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# ELECTRON BEAM DOSE CALCULATION: HYBRID PENCIL BEAM MODEL EXTENDED FOR SMALL FIELDS

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The hybrid electron pencil beam model (HPBM) can usually be applied to middle or large fields and get good results, whereas it is sometimes not so successful to apply HPBM to small fields. In this note we report some results obtained by using an extended hybrid electron pencil beam model for small fields. Considering the condition of small fields, we redefine the central-axis depth dose for mono-energetic electrons in small fields and calculate the fitted spectrum of incident electron beam for these small fields. Through these improvements, we can make the extended hybrid electron pencil beam model applicable to small fields and obtain good results. And at the same time, the extended HPBM retains the calculation accuracy and efficiency for middle and large fields.

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## 1 Introduction

The electron pencil beam model (EPBM) has been proved to be a very useful method for calculating electron dose in radiotherapy [1-4]. Although some new algorithms, such as Macro Monte Carlo (MMC), Voxel Monte Carlo (VMC), and Phase Space Evolution (PSE) method, have been recently developed and have obtained good results [5-7], EPBM is still being applied to clinical applications, its algorithm being under constant improvement and its computing accuracy being increased greatly. The pencil beam redefinition algorithm for electron dose distribution developed by Shiu and Hogstrom is a good example of such efforts and improvements [8]. Actually, as EPBM is simple and applicable, and in most cases its accuracy is acceptable as well, how to further improve its accuracy is still of significance. HPBM is also a kind of improved EPBM. By keeping the main idea of EPBM and incorporating with more theoretical results of the bipartition model of electron transport, HPBM reduces largely measurement data used to EPBM and improves both accuracy and efficiency of EPBM [9]. HPBM has been applied in radiation therapy. It can predict dose distributions with higher calculation speed even in the presence of inhomogeneity and under irregular geometry conditions. In most cases, its calculation accuracy is acceptable for radiation therapy [10]. However, we found that large errors in calculating dose

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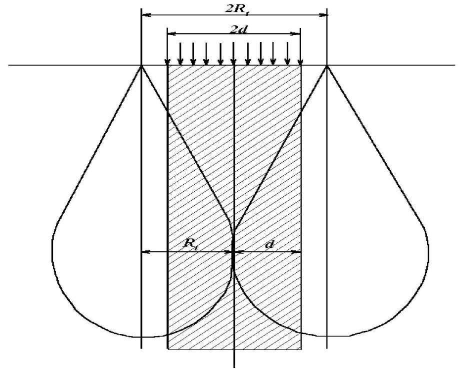


Fig. 1. Schematic diagram of small field for hybrid electron pencil-beam model

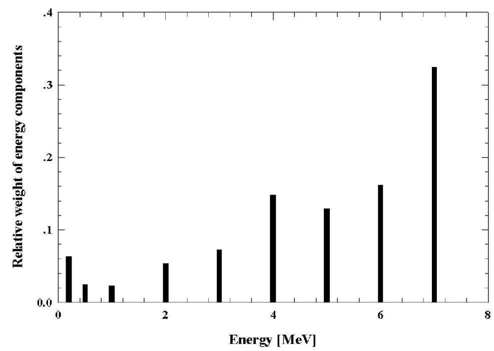


Fig. 2. The fitted spectrum for 6 MeV energy electron beam with 20 mm x 20 mm field size and 1000 mm SSD. For the estimate of the accuracy of the fitted spectrum, see the text.

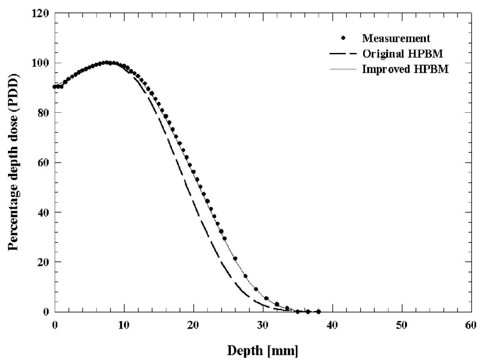


Fig. 3. Percentage depth dose comparison, the irradiation condition is same as Fig. 2.

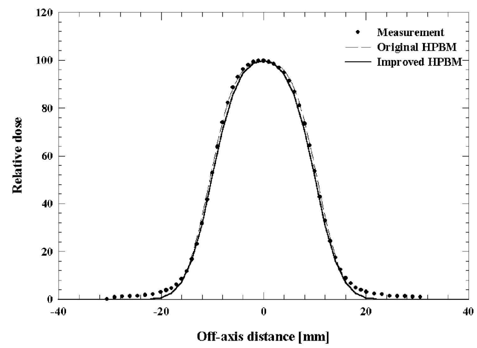


Fig. 4. Dose profile at 8 mm depth, the irradiation condition is same as Fig.2.

distribution may occur when HPBM is applied to small fields. Recently, we improved this model and obtained good calculation accuracy in small fields. Some results calculated by using the extended HPBM are presented in the paper.

## 2 Improvements in hybrid pencil beam model

In HPBM, we assumed that the Percentage Depth Dose (PDD) distribution of a finite broad electron beam approximately equals the depth dose distribution of an infinite broad beam in a considerable large region in irradiated medium. Of course, this assumption is true only when the field size is larger than the transverse diffusion distance of an electron pencil beam. Obviously, such prerequisite no longer stands for the case of small fields, especially at large penetration depths where electrons have undergone great number of interactions with medium and already had large angular spread and transverse diffusion distance. This is illustrated in Fig.1. We suppose the maximum transverse distance of an infinite narrow electron pencil-beam is $R_t$, then $2R_t$ should be the critical field size for our using above assumption. When the field size (we denote as $2d$) is smaller than $2R_t$, there will be no such region where the depth dose distribution of the small field equals to the depth dose distribution of an infinite broad beam. If we defined $D_d(z)$ as the depth dose at central axis of the small field and $D(z)$ as depth dose distribution of an infinite broad electron beam, the deviation $D(z) - D_d(z) \geq 0$ for this case is true. This result indicates that the depth dose distribution of an infinite broad electron beam is larger than the dose distribution at central-axis of the small field because in the latter case a portion of the dose at central-axis contributed from transverse diffusion electrons has been moved out. To apply HPBM to small fields, we still use the formulae [9] to describe the spatial dose distribution resulting from an electron pencil-beam with mono-energetic $E$ and uniform fluence of a small field

$$\begin{aligned} D_{sf}(x,y,z,E) = \left(\frac{D_{bm}(z,E)}{4}\right) \quad &\times \quad [erf(\frac{d+x}{\sqrt{A_h}}) + erf(\frac{d-x}{\sqrt{A_h}})] \\ &\times \quad [erf(\frac{d+y}{\sqrt{A_h}}) + erf(\frac{d-y}{\sqrt{A_h}})] \end{aligned} \tag{1}$$

Where, $D_{sf}(x,y,z,E)$ is the dose at point $(x,y,z)$ for an electron pencil beam with incident energy $E$ under the small field case. $D_{bm}(z,E)$ represents the depth dose distribution of broad beam with incident energy $E$. $erf(x)$ is the error function. $A_h$ is a characteristic transverse parameter defined in HPBM [9]

$$A_h = A_2 - (\frac{A_1}{A_0})^2(A_0 - B_0). \tag{2}$$

Here, $A_0$, $A_1$, $A_2$ and $B_0$ are defined as in previous paper [2, 9]. Generally, an incident electron beam consists of electrons with different energy components whose probability is $P(E)$. If we know the energy components $P(E)$, the spatial dose distribution can be obtained. Thus, the eq.(1) can be rewritten as

$$\begin{aligned} D_{sf}(x,y,z,E_n) = \int_0^{E_{max}} \quad & P(E)\frac{D_{bm}(z,E)}{4}[erf(\frac{d+x}{\sqrt{A_h^*}}) + erf(\frac{d-x}{\sqrt{A_h^*}})] \\ & \times[erf(\frac{d+y}{\sqrt{A_h^*}}) + erf(\frac{d-y}{\sqrt{A_h^*}})]dE, \end{aligned} \tag{3}$$

with $E_n$ and $E_{max}$ representing nominal energy and the maximum probability energy component of incident electron beam respectively.

$$A_h^* = A_2(0) + 2\sigma_{\vartheta_x}^2 z^2 + A_h(z,E). \tag{4}$$

Here, $A_2(0)$ describes the influence of penumbra on the dose profile at the surface (z=0). The second part of eq. (4), $2\sigma_{\vartheta_x}^2 z^2$, gives the transverse diffusion contributed by initial angular spread of electron beam[1]. $A_h(z,E)$ describes the transverse diffusion of electron beam given by HPBM [9]. Now we can explain eq. (3) from another point of view. At first, we use symbol $w(E)$

instead of $P(E)$. $w(E)$ is not real energy components other than a fitted energy spectrum of the incident electron beam and it can be obtained by solving an inverse problem. For a given small field, we actually can find out such a fitted initial energy spectrum $w(E)$ of incident electron beam by letting the calculated depth dose at the central axis exactly be equal to the measured dose. Thus, we have

$$D_{sf}(0,0,z) = \int_0^{E_{max}} w(E) D_{bm}(z,E) erf^2(\frac{d}{\sqrt{A_h^*(E)}}) dE \tag{5}$$

In fact, eq.(5) is the first kind of Fredholm integral equation and it is, in general, ill posed. As mentioned above, $D_{bm}(z,E)$ is the depth dose distribution of a broad electron beam with energy $E$, it can be calculated by bipartition model. For a small field, we define an equivalent energy deposition kernel as

$$D_{bm}^*(z,E) = D_{bm}(z,E) erf^2(\frac{d}{\sqrt{A_h^*(E)}}). \tag{6}$$

Then, we get

$$D_{sf}(0,0,z,E_n) = \int_0^{E_{max}} w(E) D_{bm}^*(z,E) dE. \tag{7}$$

Thus, the solution of electron dose distribution in a small field is converted to finding the fitted initial energy spectrum of electron beams. This problem has been discussed in detail in the previous paper [11]. Using the same method, i.e. regularization method, we can obtain $w(E)$. Once the $w(E)$ is determined, the dose distribution of small field can be correspondingly calculated through eq. (3) by substituting $w(E)$ for $P(E)$ only.

## 3 Results

Applying the extended HPBM mentioned above, we have calculated the energy deposition in water phantom with 20 mm x 20 mm field size and with 1000 mm source surface distance (SSD). The energy of electron beams is chosen as 6 MeV and 10 MeV. Furthermore, we made a comparison of these calculation results with experimental measurement data in water phantom. These measurements are completed by Zeng Ge in Tuebingen University hospital [12,13]. The error of measurement data is 0.5% for fields smaller than 100 mm x 100 mm. If the field size is larger than 100 mm x 100 mm and the energy is lower than 8 MeV, the errors near the field edges can be up to 1%. The relevant data for 6 MeV energy electron beams are shown from Fig.2 to Fig.6. Fig.2 shows the calculated fitted energy spectrum of electron beams with 6 MeV nominal energy. Fig.3 shows a comparison of the PDD data between extended HPBM, original HPBM and that taken from experimental measurement. Fig.4,5,6 show a comparison of dose profiles, which are at 8 mm, 12 mm, and 20 mm depth respectively, given by extended HPBM, original HPBM and experimental measurements. The other five figures, from Fig.7 to Fig.11, show a similar comparison between the improved, original HPBM and experimental measurements for the case of 10 MeV energy electron beams with 20 mm x 20 mm field size. For electron beam with both 6 MeV energy and 10 MeV energy, the accuracy of fitted energy spectrum are less than 1%. From these comparisons, it is clear that: 1) The original HPBM is not very suitable

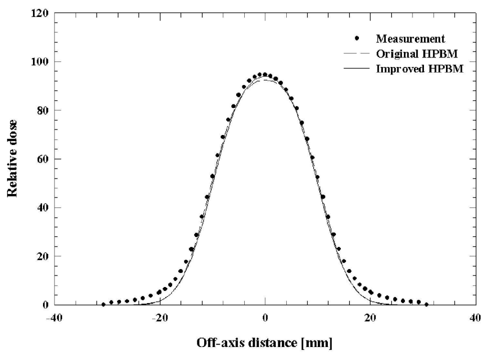


Fig. 5. Dose profile at 12 mm depth, the irradiation condition is same as Fig.2.

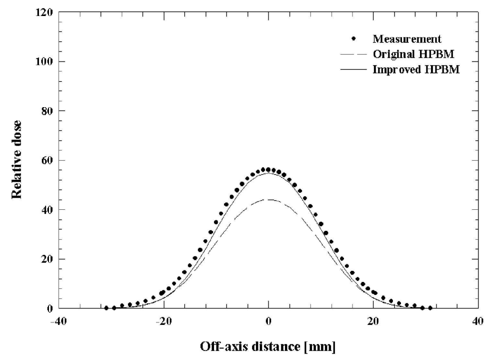


Fig. 6. Dose profile at 20 mm depth, the irradiation condition is same as Fig.2.

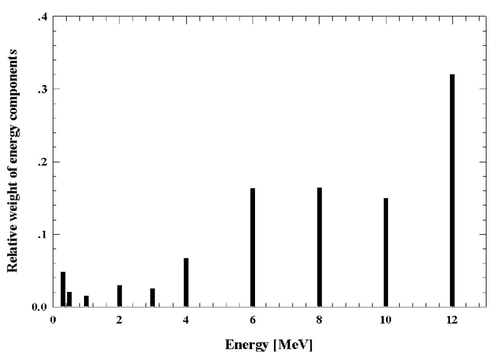


Fig. 7. The fitted spectrum for 10 MeV energy electron beam with 20 mm x 20 mm field size and 1000 mm SSD. For the estimate of the accuracy of the fitted spectrum, see the text.

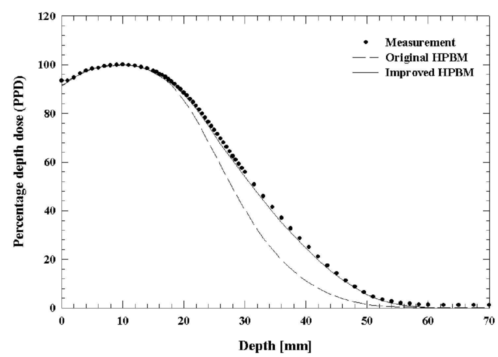


Fig. 8. Percentage depth dose comparison, the irradiation condition is same as Fig. 7.

to electron dose calculation for small fields, and 2) there is good agreement between the dose distribution calculated by the improved HPBM and that of experimental measurements. These results show a good promise of application of the improved hybrid pencil beam model to clinical cases. Though we focus on small field in this paper, the extended HPBM can also be used in middle or large field. In the case for middle or large field, the error term $erf(\frac{d}{\sqrt{A_h^*(E)}})$ in eq. (6) is about 1. Therefore, $D_{bm}^*(z,E)$ is actually equal to $D_{bm}(z,E)$. We also show a comparison between calculation result and measurement data of 8 MeV energy electron beam for 200 mm x 200 mm field size in Fig. 12.

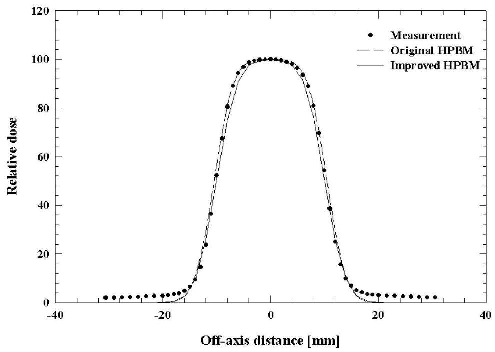


Fig. 9. Dose profile at 9 mm depth, the irradiation condition is same as Fig.7.

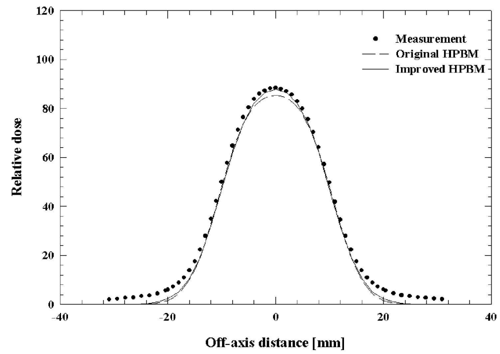


Fig. 10. Dose profile at 20 mm depth, the irradiation condition is same as Fig.7.

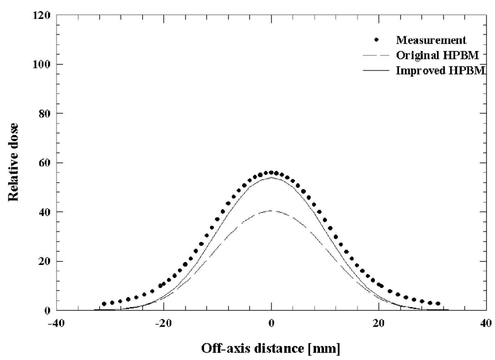


Fig. 11. Dose profile at 30 mm depth, the irradiation condition is same as Fig.7.

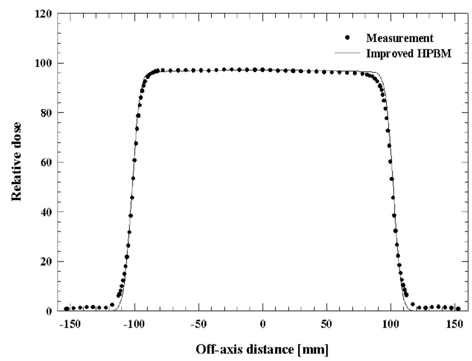


Fig. 12. Dose profile at 20 mm depth for 8 MeV energy electron beam with 200 mm x 200 mm field size and 950 mm SSD.

## 4 Discussion

When HPBM is used directly in small field case, the calculation PDD is much less than the measurement data. We will see a relative error of 12% at the 22 mm depth in the case of Fig. 3, and 15% at the 34 mm depth in the case of Fig.8. The reason is that electrons at large penetration depths have undergone great number of interaction with medium and already had large angular spread and transverse diffusion distance. Therefore, we must correct the measured PDD data when we unfold the spectrum of incident electron beam. After having done this, we get a more accuracy result. We can see a good agreement between the calculation PDD and measurement PDD. The relative error decreases and is less than 2% both for the 6 MeV energy electron beam and the 10 MeV energy electron beam (see Fig. 3 and Fig. 8). However, we still find a large error outside the filed for the 10 MeV energy electron beam (see Fig.9 and Fig.10). These results from the fact that we didn't take into account the photons contamination and electrons contamination from the collimator in this paper.

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