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CALCULATION OF THE log(ft) VALUES FOR THE ALLOWED GAMOW-TELLER TRANSITIONS IN DEFORMED NUCLEI USING THE BASIS OF WOODS-SAXON WAVE FUNCTIONS

D. I. Salamov*, **A. Küçükbursa**[1]*, **I. Maraş**†, **H. A. Aygör**†, **T. Babacan**†, **H. Bircan***

*Dumlupınar University, Department of Physics, Kütahya-Turkey

†Celal Bayar University, Department of Physics, Manisa-Turkey

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The present study investigates the log(ft) values of the allowed $\beta^{\pm}$ decay between odd-A deformed nuclei in the atomic mass regions of $125 \leq A \leq 131$ and $159 \leq A \leq 181$. Single particle energies and wave functions have been calculated with a deformed Woods-Saxon potential. Calculations have been performed within the framework of a proton-neutron quasi-particle random phase approximation (QRPA), including the schematic residual spin-isospin interaction between nucleons in the particle-hole and particle-particle channels. It has been seen that the results obtained by using the values for the fixed particle-hole and particle-particle interaction strengths, $\chi_{GT}^{ph} = 5.2/A^{0.7}\ MeV$ and $\chi_{GT}^{pp} = 0.58/A^{0.7}\ MeV$, have good agreement with the experimental observations.

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1 Introduction

Following the observation of the Fermi resonance in 1960 [1], Ikeda, Fujii and Fujita [2–6] studied this resonance theoretically and explained it through proton particle-neutron hole pair model. This model was also applied to the Gamow-Teller transition and showed that the existence of Gamow-Teller Resonance (GTR), firstly observed by Doering et al [7], is similar to the origin of the Fermi resonance. The "Gross Theory" developed by Takahashi and Yamada [8] was employed for the calculations of the β decay rates of the nuclei. Because of its statistical character, however this theory describes only the average properties of the β strength function and not the effects associated with shell structure. The macroscopic model, including the shell structures of nuclei in the β decay theory, was developed by Hamamoto [9], and Halbleib and Sorensen [10]. The Random Phase Approximation (RPA) model developed by Halbleib and Sorensen has been rather improved by other authors [11–19]. In this model, one first constructs a particle or quasi-particle basis with a pairing interaction between like nucleons, and then solves the RPA or QRPA equation with a schematic spin-isospin (for Gamow-Teller β decay) or isospin (for Fermi β decay) residual interaction. The residual interaction mentioned above plays a significant role in explaining the properties of the Gamow-Teller and Isobar Analogue Resonance (IAS). The χ_{GT} and χ_F constants are free parameters included in the spin-isospin residual interaction and isospin residual interaction, respectively. These parameters are, in general, obtained from the experimental positions IAS and GTR.

[1]E-mail address: atalaykucukbursa@hotmail.com

The spin-isospin residual interaction between nucleons plays an important role for the explanation of the GTR properties, and has also an effect on the β decay of odd nuclei [11, 20, 21], the magnetic moment [22–24], the polarization of the nuclear structure process [23–25], and the double β decay matrix elements.

The χ_{GT} constant is in general obtained from the experimental values of GTR via (n,p) and $(^3He, t)$ reactions. Different χ_{GT}'s are used in order to establish an agreement between the experimental values of GTR positions in different mass region. For example, the accepted values for the constant χ_{GT} are as follows: for heavy nuclei (^{208}Pb), $\chi_{GT} = 23/A\ MeV$ [26]; in the region of Fe nuclei, $\chi_{GT} = 15/A\ MeV$ [16]; in the region of neutron deficient Cs isotopes, $\chi_{GT} = (1.5 \div 2)(N-Z)/A\ MeV$ [14]. However, beta decay half-lives of nuclei up to A=150 have been calculated by Homma et al. [27] using the values for the fixed particle-hole and particle-particle interaction strengths, $\chi_{GT}^{ph} = 5.2/A^{0.7}\ MeV$ and $\chi_{GT}^{pp} = 0.58/A^{0.7}\ MeV$, and generally good agreement with experiment has been obtained.

In this study, we have attempted to determine the strengths of the particle-hole and particle-particle forces that would reproduce the experimental log(ft) values of $\beta^{\pm}$ decay for various odd mass deformed nuclei. The deformed Woods-Saxon potential basis has been selected for single particle basis. The issue has been solved within the framework of the proton-neutron quasi-particle random-phase approximation (QRPA), including the residual spin-isospin interaction between the nucleons in the particle-hole and particle-particle channels.

2 Model Hamiltonian

Let us consider a system of nucleons in an axially symmetric average field interacting via pairing and spin-spin interactions with a charge-exchange. In this case, Hamiltonian of the system in quasi-particle representation is given as:

$$H_0 = H_{SQP} + V_{GT}^{ph} + V_{GT}^{pp}. \tag{1}$$

H_{SQP} is the single quasi-particle (SQP) Hamiltonian and described by:

$$H_{SQP} = \sum_{s,\tau,\rho} E_s(\tau)\alpha_{s\rho}^{\dagger}(\tau)\alpha_{s\rho}(\tau), \quad \tau = n, p, \tag{2}$$

where $E_s(\tau)$ is the single quasi-particle energy of the nucleons, $\alpha_{s\rho}^{\dagger}(\alpha_{s\rho})$ is the quasi-particle creation(annihilation) operator. V_{GT}^{ph} and V_{GT}^{pp} are the residual charge-exchange spin-spin interactions in particle-hole and particle-particle channels, respectively, and given by the formula:

$$V_{GT}^{ph} = 2\chi_{GT}^{ph}\sum_{\mu}\beta_{\mu}^{+}\beta_{\mu}^{-}, \quad V_{GT}^{pp} = -2\chi_{GT}^{pp}\sum_{\mu}P_{\mu}^{+}P_{\mu}^{-}, \qquad \mu = 0, \pm 1, \tag{3}$$

with

$$\beta_{\mu}^{+} = \sum_{\substack{n,p\\ \rho,\rho'}}\langle n\rho|\sigma_{\mu} + (-1)^{\mu}\sigma_{-\mu}|p\rho'\rangle a_{n\rho}^{\dagger}a_{p\rho'}, \qquad \beta_{\mu}^{-} = (\beta_{\mu}^{+})^{\dagger},$$
$$P_{\mu}^{+} = \sum_{\substack{n,p\\ \rho,\rho'}}\langle n\rho|\sigma_{\mu} + (-1)^{\mu}\sigma_{-\mu}|p\rho'\rangle a_{n\rho}^{\dagger}a_{p\rho'}^{\dagger}, \qquad P_{\mu}^{-} = (P_{\mu}^{+})^{\dagger}, \tag{4}$$

where $a^{\dagger}_{\tau\rho}(a_{\tau\rho})$ is the nucleon creation (annihilation) operator, σ_{μ} is the spherical component of the Pauli operator. In the quasi-particle representation, the $\beta^{\pm}_{\mu}$ and $P^{\pm}_{\mu}$ operators are introduced as:

$$\beta^{+}_{\mu} = \sum_{n,p}[\frac{1}{\sqrt{2}}(\overline{d}_{np}D^{\dagger}_{np} + d_{np}D_{np}) + (\overline{b}_{np}C^{\dagger}_{np} - b_{np}C_{np})],$$
$$P^{+}_{\mu} = \sum_{n,p}[\frac{1}{\sqrt{2}}(b_{np}D^{\dagger}_{np} - \overline{b}_{np}D_{np}) + (d_{np}C^{\dagger}_{np} + \overline{d}_{np}C_{np})], \quad (5)$$

by using the definitions from [28] for the following quantities:

$$D_{np} \equiv \sum_{\rho=\pm1} \rho\alpha^{\dagger}_{n,-\rho}\alpha_{p,-\rho}, \quad D^{\dagger}_{np} \equiv \sum_{\rho=\pm1} \rho\alpha^{\dagger}_{p,-\rho}\alpha_{n,-\rho},$$
$$C_{np} \equiv \frac{1}{\sqrt{2}}\sum_{\rho=\pm1} \alpha_{p\rho}\alpha_{n,-\rho}, \quad C^{\dagger}_{np} \equiv \frac{1}{\sqrt{2}}\sum_{\rho=\pm1} \alpha^{\dagger}_{n,-\rho}\alpha^{\dagger}_{p\rho},$$
$$b_{np} \equiv \sqrt{2}\langle n|\sigma_{\mu}|p\rangle u_p v_n, \quad \overline{b}_{np} \equiv \sqrt{2}\langle n|\sigma_{\mu}|p\rangle u_n v_p,$$
$$d_{np} \equiv \sqrt{2}\langle n|\sigma_{\mu}|p\rangle u_p u_n, \quad \overline{d}_{np} \equiv \sqrt{2}\langle n|\sigma_{\mu}|p\rangle v_n v_p, \quad (6)$$

where $v_{\tau}(u_{\tau})$ is the occupation (unoccupation) amplitude which is obtained in the BCS calculations; $|n\rangle$ and $|p\rangle$ are Nilsson single particle states; D_{np} corresponds to quasi-particle scattering operator; $C^{\dagger}_{np}(C_{np})$ is a two quasi-particle creation(annihilation) operator for neutron- proton pair and it satisfies the following bosonic commutation rules in the quasi-boson approximation:

$$[C_{np}, C^{\dagger}_{n'p'}] \approx \delta_{nn'}\delta_{pp'}, \quad [C_{np}, C_{n'p'}] = 0. \quad (7)$$

Hence the effective Gamow-Teller (GT) interactions in the quasi-particle space can be written as follows:

$$V^{ph}_{GT} = V^{ph}_{CC} + V^{ph}_{DD} + V^{ph}_{CD}, \qquad V^{pp}_{GT} = V^{pp}_{CC} + V^{pp}_{DD} + V^{pp}_{CD}, \quad (8)$$

with

$$V^{ph}_{CC} = 2\chi^{ph}_{GT} \sum_{\substack{n_1,p_1\\n_2,p_2}} (\overline{b}_{n_1p_1}C^{\dagger}_{n_1p_1} - b_{n_1p_1}C_{n_1p_1})(\overline{b}_{n_2p_2}C_{n_2p_2} - b_{n_2p_2}C^{\dagger}_{n_2p_2}),$$

$$V^{pp}_{CC} = -2\chi^{pp}_{GT} \sum_{\substack{n_1,p_1\\n_2,p_2}} (d_{n_1p_1}C^{\dagger}_{n_1p_1} + \overline{d}_{n_1p_1}C_{n_1p_1})(d_{n_2p_2}C_{n_2p_2} + \overline{d}_{n_2p_2}C^{\dagger}_{n_2p_2}), \quad (9)$$

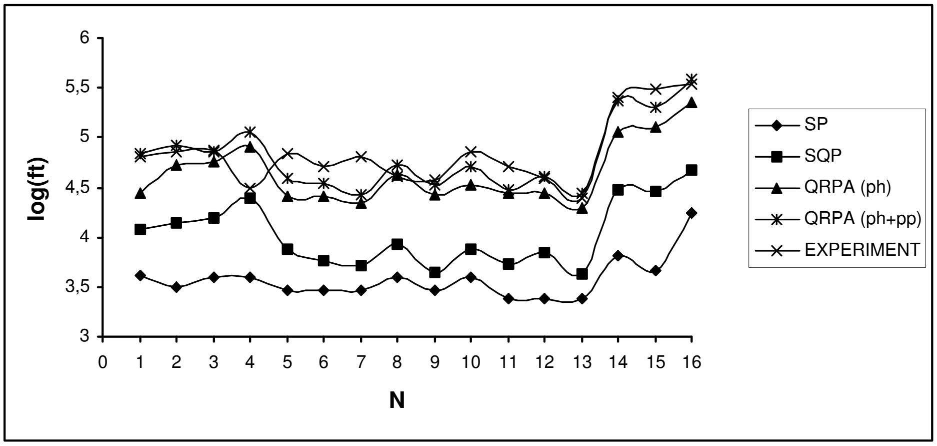

Fig. 1. The log(ft) values for the investigated nucleus in different models. The transitions are ordered in the same way in Table 1

$$V_{DD}^{ph} = \chi_{GT}^{ph} \sum_{\substack{n_1,p_1 \\ n_2,p_2}} (\overline{d}_{n_1p_1} D_{n_1p_1}^{\dagger} + d_{n_1p_1} D_{n_1p_1})(\overline{d}_{n_2p_2} D_{n_2p_2} + d_{n_2p_2} D_{n_2p_2}^{\dagger}),$$

$$V_{DD}^{pp} = -\chi_{GT}^{pp} \sum_{\substack{n_1,p_1 \\ n_2,p_2}} (b_{n_1p_1} D_{n_1p_1}^{\dagger} - \overline{b}_{n_1p_1} D_{n_1p_1})(b_{n_2p_2} D_{n_2p_2} - \overline{b}_{n_2p_2} D_{n_2p_2}^{\dagger}), \tag{10}$$

$$V_{CD}^{ph} = \sqrt{2}\chi_{GT}^{ph} \sum_{\substack{n_1,p_1 \\ n_2,p_2}} [(\overline{b}_{n_1p_1} C_{n_1p_1}^{\dagger} - b_{n_1p_1} C_{n_1p_1})(\overline{d}_{n_2p_2} D_{n_2p_2} + d_{n_2p_2} D_{n_2p_2}^{\dagger}) + hc],$$

$$V_{CD}^{pp} = -\sqrt{2}\chi_{GT}^{pp} \sum_{\substack{n_1,p_1 \\ n_2,p_2}} [(d_{n_1p_1} C_{n_1p_1}^{\dagger} + \overline{d}_{n_1p_1} C_{n_1p_1})(b_{n_2p_2} D_{n_2p_2} - \overline{b}_{n_2p_2} D_{n_2p_2}^{\dagger}) + hc]. \tag{11}$$

In these calculations, V_{DD}^{ph} and V_{DD}^{pp} are neglected because they only contribute to higher order terms. The V_{CD}^{ph} and V_{CD}^{pp}, which contain linear terms in the quasi-particle scattering operator (D_{np}), are only needed for the odd-A transitions between one quasi-particle states when the particle-phonon interaction is treated in the first order perturbation theory.

Let us first consider the Gamow-Teller interaction in even-even nuclei. In this case, Hamiltonian of the system can be written in the form of

$$H_0 = H_{SQP} + V_{CC}^{ph} + V_{CC}^{pp}. \tag{12}$$

In QRPA, the collective 1^+ states in odd-odd nuclei are considered as one-phonon excitations

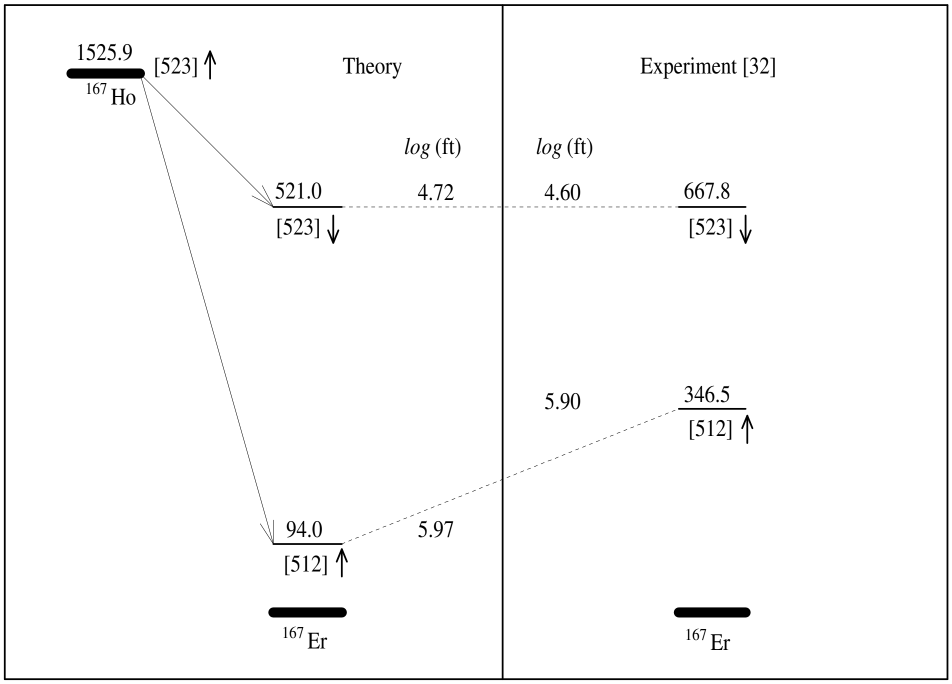

Fig. 2. Energy (in KeV) and log(ft) values for ^{167}Ho→^{167}Er β^- transitions

and described by

$$|\Psi_i\rangle = Q_i^\dagger|0\rangle = \sum_{n,p}(\psi_{np}^i C_{np}^\dagger - \varphi_{np}^i C_{np}), \tag{13}$$

where $Q_i^\dagger$ is the neutron-proton QRPA phonon creation operator, $|0\rangle$ is the phonon vacuum which corresponds to the ground state of an even-even nucleus and fulfills $Q_i|0\rangle = 0$ for all i. The two-quasi-particle amplitudes ψ_{np}^i and φ_{np}^i are normalized by:

$$\sum_{n,p}[(\psi_{np}^i)^2 - (\varphi_{np}^i)^2] = 1. \tag{14}$$

Employing the conventional procedure of QRPA and solving the equation of motion

$$[H_0, Q_i^\dagger]|0\rangle = \omega_i Q_i^\dagger|0\rangle, \tag{15}$$

we obtain the dispersion equation for the excitation energies ω_i of 1^+ states:

$$D(\omega_i) = \begin{vmatrix} 1+2\chi_{GT}^{ph}e & 2\chi_{GT}^{ph}s & -2\chi_{GT}^{pp}g & -2\chi_{GT}^{pp}h \\ 2\chi_{GT}^{ph}s & 1+2\chi_{GT}^{ph}\overline{e} & -2\chi_{GT}^{pp}\overline{h} & -2\chi_{GT}^{pp}\overline{g} \\ 2\chi_{GT}^{ph}g & 2\chi_{GT}^{ph}\overline{h} & 1-2\chi_{GT}^{pp}f & -2\chi_{GT}^{pp}s \\ 2\chi_{GT}^{ph}h & 2\chi_{GT}^{ph}\overline{g} & -2\chi_{GT}^{pp}s & 1-2\chi_{GT}^{pp}\overline{f} \end{vmatrix} = 0, \tag{16}$$

with

$$\overline{e}=\sum_{n,p}\left[\frac{\overline{b}_{np}^2}{E_{np}-\omega_i}+\frac{b_{np}^2}{E_{np}+\omega_i}\right],\quad e=\sum_{n,p}\left[\frac{b_{np}^2}{E_{np}-\omega_i}+\frac{\overline{b}_{np}^2}{E_{np}+\omega_i}\right],$$
$$\overline{f}=\sum_{n,p}\left[\frac{\overline{d}_{np}^2}{E_{np}-\omega_i}+\frac{d_{np}^2}{E_{np}+\omega_i}\right],\quad f=\sum_{n,p}\left[\frac{d_{np}^2}{E_{np}-\omega_i}+\frac{\overline{d}_{np}^2}{E_{np}+\omega_i}\right],$$
$$\overline{g}=\sum_{n,p}\left[\frac{\overline{b}_{np}\overline{d}_{np}}{E_{np}-\omega_i}-\frac{b_{np}d_{np}}{E_{np}+\omega_i}\right],\quad g=\sum_{n,p}\left[\frac{b_{np}d_{np}}{E_{np}-\omega_i}-\frac{\overline{b}_{np}\overline{d}_{np}}{E_{np}+\omega_i}\right],$$
$$\overline{h}=\sum_{n,p}\left[\frac{\overline{b}_{np}d_{np}}{E_{np}-\omega_i}-\frac{b_{np}\overline{d}_{np}}{E_{np}+\omega_i}\right],\quad h=\sum_{n,p}\left[\frac{b_{np}\overline{d}_{np}}{E_{np}-\omega_i}-\frac{\overline{b}_{np}d_{np}}{E_{np}+\omega_i}\right],$$
$$s=\sum_{n,p}b_{np}\overline{b}_{np}\left[\frac{1}{E_{np}-\omega_i}+\frac{1}{E_{np}+\omega_i}\right]$$
$$=\sum_{n,p}d_{np}\overline{d}_{np}\left[\frac{1}{E_{np}-\omega_i}+\frac{1}{E_{np}+\omega_i}\right]. \tag{17}$$

where $E_{np}=E_n+E_p$ is a two quasi-particle energy for neutron-proton pair. The two quasi-particle amplitudes then become

$$\psi_{np}^i=-\frac{\overline{b}_{np}+L_1(\omega_i)b_{np}-\frac{\chi_{GT}^{pp}}{\chi_{GT}^{ph}}(L_2(\omega_i)d_{np}+L_3(\omega_i)\,\overline{d}_{np})}{E_{np}-\omega_i}\frac{1}{\sqrt{Z(\omega_i)}},$$
$$\varphi_{np}^i=\frac{b_{np}+L_1(\omega_i)\overline{b}_{np}+\frac{\chi_{GT}^{pp}}{\chi_{GT}^{ph}}(L_2(\omega_i)\overline{d}_{np}+L_3(\omega_i)\,d_{np})}{E_{np}+\omega_i}\frac{1}{\sqrt{Z(\omega_i)}}. \tag{18}$$

Here $L_k(\omega_i)$ is defined as $L_k(\omega_i)=D_0/D_k$ (k=1,2,3) with

$$D_0=-\begin{vmatrix} 1+2\chi_{GT}^{ph}\overline{e} & -2\chi_{GT}^{pp}\overline{h} & -2\chi_{GT}^{pp}\overline{g} \\ 2\chi_{GT}^{ph}\overline{h} & 1-2\chi_{GT}^{pp}f & -2\chi_{GT}^{pp}s \\ 2\chi_{GT}^{ph}\overline{g} & -2\chi_{GT}^{pp}s & 1-2\chi_{GT}^{pp}\overline{f} \end{vmatrix},$$

$$D_1=2\chi_{GT}^{ph}\begin{vmatrix} s & -2\chi_{GT}^{pp}\overline{h} & -2\chi_{GT}^{pp}\overline{g} \\ g & 1-2\chi_{GT}^{pp}f & -2\chi_{GT}^{pp}s \\ h & -2\chi_{GT}^{pp}s & 1-2\chi_{GT}^{pp}\overline{f} \end{vmatrix},$$

$$D_2=2\chi_{GT}^{ph}\begin{vmatrix} 1+2\chi_{GT}^{ph}\overline{e} & s & -2\chi_{GT}^{pp}\overline{g} \\ 2\chi_{GT}^{ph}\overline{h} & g & -2\chi_{GT}^{pp}s \\ 2\chi_{GT}^{ph}\overline{g} & h & 1-2\chi_{GT}^{pp}\overline{f} \end{vmatrix},$$

$$D_3 = 2\chi_{GT}^{ph} \begin{vmatrix} 1+2\chi_{GT}^{ph}\overline{e} & -2\chi_{GT}^{pp}\overline{h} & s \\ 2\chi_{GT}^{ph}\overline{h} & 1-2\chi_{GT}^{pp}f & g \\ 2\chi_{GT}^{ph}\overline{g} & -2\chi_{GT}^{pp}s & h \end{vmatrix}, \tag{19}$$

and the normalization constant $Z(\omega_i)$ is determined from eq. (14).

One of the quantities, which characterize the excited Gamow-Teller 1^+ states, is the probability of $\beta^{\pm}$ transitions from these states to neighbor even-even nuclei. The $\beta^{\pm}$ transition amplitude from the even-even correlated ground state to the 1^+ excited state in the odd-odd daughter nucleus is given by:

$$\langle 1_i^+|\beta^-|0^+\rangle = -\sum_{n,p}(b_{np}\psi_{np}^i - \overline{b}_{np}\varphi_{np}^i) \equiv M_i^-,$$

$$\langle 1_i^+|\beta^+|0^+\rangle = \sum_{n,p}(\overline{b}_{np}\psi_{np}^i - b_{np}\varphi_{np}^i) \equiv M_i^+. \tag{20}$$

Let us now examine the collective Gamow-Teller interaction for odd mass nuclei. The system Hamiltonian for this case in terms of $Q_i^\dagger$ and Q_i is;

$$\begin{aligned} H &= H_0 + \sqrt{2}\chi_{GT}^{ph}\sum_{n,p,j}[(M_j^+Q_j^\dagger + M_j^-Q_j)(d_{np}D_{np}^\dagger + \overline{d}_{np}D_{np}) + hc] \\ &- \sqrt{2}\chi_{GT}^{pp}\sum_{n,p,j}[(F_j^+Q_j^\dagger + F_j^-Q_j)(b_{np}D_{np} + \overline{b}_{np}D_{np}^\dagger) + hc], \end{aligned} \tag{21}$$

where

$$\begin{aligned} F_j^+ &= \sum_{n,p}(d_{np}\psi_{np}^i + \overline{d}_{np}\varphi_{np}^i), \\ F_j^- &= \sum_{n,p}(\overline{d}_{np}\psi_{np}^i + d_{np}\varphi_{np}^i). \end{aligned}$$

In QRPA method, the wave function of the odd mass (with odd neutron) nuclei is given by

$$|\Psi_{I_nK_n}^j\rangle = \Omega_{I_nK_n}^{j\dagger}|0\rangle = [N_{I_n}^j\alpha_{I_nK_n}^\dagger + \sum_{i,I_p,K_p} R_{ij}^{I_nI_p}Q_i^\dagger\alpha_{I_pK_p}^\dagger]|0\rangle. \tag{22}$$

It is assumed that wave function (22) is formed by superposition of one and three quasiparticle (one quasiparticle + one phonon) states. The amplitudes corresponding to the states, $N_{I_n}^j$ and $R_{ij}^{I_nI_p}$, are fulfilled by the normalization condition

$$(N_{I_n}^j)^2 + \sum_{i,I_p}(R_{ij}^{I_nI_p})^2 = 1. \tag{23}$$

Solving the equation of motion

$$[H, \Omega^{j\dagger}_{I_n K_n}]|0\rangle = W^j_{I_n K_n} \Omega^{j\dagger}_{I_n K_n}|0\rangle, \tag{24}$$

the dispersion equation for excitation energies $W^j_{I_n K_n}$, corresponding to states given in eq. (22), is obtained as

$$W^j_{I_n K_n} - E_{I_n K_n} = 2 \sum_{i, I_p, K_p} \frac{\left[\chi^{ph}_{GT}(d_{I_n I_p} M^+_i + \overline{d}_{I_n I_p} M^-_i) - \chi^{pp}_{GT}(b_{I_n I_p} F^+_i - \overline{b}_{I_n I_p} F^-_i)\right]^2}{W^j_{I_n K_n} - \omega_i - E_{I_p K_p}}. \tag{25}$$

The amplitude for three quasi-particle state, $R^{I_n I_p}_{ij}$, is written in terms of the amplitude for one quasi-particle state, $N^j_{I_n}$, as follows:

$$R^{I_n I_p}_{ij} = \frac{\sqrt{2}\left[\chi^{ph}_{GT}(d_{I_n I_p} M^+_i + \overline{d}_{I_n I_p} M^-_i) - \chi^{pp}_{GT}(b_{I_n I_p} F^+_i - \overline{b}_{I_n I_p} F^-_i)\right]}{W^j_{I_n K_n} - \omega_i - E_{I_p K_p}} N^j_{I_n}, \tag{26}$$

where $N^j_{I_n}$ is calculated from eq. (23). The corresponding expressions for the nuclei with odd-proton number are formulated by performing the transformation $I_n K_n \leftrightarrow I_p K_p$ in eqs. (22)-26).

3 Gamow-Teller $\beta^{\pm}$ Transition Matrix Elements in Odd-A Nucleus

The Gamow-Teller $\beta^{\pm}$ transition matrix elements of nuclei can be given by the expression [14]:

$$M_{\beta^{\pm}} = \langle \Psi^f_{I_1 K_1} | \beta^{\pm}_{\mu} | \Psi^i_{I_2 K_2} \rangle. \tag{27}$$

The corresponding matrix elements of odd-A transitions are expressed for two different cases as follows:

a) The case in which the number of pair does not change:

$$\begin{aligned}
M_{\beta^-} &= \langle \Psi^f_{I_p K_p} | \beta^-_{\mu} | \Psi^i_{I_n K_n} \rangle \\
&= -[d_{I_n I_p} N^i_{I_n} N^f_{I_p} + \overline{d}_{I_n I_p} \sum_j R^{I_n I_p}_{ij} R^{I_n I_p}_{fj} \\
&\quad + N^i_{I_n} \sum_j R^{I_n I_p}_{fj} M^-_j + N^f_{I_p} \sum_j R^{I_n I_p}_{ij} M^+_j], \\
M_{\beta^+} &= \langle \Psi^f_{I_n K_n} | \beta^+_{\mu} | \Psi^i_{I_p K_p} \rangle \\
&= -[d_{I_n I_p} N^i_{I_p} N^f_{I_n} + \overline{d}_{I_n I_p} \sum_j R^{I_n I_p}_{ij} R^{I_n I_p}_{fj} \\
&\quad + N^i_{I_p} \sum_j R^{I_n I_p}_{fj} M^+_j + N^f_{I_n} \sum_j R^{I_n I_p}_{ij} M^-_j],
\end{aligned} \tag{28}$$

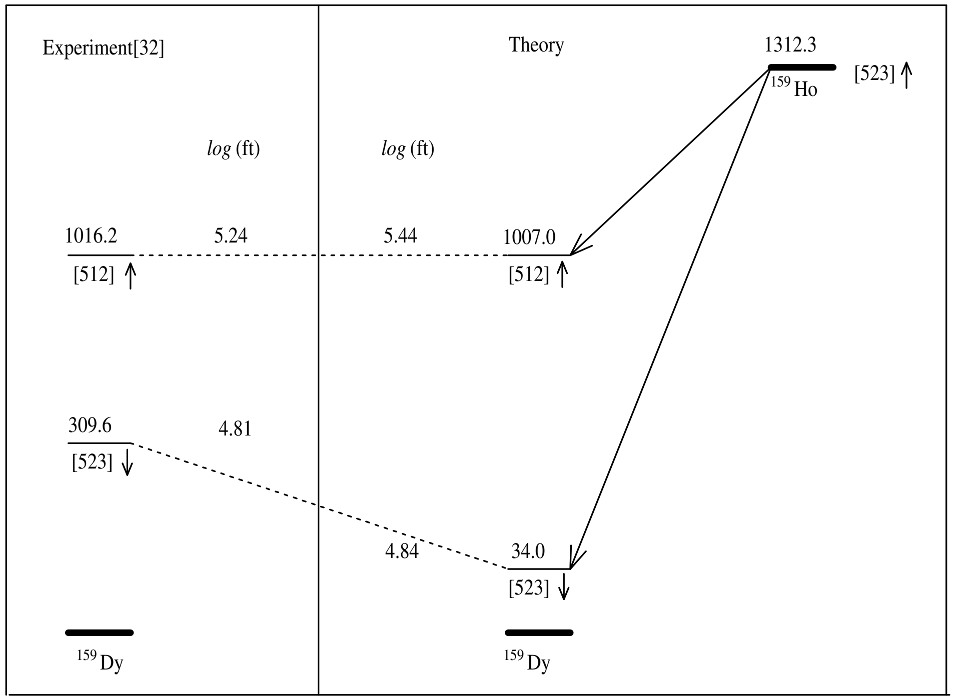

Fig. 3. Energy (in KeV) and log(ft) values for ^{159}Ho→^{159}Dy β^+ transitions

b) The case in which the number of pair changes:

$$
\begin{aligned}
M_{\beta^-} &= \langle \Psi^f_{I_n K_n} | \beta^-_\mu | \Psi^i_{I_p K_p} \rangle \\
&= -[\overline{d}_{I_n I_p} N^i_{I_p} N^f_{I_n} + d_{I_n I_p} \sum_j R^{I_n I_p}_{ij} R^{I_n I_p}_{fj} \\
&\quad + N^i_{I_p} \sum_j R^{I_n I_p}_{fj} M^-_j + N^f_{I_n} \sum_j R^{I_n I_p}_{ij} M^+_j], \\
M_{\beta^+} &= \langle \Psi^f_{I_p K_p} | \beta^+_\mu | \Psi^i_{I_n K_n} \rangle \\
&= -[\overline{d}_{I_n I_p} N^i_{I_n} N^f_{I_p} + d_{I_n I_p} \sum_j R^{I_n I_p}_{ij} R^{I_n I_p}_{fj} \\
&\quad + N^i_{I_n} \sum_j R^{I_n I_p}_{fj} M^+_j + N^f_{I_p} \sum_j R^{I_n I_p}_{ij} M^-_j],
\end{aligned} \tag{29}
$$

where $\mu = K_f - K_i$. The reduced transition probability for the $I_i K_i \to I_f K_f$ transitions on the laboratory frame is expressed by

$$
B^\pm_{GT}(I_i K_i \to I_f K_f) = \frac{g_A^2}{4\pi} (I_i K_i 1 K_f - K_i / I_f K_f)^2 |M_{\beta^\pm}|^2, \tag{30}
$$

and the ft values for these transitions are given by the following formula:

$$(ft)_{\beta^{\pm}} = D\frac{g_v^2}{4\pi B_{GT}^{\pm}} = \frac{D}{\left(\frac{g_A}{g_v}\right)^2 (I_i K_i 1 K_f - K_i / I_f K_f)^2 |M_{\beta^{\pm}}|^2}. \quad (31)$$

In our calculations we use the following constants appearing in eq. (31) of Ref. [29], $D \equiv \frac{2\pi^3\hbar^7 ln2}{g_v^2 m_e^5 c^4} = 6295s$ and $\frac{g_A}{g_v} = -1.254$ and we take $K_i = I_i$, $K_f = I_f$ for the deformed nuclei.

4 Results and Discussion

Numerical calculations have been performed for the deformed nuclei in the atomic mass regions of $125 \leq A \leq 131$ and $159 \leq A \leq 181$. Nilsson single particle energies and wave functions have been calculated with a deformed Woods-Saxon potential [30]. All energy levels from the bottom of the potential well to 8 MeV have been considered for neutrons and protons. The deformation parameters and pairing interaction constants have been chosen in accordance with Ref. [31]. The log(ft) values for the GT transition in 16 nuclei are obtained from the formula given in eq. (31). The calculations have been carried out for the transitions [523] $\uparrow\leftrightarrow$[523] $\downarrow$, [514] $\uparrow\leftrightarrow$[514] $\downarrow$ and [402] $\uparrow\leftrightarrow$[402] $\downarrow$ having lower energies. Since there are some transitions (for ^{159}Ho$\rightarrow$^{159}Dy and ^{167}Ho$\rightarrow$^{167}Er) to the excited states of the daughter nucleus, the corresponding energies (with respect to the ground states) have been calculated.

The calculated log(ft) values and final energies for the investigated nucleus within the different models are given in the Table 1. The transitions in the first 10 rows of the Table 1 correspond to [523] $\uparrow\leftrightarrow$[523] $\downarrow$ transitions. The rows from 10 to 14 show [514] $\uparrow\leftrightarrow$[514] $\downarrow$ transitions, and [402] $\uparrow\leftrightarrow$[402] $\downarrow$ transitions have been presented in the rows 14-16. In the 3rd and 4th columns of the Table 1, there has been given the comparison of the calculated energy values for the transitions to the excited states of the daughter nucleus together with the experimental ones. The theoretical results of the log(ft) values for the GT transitions in SQP, QRPA(without pp interaction) and QRPA(with pp interaction) as well as the experimental log(ft) values have been presented in the last 4 columns of the Table 1.

In Fig. 1, the log(ft) values for the investigated nuclei in different models have been compared with the experimental values. The results show that the single-particle (SP) values of the GT beta transition rate are 15-20 times larger than the corresponding experimental ones in the basis of Woods-Saxon potential as it is in the case of Nilsson potential. This number reduces up to 8-10 times when the pairing interactions between like nucleons are taken into account. However, if the effective Gamow-Teller interaction is considered and the appropriate value for the interaction constants χ_{GT}^{ph} and χ_{GT}^{pp} is chosen, it is possible to make the transition rate values closer to the experimental ones. In the calculations, the following values for these constants have been used: $\chi_{GT}^{ph} = 5.2/A^{0.7}MeV$ and $\chi_{GT}^{pp} = 0.58/A^{0.7}MeV$ [27]. As seen from Table 1, in general, the values of beta transition log(ft) increase and thus become closer to the experimental value when the particle-particle channel term is taking into account for the charge exchange spin-spin forces. Calculations show that, although the contributions of the three quasiparticle states to the one quasiparticle wave function are small as a result of the GT interactions (the norm of wave function is less than 1%), the contribution of the core which comes from the polarization

N	Transitions	$E_f(KeV)$		$log(ft)$			
		Theory	Exp.[32]	SQP	QRPA (ph)	QRPA (ph+pp)	Exp.[25,32]
1	^{159}Ho→^{159}Dy	34.0 1007	309.6 1016.2	4.07 4.69	4.44 5.07	4.84 5.44	4.81 5.24
2	^{161}Gd→^{161}Tb	263.9	417.2	4.14	4.73	4.92	4.86 4.86 ± 0.04
3	^{161}Ho→^{161}Dy	0	25.7	4.20	4.76	4.86	4.88 4.80 ± 0.20
4	^{163}Ho→^{163}Dy	33.2	0	4.40	4.91	5.05	4.50
5	^{163}Er→^{163}Ho	0	0	3.88	4.41	4.59	4.84 4.83 ± 0.01
6	^{165}Er→^{165}Ho	0	0	3.77	4.41	4.54	4.7 4.64 ± 0.02
7	^{165}Yb→^{165}Tm	222.0	160.5	3.72	4.35	4.42	4.80 4.80 ± 0.10
8	^{167}Ho→^{167}Er	94.0 521	346.5 667.8	5.31 3.92	5.90 4.62	5.97 4.72	5.90 4.80 ± 0.20
9	^{167}Yb→^{167}Tm	264.7	292.8	3.64	4.42	4.53	4.58 4.55 ± 0.05
10	^{169}Ho→^{169}Er	943.8	853.0	3.88	4.53	4.71	4.86
11	^{175}Yb→^{175}Lu	194.6	396	3.73	4.44	4.47	4.70
12	^{179}W→^{179}Ta	0	30.7	3.85	4.44	4.60	4.59
13	^{181}Os→^{181}Re	38.1	262	3.62	4.29	4.44	4.40
14	^{125}I→^{125}Te	72.7	35.5	4.48	5.05	5.37	5.40
15	^{127}Te→^{127}I	0	0	4.46	5.10	5.30	5.48
16	^{131}Cs→^{131}Xe	24.0	0	4.67	5.35	5.58	5.54

Tab. 1. The log(ft) values and final energies for the investigated nucleus within the different models. N is the ordered number of transitions

phenomena is significant. The sums of these contributions make the beta transition rate 7-8 times smaller, and thus an agreement between its theoretical and experimental values is being achieved.

The δ_2 deformation parameters of the investigated nuclei have been calculated by using the ε_2 and β_2 deformation parameters, which have been taken from Möller et al. [31]. The log(ft) values, calculated by $\delta_2 = 0.129$ value found for $^{131}_{55}Cs$ nucleus, represent 5.82 and 6.02 for particle-hole and particle-particle plus particle-hole channels, respectively. These results are higher than the experimental value (5.54). However, good agreement with the experimental value has been obtained when the following deformation parameter values of $^{131}_{55}Cs$ nucleus are used: $\delta_2^n = 0.129$ [31] and $\delta_2^p = -0.005$ (see Table 1).

As an example to β^- and β^+ Gamow-Teller transitions, the schematic form of the ^{167}Ho→^{167}Er and ^{159}Ho→^{159}Dy transitions have been presented in Fig. 2 and Fig. 3, respectively. As seen from these figures, although the energies of low-lying states in both transitions are smaller than the corresponding experimental values, the log(ft) values are in good agreement with the experimental ones. In higher states, this agreement is valid for both the energy and log(ft) values.

In summary, we have investigated the $\beta^{\pm}$ decay between ground state of odd-A nuclei in the atomic mass regions of $125 \leq A \leq 131$ and $159 \leq A \leq 181$ through proton-neutron QRPA by taking into account the residual spin-isospin interaction between the nucleons in the particle hole and particle-particle channels, and calculated the log(ft) values of decay. The results of our calculations show that the parameters with the values of $\chi_{GT}^{ph} = 5.2/A^{0.7} MeV$ and $\chi_{GT}^{pp} = 0.58/A^{0.7} MeV$, taken from [27], reproduce observed beta decay log(ft) values of the investigated even mass deformed nuclei.

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References

[1] J. D. Anderson and C. Wong: *Phys. Rev. Lett.* **7** (1961) 250
[2] K. Ikeda, S. Fujii and J. I. Fujita: *Phys. Lett.* **2** (1962) 169
[3] K. Ikeda, S. Fujii and J. I. Fujita: *Phys. Lett.* **3** (1963) 271
[4] J. I. Fujita, S. Fujii and K. Ikeda: *Phys. Rev.* **133** (1964) 549
[5] K. Ikeda: *Prog. Theor. Phys.* **31** (1964) 434
[6] J. I. Fujita and K. Ikeda: *Nucl. Phys.* **67** (1965) 145
[7] R. R. Doering, A. Galonsky, D. M. Petterson and G. F. Bertsch: *Phys. Rev. Lett.* **35** (1975) 1961
[8] K. Takahashi and M. Yamada: *Prog. Theor. Phys.* **41** (1969) 1470
[9] I. Hamamoto: *Nucl. Phys.* **62** (1965) 49
[10] J. A. Halbleib and R. A. Sorensen: *Nucl. Phys.* **A 98** (1967) 542
[11] S. I. Gabrakov, A. A. Kuliev: *JINR* **P4-5003** (1970) 1
[12] Yu. V. Gaponov, Yu. S. Lyutostanskii: *Sov. J. Nucl. Phys.* **16** (1973) 270 ;
Yu. V. Gaponov, Yu. S. Lyutostanskii: *Sov. J. Nucl. Phys.* **19** (1974) 332
[13] J. Randrup: *Nucl. Phys.* **A 207** (1973) 209
[14] S. P. Ivanova, A. A. Kuliev, D. I. Salamov: *Sov. J. Nucl. Phys.* **24** (1976) 278 ;
S. P. Ivanova, A. A. Kuliev, D. I. Salamov: *Sov. J. Izv. ANSSSR Ser. Phys.* **41** (1977) 131
[15] H. V. Klapdor, T. Oda, J. Metzinger, W. Hillebrandt and F. K. Thielemann: *Z. Phys.* **A 299** (1981) 213
[16] E. Bender, K. Muto and H. V. Klapdor: *Phys. Lett.* **B 208** (1988) 53
[17] K. Muto, E. Bender and H. V. Klapdor: *Z. Phys.* **A 333** (1989) 125
[18] P. Möller, and J. Randrup: *Nucl. Phys.* **A 514** (1990) 1
[19] K. Muto, E. Bender, T. Oda, and H. V. Klapdor-Kleingrothaus: *Z. Phys.* **A 341** (1992) 407
[20] Z. Bochnacki and S. Ogaza: *Nucl. Phys.* **A 102** (1967) 529
[21] P. Sarriguren, E. Moya de Guerra and A. Escuderos: *Phys. Rev.* **C 64** (2001) 064306-1
[22] R. J. Blin-Stoyle, M. A. Perks: *Proceed Phys. Society* **A 67** (1954) 885
[23] A. Arima, H. Hovie: *Prog. Theor. Phys.* **11** (1954) 509 ;
A. Arima, H. Hovie: *Prog. Theor. Phys.* **12** (1954) 623
[24] A. A. Kuliev, N. I. Pyatov: *Sov. J. Nucl. Phys.* **9** (1969) 313
[25] A. Bohr and B. R. Mottelson: *Nuclear Structure*, Vol.II, Benjamin, New-York 1969

[26] C. Gaarde, J. S. Larsen, C. D. Goodman, C. C. Foster, C. A. Goulding, D. J. Horen, T. Masterson, J. Rapaport, T. N. Taddeucci, and E. Sugarbaker: *Proceedings of 4th International Conference on Nuclei Far from Stability, Helsingor,* **CERN 81-09** (1981) 281

[27] H. Homma, E. Bender, M. Hirsch, K. Muto, H. V. Klapdor-Kleingrothaus, and T. Oda: *Phys. Rev.* **C 54** (1996) 2972

[28] C. Selam, T. Babacan, H. A. Aygör, H. Bircan, A. Küçükbursa, I. Maraş: *Mathematical and Computational Applications* (2003) to be published ;

H. A. Aygör, A. A. Kuliev, A. Küçükbursa, A. Yörük: *Proceeding II. Kızılırmak International Conference, Kırıkkale-Turkey* (20-22 May 1998) 460

[29] Particle Data Group, G. P. Yost et al.: *Phys. Lett.* **B 204** (1988) 1

[30] M. Cerkaski, J. Dudek, Z. Szymanski, C. G. Andersson, G. Leander, S. Aberg, S. G. Nilsson and I. Ragnarsson: *Phys. Lett.* **B 70** (1977) 9 ;

J. Dudek, W. Nazarewicz, A. Faessler: *Nucl. Phys.* **A 412** (1984) 61

[31] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki: *Atomic Data and Nuclear Data Tables* **59** (1995) 185 ;

P. Möller, J. R. Nix, and K. L. Kratz: *Atomic Data and Nuclear Data Tables* **66** (1997) 131

[32] B. Sing, J. L. Rodriguez, S. S. M. Wong, J. K. Tuli: *Nuclear Data Sheets* **84** (1998) 487