acta physica slovaca vol. 53 No. 6, 453 – 462 December 2003

A MODEL OF BINARY COLLISIONS DEPENDENCE OF JET QUENCHING IN NUCLEAR COLLISIONS AT ULTRARELATIVISTIC ENERGIES

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Received 8 September 2003 in final form 20 October 2003, accepted 22 October 2003

We describe a model of jet quenching in nuclear collisions at RHIC energies. In the model, jet quenching is to be caused by the interruption of jet formation by nucleons arriving at the position of jet formation in a time shorter than the jet formation time. Our mechanism predicts suppression of high-p_T spectra also in d+Au reactions.

PACS: 25.75.-q, 25.45.-z, 25.75.Nq

1 Introduction

Hadronic spectra at high transverse momenta from ultrarelativistic nuclear collisions result from jets produced in the early hard partonic interactions. The yields are normally expected to scale with the number of binary nucleon-nucleon collisions. However, experimental results obtained at the Relativistic Heavy Ion Collider (RHIC) contradict these expectations: in central Au+Au collisions the yields above 2 GeV/c are *strongly suppressed* with respect to binary collisions scaling [1–4]. Recent data from d+Au collisions show an *enhancement* of the yield with respect to the binary-collision scaling for $p_T \gtrsim 2\,\mathrm{GeV}/c$ [5–8] but indicate that the yield may fall below this expectation above 8 GeV/c [6].

The lack of strong suppression of high-p_T yields in deuteron-induced reactions suggests that the effect in Au+Au collisions is due to partonic energy loss in a medium with very high energy density. This mechanism has been studied by many authors [9–14].

In addition, there are some models which aim to describe high-p_T spectra from nuclear collisions by invoking different physical effects. The perturbative-QCD-improved parton model of refs. [15, 16] includes shadowing and the broadening of p_T spectra due to intrinsic parton transverse momentum. The authors conclude that the mechanism is insufficient to explain high-p_T suppression at RHIC and even at the SPS.

The mechanism of parton saturation [17–19] was argued to lead to suppression of intermediate p_T production in both Au+Au and d+Au collision systems [20, 21].

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In ref. [22], we proposed a model for the suppression of high-p_T spectra in ultrarelativistic nuclear collisions based on arguments involving *the uncertainty principle* and *formation time* [23, 24]. The mean-free path of an incident nucleon in the nucleon–nucleon centre-of-mass system (CMS) at RHIC is rather short: $\lambda \approx 0.025$ fm. When two jets are produced in a particular nucleon–nucleon collision, the next nucleon arrives at the position where the jets were created in a time interval of the order of λ/c. According to the uncertainty relation, a process with longitudinal momentum transfer Δp_L and energy transfer ΔE needs space and time intervals given by

$$\Delta z > \frac{\hbar}{\Delta p_L}\,, \qquad \Delta t > \frac{\hbar}{\Delta E}\,. \tag{1}$$

to reach completion. Hence, if the mean-free path or the mean-free time are shorter than these intervals, the created jets may be seriously influenced by nucleons arriving at the position where the process develops. For the quoted mean-free path at RHIC, this puts a limit on processes which are *not* influenced by our mechanism at $\Delta p_L \gtrsim 8\,\mathrm{GeV}/c$. The limit is, of course, not strict, since the time interval between subsequent collisions fluctuates.

In the next Section we present a slightly improved version of the model from our earlier paper [22]. The results are shown in Section 3 and we conclude in Section 4. The appendix contains a discussion of Lorentz invariance in our model. In what follows we shall work in natural units $c = \hbar = 1$.

2 The model

We shall formulate our Glauber model in the nucleon–nucleon centre-of-mass system. For the sake of simplicity, we assume that the density distribution is uniform. Technically, the model is rather similar to those for nuclear absorption of J/ψ in heavy-ion collisions [25].

A Glauber model is usually formulated in terms of "tube-on-tube" collisions, with tubes filled with nucleons[2]. For a collision of nuclei A and B at impact parameter b we have for the lengths of the colliding tubes

$$2\,L_A(s) = 2\,\gamma^{-1}\sqrt{R_A^2 - s^2}\,, \quad 2\,L_B(b,s,\theta) = 2\,\gamma^{-1}\sqrt{R_B^2 - b^2 - s^2 + 2\,b\,s\cos\theta}\,. \tag{2}$$

Here, γ is the Lorentz contraction factor for the boost of the nuclei from their own rest-frame to the CMS. All other coordinates and sizes on the r.h.s., however, are taken in the rest frames of the nuclei and their meaning is explained in Figure 1. The positions of nucleons within the colliding tubes will be denoted by z_A and z_B. The values of z_A and z_B satisfy

$$-L_A \le z_A \le L_A, \quad -L_B \le z_B \le L_B.$$

We take z_A and z_B as increasing in the direction of motion of A and B respectively in the CMS.

We will be interested in comparing the yield of jets at high transverse momentum produced in nuclear collisions to that produced in nucleon–nucleon collisions. By staying at the level

[2]The integral of the density along the "tube", i.e. in the longitudinal direction, is often called *the thickness function* $T_A(s) = \int_{-\infty}^{\infty} \rho(\sqrt{s^2+z^2})\,dz$. In our model, the density is given by $\rho_A\theta(R_A - r)$ and the thickness function corresponds to the tube length given in eq. (2) multiplied by the constant nuclear density.

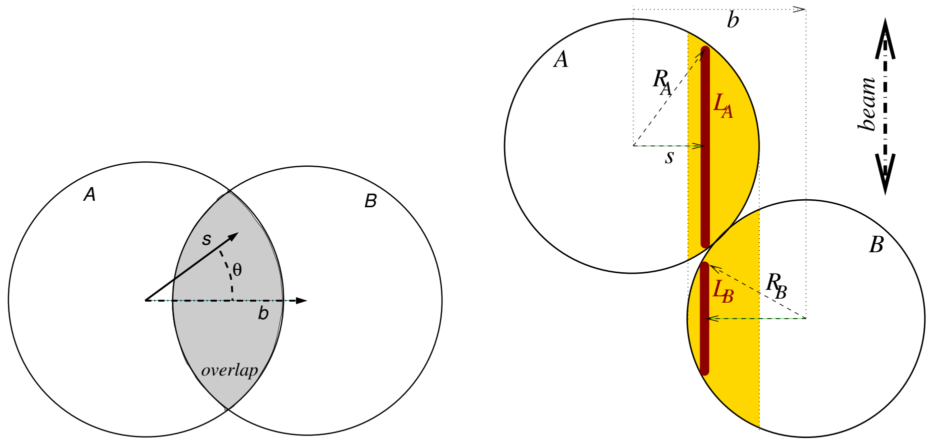

Fig. 1. Left: geometry of non-central nuclear collisions. Right: layout of tube-on-tube interaction (plotted without Lorentz contraction).

of jets and not including fragmentation, our calculation is not influenced by the form of the fragmentation function which is not entirely known for nuclear collisions. On the other hand, we must keep in mind that our results cannot be compared directly to the measured p_T-spectra.

The yield of jets in collisions of nuclei A+B at impact parameter b is defined as

$$Y_{AB}(p_T, b) = \frac{\frac{d\sigma_{AB}}{dp_T^2\, db^2}}{\frac{d\sigma_{AB}}{db^2}}\,, \tag{3}$$

while the yield in nucleon–nucleon collisions is introduced as

$$Y_{pp}(p_T) = \frac{\frac{d\sigma_{pp}}{dp_T^2}}{\sigma_{pp}}\,. \tag{4}$$

As in our previous paper [22] we intend to determine quantity

$$R_{AB}(p_T, b) = \frac{Y_{AB}(p_T, b)}{Y_{pp}(p_T)} = \sigma_{nn} \int_{\text{overlap}} s\, ds\, d\theta \int_{-L_A}^{L_A} dz_A\, \rho_A \int_{-L_B}^{L_B} dz_B\, \rho_B\, F(b, s, \theta, z_A, z_B)\,. \tag{5}$$

Here, ρ_A and ρ_B are the Lorentz-contracted nuclear densities of the nuclei A and B,

$$\rho_A = \rho_B = \rho = \gamma\, \rho_0$$

with $\rho_0 = 0.138\,\text{fm}^{-3}$. For the total inelastic nucleon–nucleon cross-section we will take

$$\sigma_{nn} = 40\,\text{mb} = 4\,\text{fm}^2\,.$$

Without the suppression factor $F(b, s, \theta, z_A, z_B)$, formula (5) would give the average number of binary collisions in an interaction of nuclei A+B at impact parameter b. The suppression factor will account for the assumed effect of jet destruction.

If jets are created in a hard interaction of two incident partons of the colliding nuclei, we shall assume a cross-section for their destruction by a subsequent incoming nucleon (in CMS) an expression

$$\sigma_a(p_T, t, t') = \sigma_0 \left(\frac{1}{1 + (p_T(t' - t))^2} \right)^2 , \tag{6}$$

where t is the time coordinate of the hard interaction and t' is the time[3] when the destroying nucleon arrives at the position of the jet[4]. The jet transverse momentum p_T is roughly equal to the energy ΔE involved in the process. Therefore, the form (6) is in line with our considerations about the jet destruction based on the uncertainty relation described in the introduction. The absorption cross-section is of order of few milibarns and were tuned by the parameter σ_0. Although formula (6) for the cross-section is written in the CMS reference frame, we demonstrate in the Appendix how it can be written in an explicitly invariant way. The improvement over our previous paper [22] lies in the time-dependent prescription for the absorption cross-section. In [22] we ignored the fact that nucleons with different distances from the origin of the jet have different chances to destroy it, and we assumed

$$\sigma_a^{\mathrm{old}}(p_T) = \sigma_0 \left(\frac{1}{1 + (p_T/p_{T0})^2} \right)^2 \qquad \text{(the model of [22])}, \tag{7}$$

with $p_{T0} = 8\,\mathrm{GeV}/c$ for collisions at RHIC.

The suppression factor $F(b, s, \theta, z_A, z_B)$ now reads

$$F(b, s, \theta, z_A, z_B) = \exp\left(- \int_{-L_A/v}^{z_A/v} \sigma_a \left(p_T, t, \frac{z_A}{v} \right) \rho_A \, v \, dt \right) \times \exp\left(- \int_{-L_B/v}^{z_B/v} \sigma_a \left(p_T, t, \frac{z_B}{v} \right) \rho_B \, v \, dt \right) , \tag{8}$$

where z_A and z_B determine the position of the hard interaction within nuclei A and B, respectively, and v is the longitudinal velocity of the colliding nuclei. The b, s, and θ-dependences of F are implicitly included in L_A and L_B. When we insert the prescription (6) into equation (8), the integrals can be performed analytically and lead to

$$F(b, s, \theta, z_A, z_B) = \exp\left[-\frac{\sigma_0 \, v \, \rho_A}{2 \, p_T} \left(\frac{f_A}{1 + f_A^2} + \arctan f_A \right) - \frac{\sigma_0 \, v \, \rho_B}{2 \, p_T} \left(\frac{f_B}{1 + f_B^2} + \arctan f_B \right) \right] , \tag{9}$$

[3] In our previous paper [22] we used the longitudinal coordinates z and z' instead of the times in our argumentation. The advantage of the new formalism is that σ_a can be written in a Lorentz invariant way, as described in the Appendix. At the energies studied, however, the numerical difference between $(t' - t)$ and $(z' - z)$ is negligible.

[4] For brevity, we talk about a "destroying nucleon" although we rather mean those of its quarks with large enough momentum. The problem with soft sea quarks is that they can be barely localised as precisely as we need for our argument, but we can assume that their energy is too small to destroy the jet.

with

$$f_A = \frac{p_T}{v}(z_A + L_A)\,, \qquad f_B = \frac{p_T}{v}(z_B + L_B)\,. \tag{10}$$

We can compare our results with our previous model of ref. [22]. The suppression factor there was obtained by inserting formula (7) for the absorption cross-section. This leads to

$$\begin{aligned} F^{\rm old}(b,s,\theta,z_A,z_B) &= \exp\left(-\int_{-L_A}^{z_A} \sigma_a^{\rm old}(p_T)\,\rho_A\,dz\right) \exp\left(-\int_{-L_B}^{z_B} \sigma_a^{\rm old}(p_T)\,\rho_B\,dz\right) \\ &= \exp\left(-\sigma_a^{\rm old}(p_T)\,\rho_A\,(z_A+L_A) - \sigma_a^{\rm old}(p_T)\,\rho_B\,(z_B+L_B)\right)\,. \end{aligned} \tag{11}$$

3 Results

In our calculation we simulated a set of nuclear collisions within the Glauber model framework and for every nucleon–nucleon interaction we determined the suppression factor according to eq. (9). By adding the suppression factors from all nucleon–nucleon interactions we obtained the total yield of jets relative to the yield from nucleon–nucleon interactions as defined in eq. (5). The set of nuclear collisions under study was always chosen appropriately to sample given centrality requirements.

The absorption cross-section parameter σ_0 was tuned to

$$\sigma_0 = 8\,\mathrm{mb} = 0.8\,\mathrm{fm}^2\,.$$

In the figures we always plot the relative yield of jets divided by the number of binary collisions. In case of no suppression this number approaches unity. As we have already mentioned, we cannot compare our results directly to experimental data because fragmentation is not included in our calculation. Nevertheless, we make some remarks on the relation of our results to data in Section 4.

Figure 2 shows the dependence of the relative yield on the number of binary collisions. We observe that scaling with the number of binary collisions is slowly recovered for large p_T. As the transverse momentum increases, the curves converge to the asymptotic value of $R_{AB}(p_T, n_{\rm coll})/n_{\rm coll} = 1$.

The p_T-dependence of the yields of jets is plotted in Figure 3 for central and peripheral Au+Au collisions and for d+Au collisions. Our mechanism leads to suppression of jet production even in d+Au collisions. We comment on the relation of this result to data in the next Section.

In Figure 4 we compare our model with that of ref. [22]. This comparison shows that the models give similar results at low transverse momenta while at high p_T the suppression is stronger in the new model. This is due to destroying nucleons which may be very close to the jet production site in the new model (i.e., $(t' - t)$ is small) and thus associated with a very large absorption cross-section. In the old model [22], on the other hand, all nucleons were effectively put a distance λ from the jet production such that there were effectively no nucleons that would be close enough to destroy very-high-p_T jets[5]. In Figure 4 we also show the result of a modified model in

[5]Mathematically, in the present model the supression factor F in eq. (9) goes like $\exp(-\mathrm{const}/p_T^2)$ for high p_T, while in the old model the factor $F^{\rm old}$ of eq. (11) approaches $\exp(-\mathrm{const}/p_T^4)$.

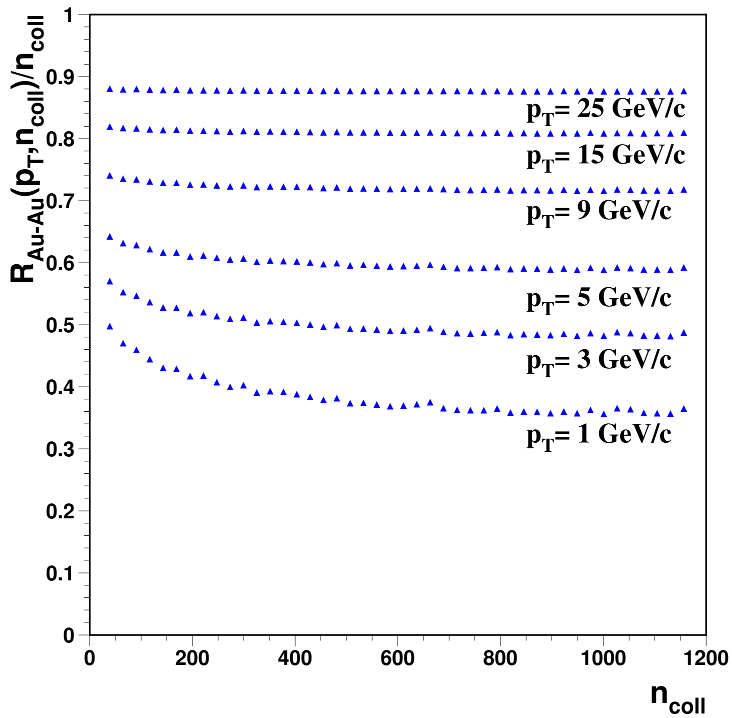

Fig. 2. The ratio $R_{\mathrm{Au+Au}}(p_T, b)/n_{\mathrm{coll}}(b)$ for central collisions at $\sqrt{s} = 200$ AGeV plotted as a function of n_{coll}. Different curves correspond to variations of p_T, as indicated.

which we forbid the nucleons within a nucleus (at rest) to be closer to each other than 1 fm, such that there are no very close destroying nucleons. This makes the suppression weaker by 5–15%, depending on p_T.

When we compare the old and the actual model at low p_T, we see the effect of suppression by subsequent nucleons folowing the first potentially destroying nucleon. While in the old model their absorption cross-sections were equal to that of the first destroying nucleon, in the new formulation these cross-sections are smaller due to the larger distance from the origin of the jet production. Therefore, the new model gives weaker suppression at low p_T than the old one.

4 Conclusions

We have described here an improved version of the jet attenuation model of [22] in which the cross-section for the jet attenuation is time-dependent as implied by (1). We have shown that results for the jet attenuation obtained by the new version do not differ very much from the previous version.

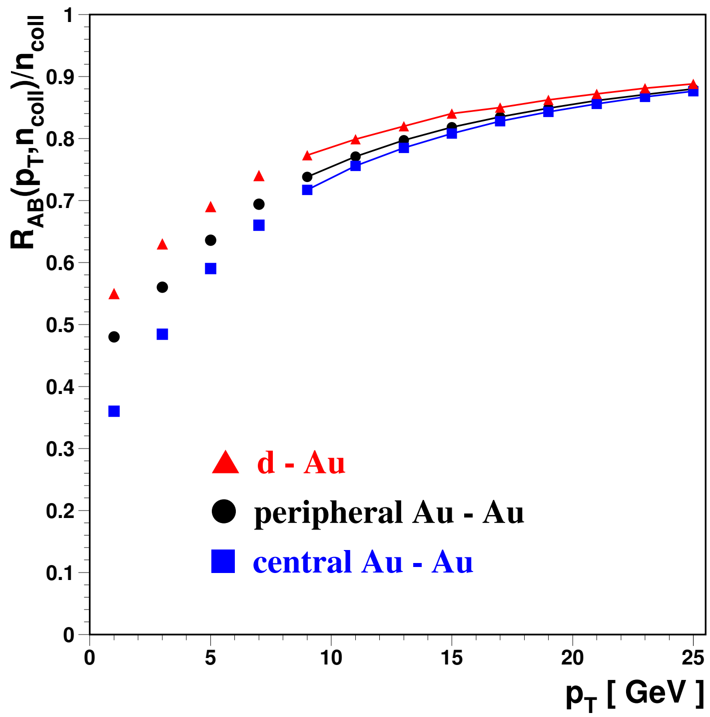

Fig. 3. The ratio $R_{\mathrm{Au+Au}}(p_T, b)/n_{\mathrm{coll}}(b)$ plotted as a function of the jet transverse momentum p_T. Calculated for collisions at $\sqrt{s} = 200\ A$GeV. Different curves refer to: central Au+Au collisions (0–5% of the total cross-section), peripheral Au+Au collisions (60–80%), and minimum bias d+Au collisions.

An important feature of our model is the lack of any thresholds under which the mechanism of jet suppression would cease. We obtain a *suppression* with respect to the scaling with the number of binary collisions even in the case of *d+Au collisions*. This differs from the recently published high-p_T data from d+Au collisions at RHIC [5–8]. An *enhancement*, i.e., a yield larger than expected from the n_{coll}-scaling was reported from the deuteron-induced reactions. This is likely to be, at least in part, a Cronin effect which results from p_T-broadening of the incoming partons. Since there is no such effect included in the model, we cannot describe this feature.

However, the data on high-p_T spectra of charged hadrons from d+Au collisions published by STAR collaboration indicate [6] that the yield normalised to the p_T-spectrum and the number of binary collisions possibly falls below unity for $p_T \gtrsim 9\,\mathrm{GeV}/c$. In this kinematic region, the Cronin effect may not be effective any more and the suppression could be described by our model. In order to indicate this, in Figure 3 we did not plot the curves in the kinematic region where we expect the dominance of the Cronin effect and highlighted only the kinematic region where our model could stay relevant as is. We plan to combine the Cronin effect with the present model in the future and confront our results with existing data.

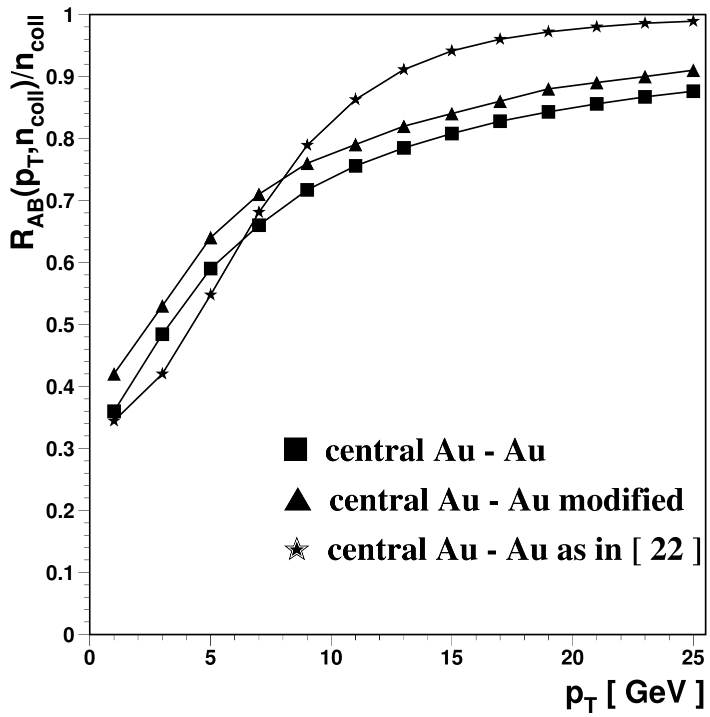

Fig. 4. The comparison of our model to that of our earlier paper [22]. The relative yield $R_{\mathrm{Au+Au}}(p_T,b)/n_{\mathrm{coll}}(b)$ plotted as a function of p_T, calculated for central Au+Au collisions (0–5% centrality cut) at $\sqrt{s}=200$ AGeV.

Acknowledgements

One of the authors (JP) is indebted to the CERN Theory Division for the hospitality extended to him. We like to thank Orlando Villalobos-Baillie for many useful discussions and careful reading of the paper. The work of JP and NP was supported in part by the Slovak Ministry of Education under the Grant No. VEGA V2F13.

A Lorentz invariant formulation of the absorption cross-section

In the earlier version of our model [22], the relative yield R, the suppression factor F, and the absorption cross-section σ_a, given by eqs. (6), (7), and (3) of that paper, respectively, are manifestly invariant under longitudinal boosts, since p_T and σ_0 are boost invariant and the products $\rho_A\,dz_A$ and $\rho_B\,dz_B$ are as well. In the present formulation, the corresponding equations (5), (6), and (8) are valid in the CMS frame, but they can be also written in a Lorentz invariant way.

Prescription (6) for the absorption cross-section was formulated in the centre-of-mass system.

In this system, it says that σ_a will be large if

$$(t' - t)^2 \lesssim \frac{1}{p_T^2} \, . \tag{12}$$

We treat only jets at central rapidity which do not move longitudinally in CMS. The positions where the jet is created and where it is hit by another nucleon are thus the same. Thus, the l.h.s. of the previous inequality can be written in a Lorentz invariant way:

$$(x'^{\mu} - x^{\mu})(x'_{\mu} - x_{\mu}) \lesssim \frac{1}{p_T^2} \, , \tag{13}$$

where x^μ and x'^μ are the four-vectors corresponding to events of jet creation and possible destruction, respectively. A Lorentz invariant formula for the absorption cross-section for midrapidity jets thus reads

$$\sigma_a(p_T, x^\mu, x'^\mu) = \sigma_0 \left(\frac{1}{1 + p_T^2 (x'^\mu - x^\mu)(x'_\mu - x_\mu)} \right)^2 \, . \tag{14}$$

We can put this in a more suitable form. We neglect the movement in transverse direction and write

$$(x'^{\mu} - x^{\mu})(x'_{\mu} - x_{\mu}) = \Delta t^2 - \Delta z^2 \tag{15}$$

We denote:

- y_j the jet rapidity in a given frame,
- y_A the nucleon rapidity, same as nucleus rapidity,
- z^* the distance of the destroying nucleon from the place where the jet was produced, measured in the rest-frame of the nucleus.

Then, the time between jet creation and its possible destruction can be written as

$$\Delta t = \frac{z^*}{\cosh y_A (\tanh y_A - \tanh y_j)} \, , \tag{16}$$

and the distance between these two events is given by

$$\Delta z = \Delta t \, v_{\text{jet}} = \frac{z^* \, \tanh y_j}{\cosh y_A (\tanh y_A - \tanh y_j)} \, . \tag{17}$$

Using these equations and after some algebra, we obtain

$$\Delta t^2 - \Delta z^2 = \frac{z^{*2}}{\sinh(y_A - y_j)} \, . \tag{18}$$

This form depends only on difference of rapidities and is thus Lorentz invariant.

If we are only interested in midrapidity jets, we can replace

$$y_j = \frac{y_A + y_B}{2}$$

where y_A and y_B are rapidities of the colliding nuclei and obtain the absorption cross section

$$\sigma_a = \sigma_0 \left(\frac{1}{1 + \frac{p_T^2 \, z^{*2}}{\sinh^2((y_A - y_B)/2)}} \right)^2 . \tag{19}$$

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