## SPECTRAL INTERFEROMETRY INCLUDING THE EFFECT OF TRANSPARENT THIN FILMS TO MEASURE DISTANCES AND DISPLACEMENTS

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A spectral-domain interferometric technique is applied for measuring mirror distances and displacements in a dispersive Michelson interferometer when the effect of transparent thin films coated onto the interferometer beam splitter and compensator is known. We employ a low-resolution spectrometer in two experiments with different amounts of dispersion in a Michelson interferometer that includes fused-silica optical sample. Knowing the thickness of the optical sample and the nonlinear phase function of the thin films, the positions of the interferometer mirror are determined precisely by a least-squares fitting of the theoretical spectral interferograms to the recorded ones. We compare the results of the processing that include and do not include the effect of transparent thin films.

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### 1 Introduction

White-light spectral interferometric techniques based on channelled spectrum detection have been widely applied for measuring distances and displacements [1–5], and in optical profilometry [6–8]. However, an optical configuration with white-light channelled spectrum detection operates in a limited distance range with the minimum distance given by the spectral bandwidth of a white-light source and the maximum distance given by the spectrometer resolving power [2].

Recently, we have demonstrated experimentally that a new technique of dispersive whitelight interferometry employing a low-resolution spectrometer and based on resolving the spectral interference fringes in a narrow wavelength range in the vicinity of the so-called equalization wavelengths can be used in substantially larger range of distances. The new technique, which needs no phase retrieving procedure to be applied, has been used for measuring distances and displacements when dispersion in a two-beam interferometer is known [9]. Moreover, we have confirmed that in contrary to standard spatial-domain white-light interferometry employing interferometers balanced for dispersion the measurement technique is characterized by the range of measurable distances dependent on the amount of dispersion in the interferometer [9, 10]. We

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have also demonstrated that processing of the recorded spectral interferograms using a least-squares method gives distances with resolution comparable to vertical resolutions of standard spatial-domain white-light profilometers [11].

More recently, a new spectral-domain white-light interferometric technique of measuring distances and displacements has been presented when the effect of dispersion in a slightly dispersive Michelson interferometer, comprising two coated plates of a beam splitter and a compensator, is known and the spectral interference fringes are resolved over a wide wavelength range [12]. A new procedure based on both processing the recorded spectral interferograms by the Fourier transform method (FTM) [13] and knowing the dispersion relation for the fused-silica plates has been utilized for determining the thickness of fused silica material and the nonlinear phase function of the coatings. Once these quantities are known, the positions of the interferometer mirror are determined precisely.

The aim of the paper is extend the application of white-light spectral interferometry for measuring mirror distances and displacements in a dispersive Michelson interferometer when the effect of transparent thin films is known. We utilize a Michelson interferometer with coated fused-silica beam splitter and compensator, and fused-silica sample of two different thicknesses. Knowing the nonlinear phase function of the coatings and the sample thickness, the positions of the interferometer mirror are determined precisely by a least-squares fitting of the theoretical spectral interferograms to the recorded ones. We compare the results of the processing that include and do not include the effect of transparent films.

#### 2 Experimental configuration

The experimental set-up used in the application of spectral-domain white-light interferometry to measure mirror distances or displacements in a dispersive Michelson interferometer is shown in Fig. 1. It consists of a white-light source, a 20 W quartz tungsten halogen (QTH) lamp, an aperture with a collimating lens, a bulk-optic Michelson interferometer with a beam splitter and a compensator, a micropositioner connected to one of the mirrors, an optical sample, a microscope objective, micropositioners, a detecting optical fibre of a 6  $\mu$ m core diameter, a miniature fibre-optic spectrometer S2000, an A/D converter and a personal computer. The beam splitter and compensator are coated parallel plates made of fused silica having a thickness of 1 cm. The optical sample is fused-silica parallel plate having a thickness of 0.5 or 1 cm. The fibre-optic spectrometer S2000 (Ocean Optics, Inc.), the design of which was reported previously, has in our case the resolution given by the effective width of the light beam from a core of the read optical fibre. We use the read optical fibre of a 50  $\mu$ m core diameter to which a Gaussian response function corresponds [14]. The spectral interferograms are recorded in the rigid positions of the interferometer with known positions of mirror 2.

#### **3** Experimental method

Let us consider the mutual interference of two beams from a broadband source at the output of the uncompensated (dispersive) Michelson interferometer with a beam splitter of the effective thickness  $t_{ef1}$ , a compensator of the effective thickness  $t_{ef2}$  and an optical sample of the thickness t (see Fig. 1). When a low-resolution spectrometer of a Gaussian response function is used



Fig. 1. Experimental set-up with a dispersive Michelson interferometer to measure distances and displacements of the interferometer mirror.

to record the spectral interferogram and when only the effect of first-order dispersion is taken into account, the recorded spectrum can be represented by [10]

$$I(\lambda) = I^{(0)}(\lambda) \left\{ 1 + V_I \exp\{-(\pi^2/2) [\Delta^{\mathrm{g}}(\lambda) \Delta \lambda_{\mathrm{R}}/\lambda^2]^2 \right\} \cos[(2\pi/\lambda) \Delta(\lambda)] \right\},\tag{1}$$

where  $I^{(0)}(\lambda)$  is the reference spectrum,  $\Delta(\lambda)$  and  $\Delta^{g}(\lambda)$  are the wavelength-dependent optical path difference (OPD) and group OPD between interfering beams, respectively,  $\Delta\lambda_{\rm R}$  is the width of the response function of the spectrometer, and  $V_{I}$  is the visibility term including the effect of the spatial integration of the detecting optical fibre on the interference fringes. When the beam splitter, the compensator and the optical sample are made of the same material, and when the overall effective thickness  $t_{\rm eff} = t_{\rm ef1} - t_{\rm ef2}$  is introduced, the OPD  $\Delta(\lambda)$  in eq. (1) is given by

$$\Delta(\lambda) = 2L + 2n(\lambda)(t_{\rm ef} - t) + \lambda\delta(\lambda)/(2\pi), \tag{2}$$

where 2L is the difference of path lengths between the interfering beams in the air,  $n(\lambda)$  is the wavelength-dependent refractive index of the material and  $\delta(\lambda)$  is the nonlinear phase function due to the effect of thin films coated onto the beam splitter and compensator [5, 12]. Similarly, the group OPD  $\Delta^{g}(\lambda)$  in eq. (1) is given by

$$\Delta^{g}(\lambda) \approx 2L + 2N(\lambda)(t_{\rm ef} - t), \tag{3}$$

where  $N(\lambda)$  is the wavelength-dependent group refractive index of the material. When a dispersive Michelson interferometer with an optical sample is considered, the equalization wavelengths  $\lambda_0$  fulfilling the relation  $\Delta^{g}(\lambda_0) = 0$  can be resolved and there is a possibility to use a simple procedure to measure the thickness  $t_{ef} - t$  [9]. When a slightly dispersive Michelson interferometer without a sample is considered, there is a possibility to use a simple procedure to measure

both the overall effective thickness  $t_{\rm ef}$  and the nonlinear phase function  $\delta(\lambda)$  [12]. Once these quantities are determined and when a dispersive Michelson interferometer with an optical sample of known thickness t is considered, it results from eqs. (2) and (3) that knowing the refractive index dispersion  $n(\lambda)$  it is possible to fit the theoretical spectral interferograms (1) to the recorded ones and to determine precisely the mirror positions L or the mirror displacements  $\Delta L$  and the mirror displacement step  $\delta L$ .

#### 4 Experimental results and discussion

First, the spectral interferograms recorded for the slightly dispersive Michelson interferometer not including the optical sample (t = 0) were processed to determine the nonlinear phase function  $\delta(\lambda)$  of thin films coated onto the interferometer beam splitter and compensator. We adjusted mirror 2 of the Michelson interferometer to resolve the spectral interference fringes in the spectral range as wide as possible. Mirror 2 was displaced manually by using the micropositioner with a constant step of 10  $\mu$ m and the spectral region from 500 to 850 nm was chosen as that in which the spectral interference fringes should be fully observable. By processing one of the recorded spectral interferograms using the FTM [13], the overall OPD  $\Delta(\lambda)$  between beams in the interferometer was determined. Using a simple procedure based on the linear dependence between the overall OPD  $\Delta(\lambda)$  and the refractive index  $n(\lambda)$  of the fused silica, the nonlinear phase function  $\delta(\lambda)$  was determined [12]. It is shown in Fig. 2.



Fig. 2. Measured nonlinear phase as a function of the wavelength.

Second, we processed the spectral interferograms recorded for the Michelson interferometer including the first fused-silica optical sample having a thickness t of approximately 0.5 cm. We started with the determination of the equalization wavelength  $\lambda_0$  as a function of the displacement  $\Delta L$  of mirror 2 in the interferometer to obtain precisely the overall thickness  $t - t_{\rm ef}$  of the fused-silica material in the interferometer. Mirror 2 was displaced with a constant step of 10  $\mu$ m and it was revealed from the recorded spectral interferograms that the equalization wavelengths

are resolved in the spectral range approximately from 528 to 826 nm. We chose a wavelength of 826.41 nm as the reference wavelength so that the corresponding mirror displacements  $\Delta L$  varied from 100 to 0  $\mu$ m. We confirmed the linear dependence of the mirror displacement on the group refractive index of the fused silica, the slope of which gave the overall thickness  $t - t_{\rm ef}$  of 5040  $\mu$ m with a standard deviation of 13  $\mu$ m.



Fig. 3. Example of the recorded spectral interferogram compared with theory (dashed line): the first fused-silica sample.



Fig. 4. The mirror positions determined from the processed spectral interferograms as a function of the adjusted mirror displacement (points) and the corresponding deviations: the first fused-silica sample.

Knowledge of the overall thickness  $t - t_{ef}$ , the refractive index dispersion  $n = n(\lambda)$  and the nonlinear phase function  $\delta(\lambda)$  enabled us to precise the determination of the mirror position

L by using eq. (2) and a least-squares fit of the theoretical spectral interferogram (1) to the recorded one. Thus Fig. 3 shows one of the recorded spectral interferograms, corresponding to the displacement of 40  $\mu$ m with the equalization wavelength having a value of 651.50 nm, which is compared with the theoretical spectral interferogram (1) represented by the dashed line and characterized by  $V_I = 0.6, \Delta \lambda_R = 3.0$  nm and  $L = 7429.650 \ \mu\text{m}$ . The reference spectrum  $I^{(0)}(\lambda)$  in eq. (1) we took was the spectrum of one of the recorded spectral interferograms not including the spectral interference fringes and L was determined with a resolution of 1 nm. We see very good agreement between theory and experiment. Using the same procedure, the mirror positions L corresponding to the other recorded spectral interferograms were determined precisely. The procedure was applied to 11 spectral interferograms and Fig. 4 shows the values of L determined as a function of the adjusted mirror displacements. We clearly see that this dependence is well fitted to the linear function characterized by a correlation factor as high as 0.99996 so that 10  $\mu$ m mirror displacements were adjusted manually with high precisions. The method measures mirror positions with resolution comparable to vertical resolutions of standard spatial-domain white-light profilometers [11]. It is interesting to determine precisely the mirror positions when the effect of the thin films represented by the nonlinear phase function  $\delta(\lambda)$  is not taken into account. Fig. 4 shows the corresponding deviations of the mirror positions thus determined from the correct mirror positions. We see that a systematic error less than 10 nm is present in the mirror position determination. Moreover, Fig. 4 shows that the dependence of the error on the mirror position, or in other words on the equalization wavelength, is closely related to the nonlinear phase function. This fact indicates that errors owing to possible variations of the overall thickness with the mirror positions are negligible.

Finally, we processed the spectral interferograms recorded for the Michelson interferometer including the second fused-silica optical sample having a thickness t of approximately 1 cm. Once again we started with the determination of the equalization wavelength  $\lambda_0$  as a function of the displacement  $\Delta L$  of mirror 2 in the interferometer to obtain precisely the overall thickness  $t - t_{\rm ef}$  of the fused-silica material in the interferometer. Mirror 2 was displaced with a constant step of 10  $\mu$ m and it was revealed from the recorded spectral interferograms revealed that the equalization wavelengths are resolved in the spectral range approximately from 523 to 729 nm. We chose a wavelength of 729.33 nm as the reference wavelength so that the corresponding mirror displacements  $\Delta L$  varied from 170 to 0  $\mu$ m. Then we confirmed the linear dependence of the mirror displacement on the group refractive index of the fused silica, the slope of which gave the overall thickness  $t - t_{\rm ef}$  of 10105  $\mu$ m with a standard deviation of 18  $\mu$ m.

Knowledge of the overall thickness  $t - t_{ef}$ , the refractive index dispersion  $n = n(\lambda)$  and the nonlinear phase function  $\delta(\lambda)$  enabled us to precise the determination of the mirror position Lby using eq. (2) and a least-square fit of the theoretical spectral interferogram (1) to the recorded one. This procedure is illustrated in Fig. 5, in which one of the recorded spectral interferograms, corresponding to the displacement of 30  $\mu$ m with the equalization wavelength having a value of 673.62 nm, is compared with the theoretical spectral interferogram (1). The theoretical spectral interferogram is shown in Fig. 5 by the dashed line and is characterized by  $V_I = 0.58$  and L =14881.969  $\mu$ m. We see very good agreement between theory and experiment. Using the same procedure, the mirror positions L corresponding to the other recorded spectral interferograms were determined precisely. The procedure was applied to 18 spectral interferograms and Fig. 6 shows the values of L determined as a function of the adjusted mirror displacements. We clearly see that this dependence is well fitted to the linear function characterized by a correlation factor as high as 0.99994 so that 10  $\mu$ m mirror displacements were adjusted manually with high precisions. Once again we determined precisely the mirror positions when the effect of the thin films is not taken into account. Fig. 6 shows the corresponding deviations of the mirror positions thus determined from the correct mirror positions that represent a systematic error less than 12 nm.



Fig. 5. Example of the recorded spectral interferogram compared with theory (dashed line): the second fused-silica sample.



Fig. 6. The mirror positions determined from the processed spectral interferograms as a function of the adjusted mirror displacement (points) and the corresponding deviations: the second fused-silica sample.

# 5 Conclusion

We have applied a white-light interferometric technique employing a low-resolution spectrometer and a dispersive Michelson interferometer with coated fused-silica optical elements for measuring distances and displacements of the interferometer mirror. We have performed processing of the recorded spectral interferograms, including the equalization wavelengths, knowing not only the effect of the nonlinear phase function of the thin film coatings, but also dispersion and the overall thickness of the interferometer optical elements. The technique, which needs no phase retrieval procedures to be applied, uses a least-squares fit of the theoretical spectral interferograms to the recorded ones to determine the positions of the interferometer mirror or its displacements. We have demonstrated within two different configurations of a dispersive Michelson interferometer that the range of measurable distances depends on the amount of dispersion present in the interferometer. We have also confirmed by this measurement technique that a systematic error affects the determination of the mirror positions when the effect of thin film coatings is not taken into account. The results of the experiment demonstrate the applicability of the technique for measuring distances and displacements with high resolution.

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