BOUNDING THE ν_τ MAGNETIC MOMENT FROM THE PROCESS $e^+e^- \to \nu\bar\nu\gamma$ IN A LEFT-RIGHT SYMMETRIC MODEL

A. Gutiérrez-Rodríguez¹, M. A. Hernández-Ruíz² ¹Facultad de Física, Universidad Autónoma de Zacatecas Apartado Postal C-580, 98068 Zacatecas, Zac. México ²Facultad de Ciencias Químicas, Universidad Autónoma de Zacatecas Apartado Postal 585, 98068 Zacatecas, Zac. México.

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A bound on the ν_{τ} magnetic moment is calculated through the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at the Z_1 -pole, and in the framework of a left-right symmetric model at LEP energies. We find that the bound is almost independent of the mixing angle ϕ of the model in the allowed experimental range for this parameter.

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1 Introduction

Neutrinos seem to be likely candidates for carrying features of physics beyond the Standard Model (SM) [1]. Apart from masses and mixings, magnetic moments and electric dipole moments are also signs of new physics and are of relevance in terrestrial experiments, in the solar neutrino problem, in astrophysics and in cosmology [2].

At the present time, all of the available experimental data for electroweak processes can be understood in the context of the Standard Model of the electroweak interactions (SM) [1], with the exception of the results of the SUPER-KAMIOKANDE experiment on the neutrinooscillations [3]. However, this is not the only experiment in disagreement with the SM. The same is true for the GALLEX, SAGE, GNO, HOMESTAKE and LSND [4] experiments. Nonetheless, the SM is still the starting point for all the extended gauge models. In other words, any gauge group with physical characteristics must have as a subgroup the $SU(2)_L \times U(1)$ group of the standard model in such a way that their predictions agree with those of the SM at low energies. The purpose of the extended theories is to explain some fundamental aspects which are not clarified in the frame of the SM. One of these aspects is the origin of the parity violation at current energies. The Left-Right Symmetric Models (LRSM), based on the $SU(2)_R \times SU(2)_L \times U(1)$ gauge group [5], give an answer to this problem by restoring the parity symmetry at high energies and giving their violations at low energies as a result of the breaking of gauge symmetry. Detailed discussions on LRSM can be found in the literature [6–8].

In 1994, T. M. Gould and I. Z. Rothstein [9] reported a bound on the tau neutrino magnetic moment which they obtained through the analysis of the process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$, near the Z_1 -resonance, by considering a massive tau neutrino and using Standard Model $Z_1e^+e^-$ and $Z_1\nu\bar{\nu}$ couplings.

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Fig. 1. The Feynman diagrams contributing to the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, in a left-right symmetric model.

At low center of mass energy $s \ll M_{Z_1}^2$, the dominant contribution to the process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ involves the exchange of a virtual photon [10]. The dependence on the magnetic moment comes from a direct coupling to the virtual photon, and the observed photon is a result of initial state Bremsstrahlung.

At higher s, near the Z_1 pole $s \approx M_{Z_1}^2$, the dominant contribution for $E_{\gamma} > 10 \text{ GeV}$ [11] involves the exchange of a Z_1 boson. The dependence on the magnetic moment now comes from the radiation of the photon observed by the neutrino or antineutrino in the final state. The Feynman diagrams which give the most important contribution to the cross section are shown in Fig. 1. We emphasize here the importance of the final state radiation near the Z_1 pole, which occurs preferentially at high E_{γ} compared to conventional Bremsstrahlung.

Our aim in this paper is to analyze the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ using recent data from the CERN e^+e^- Collider LEP [11–14] near the Z_1 boson resonance in the framework of a left-right symmetric model and attributing a magnetic moment to a massive tau neutrino. Processes measured near the resonance serve to set limits on the tau neutrino magnetic moment. In this paper, we take advantage of this fact to set bounds for $\kappa(\nu_{\tau})$ for different values of the mixing



Fig. 2. The total cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as a function of ϕ and κ (Tables 2, 3).

angle ϕ [17–19], which is in agreement with other constraints previously reported [9, 10, 14, 16].

We will do our analysis near the resonance of the Z_1 ($s \approx M_{Z_1}^2$). As a consequence, our results are independent of the mass of the additional heavy Z_2 gauge boson which appears in these kind of models. Therefore, we have the mixing angle ϕ between the left and the right bosons as the only additional parameter besides the SM parameters.

The L3 Collaboration evaluated the selection efficiency using detector-simulated $e^+e^- \rightarrow \nu \bar{\nu} \gamma(\gamma)$ events, random trigger events, and large-angle $e^+e^- \rightarrow e^+e^-$ events. A total of 14 events were found by the selection. The distributions of the photon energy and the cosine of its polar angle are consistent with SM predictions. The total number of events expected from the SM is 14.1 (see Table 1). If the photon energy is greater than half the beam energy, 2 events are selected from the data and 2.4 events are expected from the SM in the $\nu \bar{\nu} \gamma$ channel.

This paper is organized as follows: In Sect. II we describe the model with the Higgs sector having two doublets and one bidoublet. In Sect. III we present the calculus of the process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$. In Sect. IV we make the numerical computations. Finally, we summarize our results in Sect. V.

2 The Left-Right Symmetric Model (LRSM)

We consider a Left-Right Symmetric Model (LRSM) with one bidoublet Φ and two doublets χ_L , χ_R of which the vacuum expectation values break the gauge symmetry to give mass to the left and right heavy gauge bosons. This is the origin of the parity violation at low energies [6], that is, at energies available at actual accelerators. The Lagrangian for the Higgs sector of the LRSM

is [7]

$$\mathcal{L}_{LRSM} = (D_{\mu}\chi_L)^{\dagger} (D^{\mu}\chi_L) + (D_{\mu}\chi_R)^{\dagger} (D^{\mu}\chi_R) + Tr(D_{\mu}\Phi)^{\dagger} (D^{\mu}\Phi).$$
(1)

The covariant derivatives are written as

$$D_{\mu}\chi_{L} = \partial_{\mu}\chi_{L} - \frac{1}{2}ig\tau \cdot \mathbf{W}_{L}\chi_{L} - \frac{1}{2}ig'B\chi_{L},$$

$$D_{\mu}\chi_{R} = \partial_{\mu}\chi_{R} - \frac{1}{2}ig\tau \cdot \mathbf{W}_{R}\chi_{R} - \frac{1}{2}ig'B\chi_{R},$$

$$D_{\mu}\Phi = \partial_{\mu}\Phi - \frac{1}{2}ig(\tau \cdot \mathbf{W}_{L}\Phi - \Phi\tau \cdot \mathbf{W}_{R}).$$
(2)

In this model there are seven gauge bosons: the charged $W_{L,R}^1$, $W_{L,R}^2$ and the neutral $W_{L,R}^3$, B. The gauge couplings constants g_L and g_R of the $SU(2)_L$ and $SU(2)_R$ subgroups respectively, are equal: $g_L = g_R = g$, since manifest left-right symmetry is assumed [20]. g' is the gauge coupling for the U(1) group.

The transformation properties of the Higgs bosons under the group $SU(2)_L \times SU(2)_R \times U(1)$ are $\chi_L \sim (1/2, 0, 1), \chi_R \sim (0, 1/2, 1)$ and $\Phi \sim (1/2, 1/2^*, 0)$. After spontaneous symmetry breaking, the ground states are of the form

$$\langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}, \tag{3}$$

breaking the symmetry group to the $U(1)_{em}$ and giving mass to the gauge bosons and fermions, with the photon remaining massless. In Eq. (3), v_L , v_R , k and k' are the vacuum expectation values. The part of the Lagrangian that contains the mass terms for the charged boson is

$$\mathcal{L}_{mass}^{C} = (W_{L}^{+} \ W_{R}^{+}) M^{C} \left(\begin{array}{c} W_{L}^{-} \\ W_{R}^{-} \end{array}\right), \tag{4}$$

where $W^{\pm} = \frac{1}{\sqrt{2}} (W^1 \mp W^2)$.

The mass matrix M^C is

$$M^{C} = \frac{g^{2}}{4} \begin{pmatrix} v_{L}^{2} + k^{2} + k^{'2} & -2kk^{'} \\ -2kk^{'} & v_{R}^{2} + k^{2} + k^{'2} \end{pmatrix}.$$
(5)

This matrix is diagonalized by an orthogonal transformation which is parametrized [20] by an angle ζ . This angle has been restricted to have a very small value from the hyperon β decay data [21].

Similarly, the part of the Lagrangian that contains the mass terms for the neutral bosons is

$$\mathcal{L}_{mass}^{N} = \frac{1}{8} (W_L^3 \ W_R^3 \ B) M^N \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}, \tag{6}$$

where the matrix M^N is given by

$$M^{N} = \frac{1}{4} \begin{pmatrix} g^{2}(v_{L}^{2} + k^{2} + k^{'2}) & -g^{2}(k^{2} + k^{'2}) & -gg^{'}v_{L}^{2} \\ -g^{2}(k^{2} + k^{'2}) & g^{2}(v_{R}^{2} + k^{2} + k^{'2}) & -gg^{'}v_{R}^{2} \\ -gg^{'}v_{L}^{2} & -gg^{'}v_{R}^{2} & g^{'2}(v_{L}^{2} + v_{R}^{2}) \end{pmatrix}.$$
 (7)

Since the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is neutral, we center our attention on the mass terms of the Lagrangian for the neutral sector, Eq. (6).

The matrix M^N for the neutral gauge bosons is diagonalized by an orthogonal transformation which can be written in terms of the angles θ_W and ϕ [22]

$$U^{N} = \begin{pmatrix} c_{W}c_{\phi} & -s_{W}t_{W}c_{\phi} - r_{W}s_{\phi}/c_{W} & t_{W}(s_{\phi} - r_{W}c_{\phi}) \\ c_{W}s_{\phi} & -s_{W}t_{W}s_{\phi} + r_{W}c_{\phi}/c_{W} & -t_{W}(c_{\phi} + r_{W}s_{\phi}) \\ s_{W} & s_{W} & r_{W} \end{pmatrix},$$
(8)

where $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $t_W = \tan \theta_W$ and $r_W = \sqrt{\cos 2\theta_W}$, with θ_W being the electroweak mixing angle. Here, $c_{\phi} = \cos \phi$ and $s_{\phi} = \sin \phi$. The angle ϕ is considered as the angle that mixes the left and right handed neutral gauge bosons $W_{L,R}^3$. The expression that relates the left and right neutral gauge bosons $W_{L,R}^3$ and B with the physical bosons Z_1 , Z_2 and the photon is:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix} = U^N \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}.$$
(9)

The diagonalization of (5) and (7) gives the mass of the charged $W_{1,2}^{\pm}$ and neutral $Z_{1,2}$ physical fields:

$$M_{W_{1,2}}^2 = \frac{g^2}{8} [v_L^2 + v_R^2 + 2(k^2 + k^{\prime 2}) \mp \sqrt{(v_R^2 - v_L^2)^2 + 16(kk^{\prime})^2}], \tag{10}$$

$$M_{Z_1,Z_2}^2 = B \mp \sqrt{B^2 - 4C},\tag{11}$$

respectively, with

$$B = \frac{1}{8} [(g^2 + g^{'2})(v_L^2 + v_R^2) + 2g^2(k^2 + k^{'2})],$$

$$C = \frac{1}{64}g^{2}(g^{2} + 2g^{'2})[v_{L}^{2}v_{R}^{2} + (k^{2} + k^{'2})(v_{L}^{2} + v_{R}^{2})]$$

Taking into account that $M_{W_2}^2 \gg M_{W_1}^2$, from the expressions for the masses of M_{Z_1} and M_{Z_2} , we conclude that the relation $M_{W_1}^2 = M_{Z_1}^2 \cos^2 \theta_W$ still holds in this model. From the Lagrangian of the LRSM, we extract the terms for the neutral interaction of a

fermion with the gauge bosons $W_{L,R}^3$ and B:

$$\mathcal{L}_{int}^{N} = g(J_{L}^{3}W_{L}^{3} + J_{R}^{3}W_{R}^{3}) + \frac{g'}{2}J_{Y}B.$$
(12)

Explicitly, the interaction Lagrangian for $Z_1 \rightarrow f\bar{f}$ [23] is

$$\mathcal{L}_{int}^{N} = \frac{g}{c_{W}} Z_{1}[(c_{\phi} - \frac{s_{W}^{2}}{r_{W}} s_{\phi})J_{L} - \frac{c_{W}^{2}}{r_{W}} s_{\phi}J_{R}],$$
(13)

where the left (right) current for the fermions are

$$J_{L,R} = J_{L,R}^3 - \sin^2 \theta_W J_{em},$$

and

$$J_{em} = J_L^3 + J_R^3 + \frac{1}{2}J_Y,$$

is the electromagnetic current. From (13) we find the amplitude \mathcal{M} for the decay of the Z_1 boson with polarization ϵ^{λ} into an fermion-antifermion par:

$$\mathcal{M} = \frac{g}{c_W} [\bar{u}\gamma^\mu \frac{1}{2} (ag_V - bg_A\gamma_5)v]\epsilon^\lambda_\mu,\tag{14}$$

with

$$a = c_{\phi} - \frac{s_{\phi}}{r_W},$$

$$b = c_{\phi} + r_W s_{\phi}.$$
(15)

In the following section, we make the calculations for the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ by using the expression (14) for the transition amplitude.

3 The Total Cross Section

We calculate the total cross section of the process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ using the Breit-Wigner resonance form [24, 25]

$$\sigma(e^+e^- \to \nu\bar{\nu}\gamma) = \frac{4\pi(2J+1)\Gamma_{e^+e^-}\Gamma_{\nu\bar{\nu}\gamma}}{(s-M_{Z_1}^2)^2 + M_{Z_1}^2\Gamma_{Z_1}^2},$$
(16)

where $\Gamma_{e^+e^-}$ is the decay rate of Z_1 to the channel $Z_1 \to e^+e^-$ and $\Gamma_{\nu\bar{\nu}\gamma}$ is the decay rate of Z_1 to the channel $Z_1 \to \nu\bar{\nu}\gamma$.

In the next subsection we calculate the widths of Eq. (16).

3.1 Width of
$$Z_1 \rightarrow e^+e^-$$

In this section we calculate the total width of the reaction

$$Z_1 \to e^+ e^-, \tag{17}$$

in the context of the left-right symmetric model which is described in Section II.

The expression for the amplitude \mathcal{M} of the process $Z_1 \rightarrow e^+e^-$, due only to the Z_1 exchange, according to the diagrams depicted in Fig. 1, and using the expression for the amplitude given in Eq. (14), is:

$$\mathcal{M} = \frac{g}{\cos\theta_W} [\bar{u}(e^-)\gamma^\mu \frac{1}{2}(a-b\gamma_5)v(e^+)]\epsilon^\lambda_\mu,\tag{18}$$

where the coupling constants a and b are given in Eq. (15).

After making all the necessary calculations, we obtain the total width of the reaction $Z_1 \rightarrow e^+e^-$

$$\Gamma_{(Z_1 \to e^+e^-)} = \frac{G_F M_{Z_1}^3}{6\pi\sqrt{2}} \sqrt{1 - 4\eta} [(1 + 2\eta)a^2 (g_V^e)^2 + (1 - 4\eta)b^2 (g_A^e)^2], \tag{19}$$

where $\eta = m_e^2/M_{Z_1}^2 = 0$ in our case, with $m_e = 0$.

We take $g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$ and $g_A^e = -\frac{1}{2}$, according to the experimental data [24], so that the total width is

$$\Gamma_{(Z_1 \to e^+e^-)} = \frac{\pi \alpha}{2M_{Z_1}^2 x_W(1 - x_W)} \frac{M_{Z_1}^3}{12\pi\sqrt{2}} \left[\frac{1}{2}(a^2 + b^2) - 4a^2 x_W + 8a^2 x_W^2\right],\tag{20}$$

where $x_W = \sin^2 \theta_W$ and $\alpha = e^2/4\pi$ is the fine structure constant.

3.2 Width of $Z_1 \rightarrow \nu \bar{\nu} \gamma$

The expression for the amplitude \mathcal{M} of the process $Z_1 \rightarrow \nu \bar{\nu} \gamma$, due only to the Z_1 exchange, according to the diagrams depicted in Fig. 1, using the expression for the amplitude given in Eq. (14) and assuming that a massive Dirac neutrino is characterized by two phenomenological parameters, *i.e.* a magnetic moment $\mu_{\nu_{\tau}} = \kappa \mu_B$ (expressed in units of the Bohr magneton μ_B) and a charge radius $\langle r^2 \rangle$, is given by

$$\mathcal{M}_{a} = [\bar{u}(p_{\nu})\Gamma^{\alpha}\frac{i}{(\ell-m_{\nu})}(-\frac{ig}{4\cos\theta_{W}}\gamma^{\beta}(a-b\gamma_{5}))v(p_{\bar{\nu}})]\epsilon^{\lambda}_{\alpha}(\gamma)\epsilon^{\lambda}_{\beta}(Z_{1}), \qquad (21)$$

and

$$\mathcal{M}_{b} = [\bar{u}(p_{\nu})(-\frac{ig}{4\cos\theta_{W}}\gamma^{\beta}(a-b\gamma_{5}))\frac{i}{(\not{k}-m_{\nu})}\Gamma^{\alpha}v(p_{\bar{\nu}})]\epsilon^{\lambda}_{\alpha}(\gamma)\epsilon^{\lambda}_{\beta}(Z_{1}), \qquad (22)$$

where

$$\Gamma^{\alpha} = eF_1(q^2)\gamma^{\alpha} + \frac{ie}{2m_{\nu}}F_2(q^2)\sigma^{\alpha\mu}q_{\mu}, \qquad (23)$$

is the neutrino electromagnetic vertex, e is the charge of the electron, q is the momentum transfer and $F_{1,2}(q^2)$ are the electromagnetic form factors of the neutrino. Explicitly [26],

$$\begin{split} F_1(q^2) &= \frac{1}{6} q^2 \langle r^2 \rangle, \\ F_2(q^2) &= -\mu_\nu \frac{m_\nu}{m_e}, \end{split}$$

as already mentioned, $\langle r^2 \rangle$ is the neutrino mean-square charge radius and the magnetic moment κ is $\kappa \mu_B = eF_2(0)/2m_\nu$. While the coupling constants are given by

$$a = \cos \phi - \frac{\sin \phi}{\sqrt{\cos 2\theta_W}},$$

$$b = \cos \phi + \sqrt{\cos 2\theta_W} \sin \phi$$

where ϕ is the mixing parameter of the LRSM [17,18] and $\epsilon_{\alpha}^{\lambda}$, $\epsilon_{\beta}^{\lambda}$ are the vector of polarization of photon and of the boson Z_1 , respectively. l(k) stands by the momentum of the virtual neutrino (antineutrino).

After applying some of the theorems of traces of the Dirac matrices and of sum and average over the initial and final spin, the square of the matrix elements becomes

$$\sum_{s} |\mathcal{M}_{T}|^{2} = \frac{g^{2}}{4\cos^{2}\theta_{W}} \kappa^{2} \mu_{B}^{2} [(a^{2} + b^{2})(s - 2\sqrt{s}E_{\gamma}) + a^{2}E_{\gamma}^{2}\sin^{2}\theta_{\gamma}].$$
(24)

Our following step, now that we know the square of the Eq. (24) transition amplitude, is to calculate the total width of $Z_1 \rightarrow \nu \bar{\nu} \gamma$.

The expression for the differential rate of decay for a particle that decays in three is expressed by

$$d\Gamma = \frac{1}{2M_Z} \sum_{s} |\mathcal{M}_T|^2 \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} \frac{d^3 \mathbf{p}_3}{2E_3(2\pi)^3} (2\pi)^4 \delta^4(p - p_1 - p_2 - p_3).$$
(25)

Applying this expression to our process we obtain:

$$\Gamma_{(Z_1 \to \nu \bar{\nu} \gamma)} = \int \frac{\alpha \kappa^2 \mu_B^2}{x_W (1 - x_W) (2\pi)^2 M_{Z_1}} [(a^2 + b^2)(s - 2\sqrt{s}E_\gamma) + a^2 E_\gamma^2 \sin^2 \theta_\gamma] E_\gamma dE_\gamma dE_\gamma d\cos \theta_\gamma,$$
(26)

where $\mu_B = e\hbar/2m_e$ is the Bohr magneton and E_{γ} , $\cos\theta_{\gamma}$ are the energy and scattering angle of the photon.

The substitution of (20) and (26) in (16) gives

$$\sigma(e^+e^- \to \nu\bar{\nu}\gamma) = \int \frac{\alpha^2 \kappa^2 \mu_B^2}{192\pi} [\frac{a^2/2 + b^2/2 - 4a^2 x_W + 8a^2 x_W^2}{x_W^2 (1 - x_W)^2}] \\ [\frac{(a^2 + b^2)(s - 2\sqrt{s}E_{\gamma}) + a^2 E_{\gamma}^2 \sin^2 \theta_{\gamma}}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2}] E_{\gamma} dE_{\gamma} d\cos \theta_{\gamma}.$$
(27)

Using the same notation as in Ref. [9], we find that the magnetic moment coupling as well as the mixing angle ϕ , which is a parameter of the LRSM contribute to the cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ in the form

$$\sigma(e^+e^- \to \nu\bar{\nu}\gamma) = \int \frac{\alpha^2 \kappa^2}{192\pi} \mu_B^2 \ C[\phi, x_W] \ F[\phi, s, E_\gamma, \cos\theta_\gamma] E_\gamma dE_\gamma d\cos\theta_\gamma, \tag{28}$$

where E_{γ} , $\cos \theta_{\gamma}$ are the energy and scattering angle of the photon.

The kinematics is contained in the function

$$F[\phi, s, E_{\gamma}, \cos \theta_{\gamma}] \equiv \frac{(a^2 + b^2)(s - 2\sqrt{s}E_{\gamma}) + a^2 E_{\gamma}^2 \sin^2 \theta_{\gamma}}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2},$$
(29)

while the coefficient C is

$$C[\phi, x_W] \equiv \frac{a^2/2 + b^2/2 - 4a^2 x_W + 8a^2 x_W^2}{x_W^2 (1 - x_W)^2},$$
(30)

where $x_W \equiv \sin^2 \theta_W$.

Evaluating the limit when the mixing angle is $\phi = 0$, the expression for a and b is reduced to a = b = 1 and Eq. (28) is reduced to the expression (3) given in Ref. [9].

4 Results

In order to evaluate the integral of the total cross section as a function of mixing angle ϕ , we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over θ_{γ} from 44.5° to 135° and E_{γ} from 15 GeV to 100 GeV for various fixed values of the mixing angle ϕ . Using the following numerical values: $\sin^2 \theta_W = 0.2314$, $M_{Z_1} = 91.187$ GeV, $\Gamma_{Z_1} = 2.49$ GeV, we obtain the cross section $\sigma = \sigma(\phi, \kappa)$. We show the cuts used in our calculations in Table 1.

The sets of data of Table 1 take into account the background of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. The principal background is radiative Bhabha scattering, $e^+e^- \rightarrow e^+e^-\gamma$, where the photon is detected at wide angles and the electrons remain undetected at low angles close to or within the beam-pipe. Other background processes are: $e^+e^- \rightarrow \gamma\gamma\gamma$, $e^+e^- \rightarrow e^+e^-X$, $(X = \pi^0, \eta, \eta', f_2(1270))$, $e^+e^- \rightarrow \mu^+\mu^\gamma$, $e^+e^- \rightarrow \tau^+\tau^-\gamma$ and $e^+e^- \rightarrow e^+e^-l^+l^-\gamma$ $(l = e, \mu)$ [14].

Exp.	$\mathcal{L}(pb^{-1})$	$ heta_{min}, heta_{max}$	$E_{min}(GeV)$	Events	Ref.
L3 1990	3.8	45, 135	10	0	[11]
1991	11.2	45, 135	22.8	0	[12]
ALEPH 1991	8.3	42, 138	17	0	[13]
L3 1994	137	44.5, 135	15	14.1	[14]

Table 1. Sets representative of experimental data from the process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$.

According to the experimental data, the allowed range for the mixing angle between Z_1 and Z_2 is

$$-9 \times 10^{-3} \le \phi \le 4 \times 10^{-3},\tag{31}$$

with a 90 % C. L. [17–19].

As was discussed in Ref.[9], $N \approx \sigma(\phi, \kappa)\mathcal{L}$. Using the Poisson statistic [14, 15], we require that $N \approx \sigma(\phi, \kappa)\mathcal{L}$ be less than 14.1, according to the data in Table 1. Using this fact, we put a bound for the tau neutrino magnetic moment as a function of the ϕ mixing parameter. We show the value of this bound for some values of the ϕ parameter in Tables 2 and 3.

ϕ	$\kappa(10^{-6})$
-0.009	2.77
-0.005	2.76
0	2.73
0.004	2.72

Table 2. Bounds on the ν_{τ} magnetic moment for different values of the ϕ mixing angle before the Z_1 resonance.

These results compare favorably with the bounds obtained in the references [9, 10]. However, the derived bounds in Table 2 could be improved by including data from the entire Z_1 resonance [9] as is shown in Table 3.

ϕ	$\kappa(10^{-6})$
-0.009	2.02
-0.005	2.01
0	2.00
0.004	1.98

Table 3. Bounds on the ν_{τ} magnetic moment for different values of the ϕ mixing angle in the Z_1 resonance.

We end this section by plotting the total cross section in Fig. 2 as a function of the mixing angle ϕ , for the bounds of the magnetic moment given in Tables 2, 3. We observe in Fig. 2 that for $\phi = 0$, we reproduce the data previously reported by R. Escribano and E. Massó [27]. Also, we observe that the total cross section increases constantly and reaches its maximum value for $\phi = 0.004$.

5 Conclusions

We have determined a bound on the magnetic moment of a massive tau neutrino in the framework of a left-right symmetric model as a function of the mixing angle ϕ , as is shown in Table 2 and Table 3.

Other upper limits on the tau neutrino magnetic moment reported in the literature are:

$$\mu_{\nu_{\tau}} < 3.3 \times 10^{-6} \mu_B \quad 90 \% \text{ C.L.},$$
(32)

$$\mu_{\nu_{\tau}} < 2.7 \times 10^{-6} \mu_B \quad 95 \ \% \text{ C.L.},$$
(33)

$$\mu_{\nu_{\tau}} < 1.83 \times 10^{-6} \mu_B \quad 90 \% \text{ C.L.},$$
(34)

$$\mu_{\nu_{\tau}} < 5.4 \times 10^{-7} \mu_B \quad 90 \% \text{ C.L.},$$
(35)

The bound given by (32) comes from the search for energetic single photon production in Z_1 decays from L3 collaboration [14]. From measurements of the Z_1 invisible width at LEP [27], the bound given by (33) is obtained. The bound given by (34) comes from the analysis of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at the Z_1 -pole, in a class of E_6 inspired models with a light additional neutral vector boson [16]. In Ref. [28], Cooper-Sarkar *et al.* have shown how the BEBC (Big European Bubble Chamber) beam dump elastic scattering experiments also provide a bound on the tau neutrino diagonal moment of a stable ν_{τ} , given by (35), thus severely restricting the cosmological annihilation scenario [29]. Our results in Table 3 confirm the bound obtained in (34).

The bound applies to Dirac as well as Majorana transition moments. However, transition moments involving ν_e and ν_{μ} flavors are more strongly bounded by other accelerator experiments. These limits are $\kappa_{tran} \leq 1.08 \times 10^{-9}$ for ν_e , and $\kappa_{tran} \leq 7.4 \times 10^{-9}$ for ν_{μ} [30]. However, a beam dump search for radiative decays gives a limit on the ν_{τ} transition magnetic moment of $\kappa_{tran} \leq 1.1 \times 10^{-9} (\frac{MeV}{m_{\nu_{\tau}}})^2$ [31].

In summary, we conclude that the estimated bound for the tau neutrino magnetic moment is almost independent of the experimental allowed values of the ϕ parameter of the model. In the limit $\phi = 0$, our bound takes the value previously reported by R. Escribano and E. Massó [27].

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