice 3,..., Alice N perform the unitary operation given by Eq.(17) on their qudits for all results of measurements they receive from Bobs then N Alices will share N qudits in the state

$$
\begin{aligned}
\left|\alpha^{1,\,2,\ldots,N}\right\rangle = \sum_{k_1=0}^{d_1-1}\sum_{k_2=0}^{d_2-1}\ldots\sum_{k_N=0}^{d_N-1} \alpha^1_{k_1}\alpha^2_{k_2}\ldots\alpha^N_{k_N}\left|k_N p_N + \ldots + k_2 p_2 + k_1 p_1\right\rangle \\
\times \left|k_N p_N + \ldots + k_2 p_2 + k_1 p_1\right\rangle \ldots \left|k_N p_N + \ldots + k_2 p_2 + k_1 p_1\right\rangle .
\end{aligned}
\tag{45}
$$

They can now perform the steps described in part 3. At the end of the protocol these N Alices will have the state (44). Thus the quantum states have been teleported from Bobs to Alices. Because some of the parties can be in fact the same party (e.g. Alice 1 = Bob 2) we have a kind of quantum teleportation network which enables exchange of quantum information between $M \leq 2N$ parties as long as all the quantum information can be encoded in a qudit of dimension d. It should be noted that in the case when we have M (not $2N$) parties the parties should share only M qudits in entangled state of the GHZ-class, because one can entangle additional qudits with the use of XOR gate defined as

$$
XOR\left|k\right\rangle\left|l\right\rangle = \left|k\right\rangle\left|l\oplus k\right\rangle .
\tag{46}
$$

One simply prepares additional qudit in the state $|0\rangle$ and applies XOR gate to his qudit from entangled state (control qudit) and additional qudit (target qudit) and so on.

Let us now discuss very simple example. We have two parties: Bob 1 = Alice 2 (I will call him/her Bob) and Bob 2 = Alice 1 (I will call him/her Alice). They share two qudits of dimension d in the maximally entangled state. Bob has a qudit of dimension d_1 and wants to teleport it to Alice. Alice has a qudit of dimension d_2 ($d_1 d_2 = d$) and wants to teleport it to Bob. They will achieve this aim if both Bob and Alice entangle additional qudits of dimension d to their two qudits in the maximally entangled state and then perform the protocol just described. One thus sees that a single pair of maximally entangled qudits can serve as a two-way quantum channel in the sense that the same pair can be used to teleport one qudit from Alice to Bob and one qudit from Bob to Alice.

5 Summary

General protocols for quantum teleportation between multiple parties with the use of qudits has been proposed. The protocols enable teleportation from one party to many parties and from many parties to one party with the use of qudits in entangled state of the GHZ-class. The protocol which enables teleportation from many senders to many receivers was also constructed. As a special case we considered teleportation of two qudits in two opposite opposite directions with the use of one pair of qudits in maximally entangled state.

References

[1] C.H. Bennett et al.: *Phys Rev. Lett.* **70** (1993) 1895

[2] A. Karlsson, M. Bourennane: *Phys. Rev. A* **58** (1998) 4394

[3] M. Hillery, V. Bužek, A. Berthiaume: *Phys. Rev. A* **59** (1999) 1829

[4] A. Grudka A. Wójcik: *Acta Phys. Pol. A* **102** (2002) 703

[5] W. Dur, J.I. Cirac: *J. Mod. Opt.* **47** (2000) 247

[6] M. Murao, M.B. Plenio, V. Vedral: *Phys. Rev. A* **61** (2000) 032311

[7] V.N. Gorbachev, A.I. Trubilko: *JETP* **118** (2001) 1036

[8] V.N. Gorbachev, A.I. Zhiliba, A.I. Trubilko, A.A. Rodichkina: *Quantum Inform. Comput.* **2** (2002) 367

[9] V.N. Gorbachev, A.I. Trubilko, A.A. Rodichkina, A.I. Zhiliba: *Phys. Lett. A* **314** (2003) 267

[10] I. Ghiu: *Phys. Rev. A* **67** (2003) 012323