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DETERMINATION OF AMPLITUDES IN NEUTRAL PION PHOTOPRODUCTION AND COMPARISON WITH PARTIAL WAVES ANALYSIS IN THE ENERGY RANGE OF 1.3 TO 2.1 GeV

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The magnitude of four independent amplitudes are obtained in neutral pion photoproduction in the energy range of 1300 to 2100 MeV incident photon. Differential cross section and three polarization parameters are required for such amplitudes reconstruction at different pion scattering angles. Results of the direct amplitudes reconstruction have been compared with the solution of partial wave analysis SM95 and SM00K at all energies. This analysis indicates that we have a fair agreement between the present work and the results of partial wave analysis at meny angles.

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1 Introduction

The direct reconstruction of amplitudes in elastic scattering is independent from dynamical models and by taking advantage of the measured polarization parameters has been widely used in the last three decades [1]. The non-dynamical spin polarization analysis of the complex reaction amplitudes not only enables us to obtain dynamical information, it is also capable of checking the validity of conservation laws such as time reversal, parity and identical particles in hadronic interactions [2]. Such analysis provides the minimum number of experiments that are required for a complete determination of the magnitudes and phases of the amplitudes that are called complete set [3].
One of the best ways for checking the validity of model-dependent contributions to partial waves analysis is to compare the partial waves analysis amplitude predictions with the direct reconstruction from amplitude analysis. From this point of view, the amplitude analysis is to be considered as a complementary method with respect to the partial waves analysis. Agreement of the results will then support the partial waves analysis which may be used to conveniently predict other measured quantities. A disagreement will either suggest a possible anomaly in the database, or will throw a doubt on the validity of the partial waves analysis solutions [4].
In order to obtain the spin amplitudes of an elastic interaction, one has to find their relations to the experimental observables by constructing the interaction matrix. Such a relationship is normally complicated. The way to get rid of such complications is to use the optimal formalism [5]. In this

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formalism the interaction matrix is diagonalized as much as possible without loss of generality. Numerous important polarization experiments have been performed in the particle physics in the late seventies and eighties such as

$$(1/2 + 1/2 \longrightarrow 1/2 + 1/2) \text{ (nucleon -nucleon scattering)}$$
$$(1 + 1/2 \longrightarrow 0 + 1/2) \text{ (pion photoproduction)}$$
$$(0 + 1/2 \longrightarrow 0 + 1/2) \text{ (pion-nucleon scattering).}$$

Some of these experiments have been repeated in the nineties with better polarization technology [6] but, in the case of pion photoproduction, the last series of experiments providing a complete set is given in Ref. [7]. Although some more recent experiments have been performed on this reaction they are all incomplete, and this is why the data of Ref. [7] has been used in this analysis regardless of its oldness. In this paper all effort have been made to reconstruct the magnitudes of the four independent hybrid reaction amplitudes for the interaction $(1 + 1/2 \longrightarrow 0 + 1/2)$.
The magnitudes of these amplitudes are obtained in terms of four experimental parameters, namely $d\sigma/d\Omega$(the unpolarized differential cross-section), $\Sigma(\theta)$(the conventional polarized photon asymmetry parameter), $T(\theta)$(the conventional polarized target asymmetry parameter) and $P(\theta)$(the recoil nucleon polarization).

2 Formalism

The reaction matrix for the general interaction $S_A + S_B \longrightarrow S_C + S_D$ where S_A, S_B, S_C and S_D are spin of the particles A, B, C and D respectively, is given [8]

$$M = \sum_l \sum_\lambda \sum_L \sum_\Lambda D(\lambda l, \Lambda L) S^{\lambda l} \otimes S^{\Lambda L}, \tag{1}$$

in which D's are complex amplitudes containing dynamics and $S^{\lambda l}$ are the spin momentum tensor corresponding to the particles with spin S_A and S_C, and similarly $S^{\Lambda L}$ are for the particles with spin S_B and S_D. l, λ, L and Λ are spin components along quantization axis for each particle. This compact form of reaction matrix is obtained from the standard factorization procedure [9]. The reaction is characterized by an initial density matrix ρ_i; and a final density matrix ρ_f. Both ρ_i and ρ_f describe the spin polarization states of particles before and after interaction. The overall initial density matrix is given in terms of outer products of the constituent initial density matrices, namely

$$\rho_i = \rho_i^{uv} \otimes \rho_i^{UV}, \tag{2}$$

and the final density matrix is given in terms of the initial density matrix

$$\rho_f = M(\rho_i^{uv} \otimes \rho_i^{UV})M^\dagger. \tag{3}$$

The experimental observables are given in terms of the expectation values of a certain spin-momentum tensor Q in the final states. The corresponding observable in the final state is given by

$$Q = Q^{\psi\omega} \otimes Q^{\Psi\Omega}, \tag{4}$$

where $Q^{\psi\omega}$ is a $(2S_C + 1) \times (2S_C + 1)$ matrix which describes the final state of S_C and $Q^{\Psi\Omega}$ is a $(2S_D + 1) \times (2S_D + 1)$ matrix which describes the final state of S_D.

3 Optimal conventions

In order to simplify the relations between the observables and the bilinear combination of the amplitudes, optimal conditions are introduced. In this formalism the initial and final density matrices as well as the spin momentum tensors are defined in a way such that the elements of the general interaction matrix are as diagonal as possible. The best way is to choose the density matrices and spin momentum tensors to be all hermitian [5], and to restrict the choice of quantization axis of each particle [10].
In the case of pion photoproduction, only parity is conserved, reducing the number of independent amplitudes from twelve to six. We shall use the hybrid frame [10] for simplicity. In this frame the polarization direction of the photon is in the reaction plane and in the direction of its momentum, but the polarization of nucleon is in the perpendicular direction to the reaction plane. Since the photon has only two directions of polarization the number of independent amplitudes is reduced from six to four and only the following spin amplitudes remain: $D(+,11)$ and $D(+,22)$ are spin non-flip whereas $D(+,12)$ and $D(+,21)$ are spin flip amplitudes, where the numbers 1 and 2 correspond to $+1/2$ and $-1/2$ components of the nucleon spin respectively.

4 Amplitude Determination

Let us we define the amplitudes in terms of the following parameters

$$D(+,11) = h_1 e^{i\alpha_1}, \tag{5}$$

$$D(+,21) = h_2 e^{i\alpha_2}, \tag{6}$$

$$D(+,12) = h_3 e^{i\alpha_3}, \tag{7}$$

$$D(+,22) = h_4 e^{i\alpha_4}. \tag{8}$$

Then in terms of h_i and α_i the experimental observables $\Sigma(\theta)$, $T(\theta)$, $P(\theta)$ and differential cross-section take the form

$$d\sigma/d\Omega = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2, \tag{9}$$

$$d\sigma/d\Omega * \Sigma(\theta) = |h_1|^2 + |h_2|^2 - |h_3|^2 - |h_4|^2, \tag{10}$$

$$d\sigma/d\Omega * T(\theta) = |h_1|^2 - |h_2|^2 - |h_3|^2 + |h_4|^2, \tag{11}$$

$$d\sigma/d\Omega * P(\theta) = |h_1|^2 - |h_2|^2 + |h_3|^2 - |h_4|^2. \tag{12}$$

We can now obtain the magnitudes of the four independent amplitudes in terms of the experimental observables $\Sigma(\theta)$, $T(\theta)$, $P(\theta)$ and the differential cross section

$$|h_1| = \frac{\sqrt{d\sigma/d\Omega}}{2}[1 + \Sigma(\theta) + T(\theta) + P(\theta)]^{1/2}, \tag{13}$$

$$|h_2| = \frac{\sqrt{d\sigma/d\Omega}}{2}[1 + \Sigma(\theta) - T(\theta) - P(\theta)]^{1/2}, \tag{14}$$

$$|h_3| = \frac{\sqrt{d\sigma/d\Omega}}{2}[1 - \Sigma(\theta) - T(\theta) + P(\theta)]^{1/2}, \quad (15)$$

$$|h_4| = \frac{\sqrt{d\sigma/d\Omega}}{2}[1 - \Sigma(\theta) + T(\theta) - P(\theta)]^{1/2}. \quad (16)$$

The values of these experimental observables are collected from Daresbury laboratory [7]. These magnitudes of four independent amplitudes are unambiguously constructed for the energy range of 1300 to 2100 MeV and four of these are plotted versus the neutral pion scattering angles from 25° to 110° in Figs. 1 to 4. In these figures we also compare the present work (p.w.) with the solution of partial wave analysis SM95 and SM00K [11].

5 Conclusion

The pion-photoproduction scattering amplitude analysis was performed using complete sets of observables at nine photon energies from 1300 to 2100 MeV at several angles. At these energies, the differential cross-section which was not measured, was taken from SM95. The results are listed in Tables 1 to 9 and four of these are plotted in Figs. 1 to 4. This analysis has indicated that at the angles less than 65° and the energies below 1600 MeV the non flip spin amplitude $D(+,11)$ is larger than the spin flip amplitude $D(+,21)$. At the energies above 1600 MeV $D(+,21)$ is larger than the $D(+,11)$. At the angles from 65° to 90° the spin flip amplitude $D(+,21)$ is larger than the spin non-flip amplitude $D(+,11)$ at all energies between 1300 to 2100 MeV. At the angles between 95° and 115° the non-flip spin amplitude $D(+,11)$ is larger than spin flip amplitude $D(+,21)$and at the angles between 25° to 95° the magnitude of spin flip amplitude $D(+,12)$ is comparable to the non-flip spin amplitude $D(+,22)$. The conclusion one can make is that the probability of spin to flip is low at small angles, increases at the intermediate angles (between 45° to 95°) and then decreases. One interesting observation is that at all the scattering angles the non-flip spin up amplitude dominates over the non-flip spin down amplitude. This means between the two non-flip amplitudes the one which is initially up is more probable to remain up than the other one which is initially down to remain spin down. In general one can say that the initial orientation of spin polarization of the target nuclei plays a significant role in the probability of spin to flip.

As indicated at figure 1 to 4 the results of the present work amplitude reconstruction and the results of two partial waves analysis SM95 and SM00K are compared at all energies and scattering angles. At several energies we observe a fluctuations and deviations of the present results from partial waves. Those are most probably due to the small absolute values of certain observables. A fair agreement of the present results and the partial waves predictions at several energies and scattering angles is found nonetheless.

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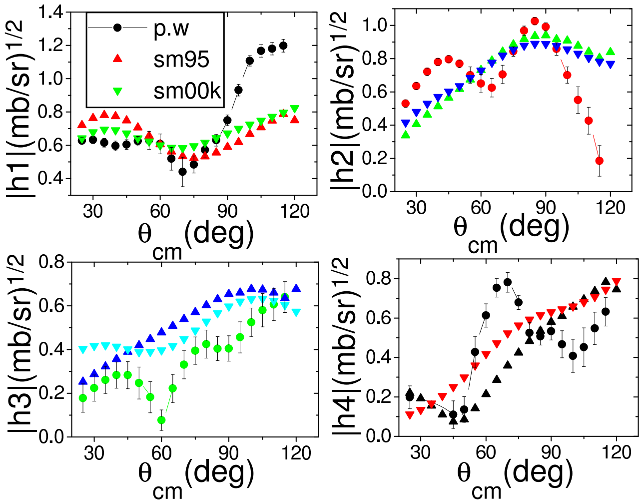

Fig. 1. The magnitudes of four independent amplitudes as a function of pion center of mass scattering angles at fixed photon energies (1600 MeV) for neutral pion photoproduction.

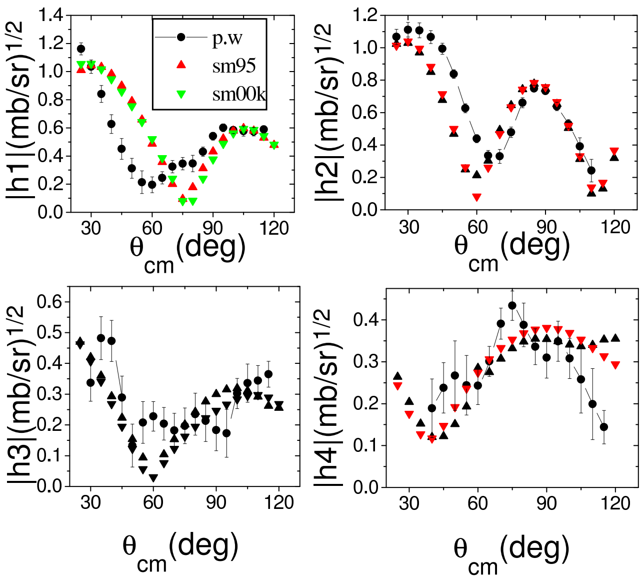

Fig. 2. The magnitudes of four independent amplitudes as a function of pion center of mass scattering angles at fixed photon energies (1900 MeV) for neutral pion photoproduction.

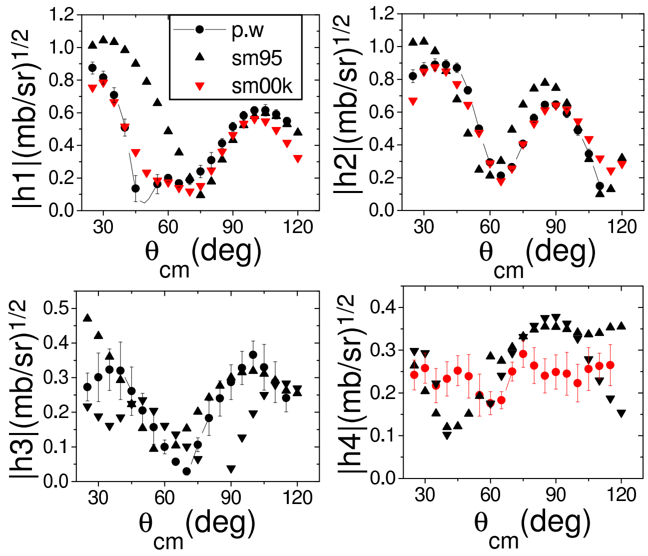

Fig. 3. The magnitudes of four independent amplitudes as a function of pion center of mass scattering angles at fixed photon energies (2000 MeV) for neutral pion photoproduction.

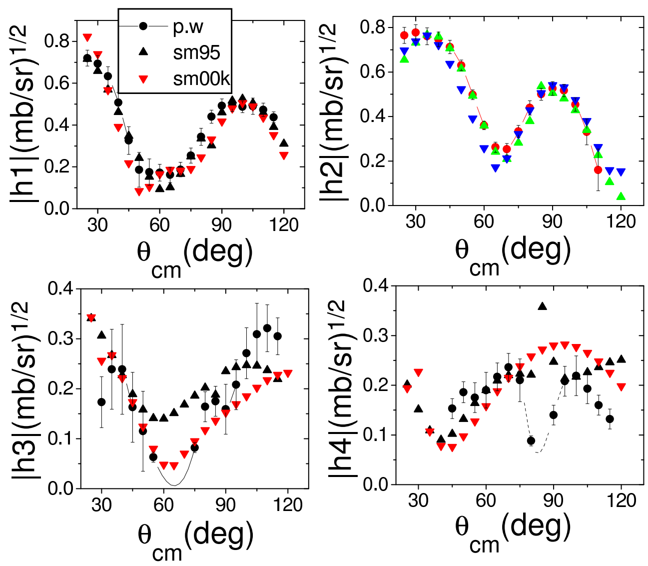

Fig. 4. The magnitudes of four independent amplitudes as a function of pion center of mass scattering angles at fixed photon energies (2100 MeV) for neutral pion photoproduction.

$$\mathbf{E = 1300MeV}$$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
35	1.872	0.095	1.905	0.093	0.514	0.210	—	—
40	2.158	0.087	1.838	0.102	0.338	0.150	—	—
45	2.508	0.089	1.544	0.144	0.617	0.300	—	—
50	2.711	0.086	1.219	0.192	0.794	0.294	—	—
55	2.510	0.150	1.434	0.263	0.501	0.300	0.518	0.060
60	2.055	0.192	1.864	0.211	0.909	0.319	1.310	1.070
65	1.811	0.155	2.116	0.133	1.060	0.231	1.377	0.061
70	1.859	0.156	2.194	0.132	0.939	0.265	0.928	0.096
75	1.710	0.143	2.392	0.102	1.047	0.184	0.663	0.110
80	1.309	0.190	2.627	0.095	1.266	0.151	0.939	0.084
85	1.074	0.179	2.628	0.073	1.374	0.168	1.142	0.094
90	1.303	0.147	2.409	0.080	1.426	0.158	1.152	0.098
95	1.769	0.131	2.066	0.112	1.507	0.223	1.002	0.100
100	2.118	0.106	1.702	0.132	1.653	0.193	0.784	0.110
105	2.227	0.151	1.348	0.249	1.749	0.171	0.778	0.094
110	2.080	0.153	0.965	0.331	—	—	1.085	0.110
115	1.894	0.158	—	—	—	—	—	—

Tab. 1. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$$\mathbf{E = 1400MeV}$$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.811	0.046	0.967	0.039	0.098	0.030	0.242	0.042
30	0.901	0.042	0.939	0.041	0.136	0.020	—	—
35	0.990	0.031	0.909	0.034	0.177	0.031	—	—
40	1.070	0.03	0.863	0.038	0.237	0.042	—	—
45	1.140	0.032	0.773	0.048	0.330	0.071	—	—
50	1.197	0.033	0.679	0.058	0.354	0.042	0.098	0.045
55	1.240	0.057	0.684	0.104	0.114	0.060	0.399	0.032
60	1.370	0.056	0.507	0.152	0.557	0.080	—	—
65	1.023	0.057	1.063	0.055	—	—	0.639	0.091
70	0.743	0.084	1.286	0.049	0.293	0.092	0.661	0.095
75	0.586	0.092	1.419	0.038	0.405	0.085	0.628	0.086
80	0.639	0.089	1.464	0.039	0.449	0.096	0.577	0.099
85	0.697	0.062	1.437	0.030	0.590	0.073	0.576	0.075
90	0.818	0.054	1.349	0.033	0.706	0.063	0.590	0.075
95	1.081	0.053	1.189	0.048	0.734	0.078	0.515	0.091
100	1.306	0.044	0.991	0.058	0.710	0.080	0.435	0.078
105	1.355	0.063	0.799	0.106	0.770	0.110	0.500	0.092
110	1.270	0.064	0.616	0.132	0.886	0.092	0.641	0.11
115	1.159	0.066	0.377	0.204	0.976	0.079	0.751	0.102

Tab. 2. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 1500MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.920	0.028	0.708	0.037	0.422	0.062	0.273	0.096
30	0.975	0.031	0.846	0.036	0.418	0.073	0.185	0.165
35	1.021	0.036	0.994	0.037	0.404	0.090	—	—
40	1.076	0.039	1.105	0.038	0.394	0.106	—	—
45	1.164	0.041	1.138	0.042	0.407	0.070	—	—
50	1.266	0.042	1.081	0.049	0.404	0.075	0.402	0.131
55	1.363	0.069	0.954	0.099	0.328	0.094	0.721	0.131
60	1.359	0.075	0.914	0.112	0.289	0.098	0.954	0.107
65	1.152	0.073	1.117	0.076	0.409	0.095	1.091	0.078
70	0.861	0.104	1.387	0.064	0.563	0.087	1.080	0.083
75	0.769	0.086	1.558	0.042	0.600	0.080	0.971	0.068
80	0.849	0.080	1.635	0.042	0.586	0.075	0.836	0.081
85	0.911	0.058	1.634	0.033	0.653	0.082	0.749	0.071
90	1.036	0.051	1.554	0.034	0.652	0.081	0.748	0.071
95	1.266	0.046	1.349	0.043	0.662	0.088	0.726	0.081
100	1.448	0.039	1.079	0.052	0.698	0.081	0.698	0.081
105	1.507	0.054	0.833	0.098	0.846	0.097	0.725	0.113
110	1.438	0.051	0.534	0.137	0.937	0.078	0.722	0.101
115	1.389	0.049	—	—	1.045	0.065	0.730	0.093

Tab. 3. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 1600MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.627	0.018	0.530	0.022	0.178	0.065	0.198	0.058
30	0.632	0.021	0.635	0.021	0.224	0.060	—	—
35	0.615	0.026	0.721	0.022	0.261	0.062	—	—
40	0.596	0.029	0.779	0.022	0.283	0.060	—	—
45	0.604	0.029	0.796	0.022	0.283	0.062	0.110	0.070
50	0.625	0.029	0.769	0.023	0.247	0.073	0.136	0.065
55	0.630	0.054	0.701	0.049	0.182	0.071	0.427	0.080
60	0.608	0.060	0.650	0.056	0.077	0.047	0.613	0.059
65	0.519	0.068	0.625	0.056	0.222	0.064	0.753	0.047
70	0.440	0.088	0.705	0.055	0.331	0.065	0.781	0.050
75	0.484	0.051	0.843	0.029	0.395	0.063	0.679	0.036
80	0.572	0.048	0.968	0.028	0.425	0.064	0.525	0.052
85	0.630	0.035	1.025	0.022	0.404	0.055	0.507	0.044
90	0.750	0.032	0.991	0.024	0.405	0.058	0.533	0.044
95	0.931	0.035	0.858	0.038	0.456	0.071	0.467	0.069
100	1.107	0.031	0.701	0.049	0.525	0.065	0.408	0.084
105	1.167	0.038	0.551	0.080	0.580	0.076	0.452	0.098
110	1.180	0.038	0.427	0.081	0.606	0.074	0.548	0.082
115	1.198	0.038	0.185	0.092	0.640	0.071	0.632	0.072

Tab. 4. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 1700MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.725	0.070	1.221	0.042	—	—	0.487	0.075
30	0.783	0.080	1.282	0.049	—	—	0.459	0.091
35	0.840	0.089	1.272	0.059	—	—	0.358	0.074
40	0.808	0.099	1.266	0.063	0.189	0.040	0.288	0.042
45	0.664	0.090	1.339	0.069	0.320	0.034	0.280	0.033
50	0.492	0.071	1.373	0.066	0.298	0.034	0.384	0.071
55	0.519	0.092	1.262	0.098	0.367	0.033	0.497	0.025
60	0.628	0.082	1.044	0.110	0.467	0.025	0.612	0.035
65	0.607	0.084	0.853	0.123	0.389	0.029	0.802	0.094
70	0.484	0.076	0.780	0.121	0.253	0.037	0.892	0.087
75	0.424	0.092	0.850	0.072	0.138	0.044	0.780	0.078
80	0.463	0.057	0.930	0.060	0.166	0.034	0.526	0.105
85	0.537	0.056	0.912	0.033	0.212	0.040	0.340	0.089
90	0.630	0.046	0.809	0.036	0.288	0.051	0.309	0.094
95	0.724	0.046	0.711	0.046	0.329	0.100	0.320	0.071
100	0.803	0.043	0.655	0.053	0.375	0.093	0.308	0.046
105	0.880	0.069	0.609	0.099	0.402	0.041	0.365	0.075
110	0.959	0.071	0.558	0.122	0.397	0.042	0.480	0.084
115	1.037	0.074	0.492	0.156	0.400	0.085	0.594	0.082

Tab. 5. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 1800MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.890	0.038	—	—	—	—	0.136	0.07
30	0.839	0.043	0.902	0.038	—	—	—	—
35	0.757	0.049	0.947	0.038	0.288	0.080	—	—
40	0.630	0.054	0.955	0.039	0.376	0.090	—	—
45	0.467	0.074	0.946	0.036	0.330	0.070	0.110	0.030
50	0.354	0.084	0.933	0.037	0.280	0.060	0.280	0.040
55	0.394	0.089	0.873	0.034	0.302	0.080	0.351	0.080
60	0.436	0.074	0.737	0.048	0.314	0.070	0.432	0.074
65	0.368	0.092	0.602	0.053	0.242	0.060	0.541	0.063
70	0.287	0.118	0.567	0.060	0.150	0.050	0.602	0.056
75	0.354	0.067	0.584	0.058	0.211	0.041	0.509	0.046
80	0.440	0.052	0.617	0.038	0.281	0.082	0.333	0.069
85	0.444	0.030	0.638	0.036	0.236	0.056	0.279	0.048
90	0.415	0.029	0.642	0.021	0.136	0.043	0.319	0.038
95	0.441	0.033	0.613	0.020	0.199	0.074	0.288	0.051
100	0.482	0.027	0.517	0.028	0.272	0.047	0.214	0.060
105	0.523	0.037	0.398	0.032	0.306	0.062	0.153	0.032
110	0.570	0.034	0.278	0.069	0.314	0.061	0.153	0.041
115	0.644	0.033	0.153	0.126	0.338	0.063	0.166	0.037

Tab. 6. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 1900MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	1.162	0.043	1.067	0.047	—	—	—	—
30	1.035	0.048	1.111	0.045	0.337	0.060	—	—
35	0.841	0.060	1.107	0.046	0.482	0.070	—	—
40	0.627	0.067	1.066	0.040	0.473	0.067	0.189	0.070
45	0.451	0.074	0.994	0.034	0.289	0.070	0.238	0.060
50	0.313	0.071	0.838	0.027	0.133	0.070	0.267	0.083
55	0.214	0.081	0.626	0.028	0.208	0.068	0.244	0.071
60	0.196	0.056	0.439	0.025	0.228	0.048	0.243	0.045
65	0.245	0.044	0.334	0.033	0.204	0.053	0.301	0.036
70	0.324	0.044	0.330	0.043	0.182	0.056	0.391	0.037
75	0.346	0.044	0.478	0.032	0.197	0.058	0.434	0.035
80	0.347	0.059	0.661	0.031	0.235	0.068	0.388	0.052
85	0.431	0.033	0.749	0.019	0.213	0.067	0.336	0.043
90	0.541	0.028	0.733	0.021	0.183	0.067	0.310	0.049
95	0.601	0.028	0.638	0.026	0.173	0.078	0.349	0.048
100	0.584	0.025	0.529	0.028	0.302	0.049	0.308	0.048
105	0.577	0.036	0.391	0.053	0.336	0.062	0.258	0.080
110	0.573	0.030	0.242	0.070	0.344	0.049	0.199	0.085
115	0.589	0.026	—	—	0.365	0.042	0.144	0.040

Tab. 7. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 2000MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.874	0.037	0.819	0.039	0.273	0.040	0.242	0.035
30	0.815	0.040	0.866	0.037	0.301	0.070	0.258	0.035
35	0.708	0.045	0.889	0.036	0.323	0.060	0.217	0.040
40	0.509	0.052	0.889	0.030	0.320	0.083	0.233	0.040
45	0.135	0.080	0.870	0.029	0.261	0.070	0.252	0.035
50	—	—	0.733	0.023	0.205	0.050	0.239	0.051
55	0.162	0.059	0.499	0.019	0.157	0.050	0.195	0.049
60	0.200	0.023	0.293	0.016	0.100	0.020	0.176	0.027
65	0.167	0.022	0.212	0.017	0.057	0.010	0.183	0.020
70	0.188	0.029	0.265	0.021	0.029	0.010	0.250	0.022
75	0.240	0.038	0.407	0.022	0.106	0.020	0.291	0.031
80	0.309	0.048	0.563	0.026	0.183	0.050	0.264	0.042
85	0.412	0.030	0.644	0.019	0.240	0.051	0.240	0.040
90	0.514	0.028	0.646	0.022	0.287	0.050	0.249	0.040
95	0.582	0.027	0.593	0.027	0.328	0.048	0.245	0.050
100	0.614	0.024	0.488	0.030	0.366	0.040	0.223	0.044
105	0.615	0.035	0.345	0.030	0.330	0.066	0.256	0.050
110	0.590	0.029	0.149	0.020	0.287	0.059	0.263	0.030
115	0.549	0.023	—	—	0.241	0.040	0.265	0.048

Tab. 8. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

$\mathbf{E = 2100MeV}$

θ	$D(+,11)$	$dD(+,11)$	$D(+,21)$	$dD(+,21)$	$D(+,12)$	$dD(+,12)$	$D(+,22)$	$dD(+,22)$
25	0.720	0.038	0.765	0.036	—	—	—	—
30	0.693	0.039	0.778	0.035	0.173	0.051	—	—
35	0.633	0.046	0.761	0.038	0.239	0.080	—	—
40	0.507	0.049	0.747	0.033	0.239	0.090	—	—
45	0.326	0.066	0.713	0.030	0.163	0.070	0.153	0.020
50	0.186	0.083	0.630	0.024	0.115	0.080	0.186	0.021
55	0.175	0.063	0.498	0.022	0.063	0.010	0.175	0.030
60	0.172	0.040	0.360	0.019	—	—	0.190	0.036
65	0.162	0.037	0.262	0.023	—	—	0.217	0.028
70	0.187	0.035	0.253	0.026	—	—	0.236	0.028
75	0.254	0.036	0.333	0.027	0.082	0.010	0.210	0.043
80	0.344	0.041	0.439	0.032	0.164	0.030	0.088	0.010
85	0.440	0.031	—	—	0.175	0.030	—	—
90	0.492	0.033	0.500	0.027	0.159	0.050	0.140	0.020
95	0.494	0.030	0.526	0.031	0.208	0.050	0.208	0.030
100	0.487	0.028	0.517	0.028	0.271	0.051	0.219	0.040
105	0.490	0.039	0.454	0.030	0.309	0.062	0.193	0.030
110	0.473	0.032	0.331	0.058	0.321	0.047	0.160	0.020
115	0.437	0.026	0.160	0.094	0.305	0.037	0.132	0.020

Tab. 9. The magnitudes of the four independent amplitudes in the hybride frame for different values of pion center of mass scattering angles and incident photon energies for neutral pion photoproduction.

References

[1] N. Ghahramany, G. Forozani: *Phys. Rev. C* **61** (2000) 1086 , J. Bystricky, C. Lechanoine-Leluc, F. Lehar: *Eur. Phys. J. C* **4** (1998) 607 , C. Lechanoine-Leluc, F. Lehar: *Rev. Mod. Phys.* **65** (1993) 47 , M.W. Mcnaughton *et al.*: *Phys. Rev. C* **41** (1990) 2809 , R. Hausammann *et al.*: *Phys. Rev. D* **40** (1989) 22 , I.P. Auer *et al.*: *Phys. Rev. D* **34** (1986) 1 N. Ghahramany, G. Goldstein, M. Moravcsik: *Phys. Rev. D* **28** (1983) 1086 , J. Bystricky, F. Lehar, P. Winternitz: *Lect. Notes Phys.* **87** (1978) 509

[2] M. Moravcsik: *Phys. Rev. D* **27** (1983) 289

[3] G. Martin *et al.*: *Nucl. Phys. A* **487** (1988) 563

[4] J. Ball *et al.*: *Il Nuovo Cimento A* **111** (1998) 13

[5] G. Goldstein, M. Moravcsik: *Ann. Phys. (N. Y.)* **98** (1976) 128 , G. Goldstein, M. Moravcsik: *Nuovo Cim. A* **84** (1984) 39

[6] C. Allgower *et al.*: *Eur. Phys. J. C* **1** (1998) 131 , C. Allgower *et al.*: *Nucl. Phys. A* **637** (1998) 231 , C. Allgower *et al.*: *Eur. Phys. J. C* **5** (1998) 453

[7] P. J. Bussey *et al.*: *Nucl. Phys. B* **154** (1979) 492 , P.J. Bussey *et al.*: *Nucl. Phys. B* **159** (1979) 383 , P.J. Bussey *et al.*: *Nucl. Phys. B* **154** (1979) 205 , P.J. Bussey *et al.*: *Nucl. Phys. B* **169** (1979) 403

[8] M. Moravcsik, R. Reid: *Ann. Phys. (NY)* **84** (1974) 535

[9] P. Chonka, M. Moravcsik: *Phys. Rev. D* **1** (1970) 1821

[10] G. Goldstein, M. Moravcsik: *Ann. Phys.* **142** (1982) 219

[11] A. R. Arndt *et al.*: *Phys. Rev. C* **66** (2002) 055213 and references therein