acta physica slovaca vol. 53 No. 2, 165 – 172 April 2003

OPTIMAL FUNCTIONS FOR PEAK SEARCH METHODS BASED ON SPECTRUM CONVOLUTION

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Received 5 October 2002, in final form 23 December 2002, accepted 7 January 2003

Any peak search method for high resolution gamma-ray spectroscopy based on convoluted spectra is faced with the choice of the convolution functions. We derive the optimal convolution functions for extraction of the most important parameters of a peak — the area of the peak, its position and the width. It is shown that such functions strongly depend on the signal-to-background ratio. For small peaks on high background the functions are well approximated by derivatives of the peak shape itself. For peaks on low background the optimal convolution functions are very simple, approaching in the limit of vanishing background the linear or quadratic shape.

PACS: 29.30.Kv, 29.85.+c

1 Introduction

In a recent work [1] we described an efficient peak search method for high-resolution gamma-ray spectra based on spectrum convolution. We showed that the convoluted spectrum, being free of background component, may be used not only to determine the peak region but also to deduce the most vital parameters of the peak itself. Such a method makes it possible in the case of Gaussian shaped peaks to obtain simple analytical expressions for the uncertainties of the peak parameters, which is very important in later stages of the peak search algorithm, dealing with doublet peaks. The assumption of a specific (Gaussian) peak shape is not critical. A strong support for this statement comes from the IAEA 1998 intercomparison study [2], which demonstrated that the peak shape model dependency, although expected prior to the study, was not found in practice. All the programs tested there reported on average the same peak areas, within 1%, independently of different peak shape models they used. Our experience with the analyses where absolute areas are important [3] supports the above conclusion.

The use of the convolution method to locate the peaks is a well established approach and is utilised in many of the available codes. It started with Mariscotti [4, 5], who used a smoothed second derivative of the spectrum to suppress the background and enhance the peaks. The so called correlation technique emerged later [6], using the second derivative of the Gaussian as the convolution function, called also the correlator. Robertson et al. [7] gave an optimal function, which minimised a specially defined peak area-to-noise ratio. These and alternative functions were carefully analysed by Hnatowicz [8].

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We suggest here an approach to the peak search algorithms in which the optimal convolution functions are used to extract the peak area, position and width from the convoluted spectrum alone. This is a novelty, since the use of the convolution method has so far been limited to peak position determination [9]. The approach keeps the statistical errors down to the optimal level for all the relevant quantities. The resulting method is a form of the Wiener filter applied to the spectrum in order to recover the desired parameters from it.

In what follows we first derive the optimal convolution functions for the peak area, position and width for an arbitrary shape of a peak or the underlying background. We then examine the obtained functions by assuming the peak shape to be a Gaussian one in order to show the characteristic features of the optimal correlators for high or low backgrounds. We compare the function with functions intuitively chosen in previous works [4–8] and suggest their modifications for detection of small peak on high background or strong peaks on low background.

2 The convolution method

The method in question for determining the peak parameters from the measured spectrum $p(x)$ is based on the transformation

$$I(y) = \int_{y-\beta}^{y+\beta} p(x) f(x-y) dx \,. \tag{1}$$

The interval of the integration is finite, $[y-\beta, y+\beta]$. A proper choice of the function $f(x-y)$ is the key to the successful implementation of the method. We assume that an ideal spectrum without statistical fluctuations, which we denote by $p_0(x)$, exhibits a line z on a background $\bar{n}(x)$

$$p_0(x) = Az(x, a, \sigma) + \bar{n}(x) \,. \tag{2}$$

Here, $z(x, a, \sigma)$ represents the shape of the total absorption peak with the unknown width σ and the unknown amplitude A proportional to the intensity of the line. We later assume that the shape is a Gaussian one given by

$$z(x, a, \sigma) = \exp\left(-\frac{(x-a)^2}{2\sigma^2}\right) . \tag{3}$$

Here, a represents the peak position and σ the standard deviation. For the moment we do not need to assume any specific shape for the peak.

The function $f(x-y)$ is chosen so as to filter out the background $\bar{n}(x)$. This is achieved by the requirement

$$\int_{y-\beta}^{y+\beta} \bar{n}(x) f(x-y) dx = 0 \,. \tag{4}$$

We next assume that $I(y)$ reaches its maximum value at $y = a$. Since the background is filtered out by the transformation (1), the area of the peak can be deduced from $I(a)$ as

$$P = \alpha I(a) \,, \tag{5}$$

where α has to be calculated knowing the detailed shapes of the functions $f(x-y)$ and $z(x, a, \sigma)$ [1]. The shape of the function $f(x-y)$ can be obtained from a restrictive requirement that the statistical uncertainty of the peak position a, the width σ or the area P be minimal. It is our task to find such optimal functions.

3 Optimal peak area correlator

In order to find this optimal function we treat the experimental spectrum $p(x)$ as a stochastic quantity

$$p(x) = p_0(x) + w(x) \,, \tag{6}$$

where $w(x)$ represents the statistical fluctuation of the number of counts in the channel x and can be treated as a random variable with a zero mean

$$\langle w(x) \rangle = 0$$

and variance

$$\langle w^2(x) \rangle = p_0(x) \,.$$

In addition we assume that there is no correlation between the fluctuations in different channels of the measured (original) spectrum, so that

$$\langle w(x) w(x') \rangle = 0$$

for all $x \neq x'$. The brackets $\langle\rangle$ denote the ensemble average of a quantity.

The relative uncertainty U_P of the area P is defined as

$$U_P = \frac{\langle (P-P_0)^2 \rangle}{P_0^2} \,, \tag{7}$$

where by P_0 we denote the area obtained using the ideal spectrum $p_0(x)$ with a known $\bar{n}(x)$. Using Eqs. (5) and (1) and taking into account the properties of the random function $w(x)$, we arrive at

$$U_P(f) = \frac{\int_{a-\beta}^{a+\beta} p_0(x) f^2(x-a) dx}{\left(\int_{a-\beta}^{a+\beta} p_0(x) f(x-a) dx\right)^2} \,. \tag{8}$$

We now search for the function $f(x-a)$ which minimises the functional U_P and is subject to the condition given by Eq. (4). The solution is obtained employing the standard techniques of variational calculus [10]. We study the functional U_P by setting $f(x-a) = f_0(x-a) + \epsilon(x)$ where the function $f_0(x-a)$ is the solution of the problem and $\epsilon(x)$ a small variation, obeying the boundary conditions $\epsilon(a-\beta) = \epsilon(a+\beta) = 0$. The condition $g(f) = \int_{a-\beta}^{a+\beta} \bar{n}(x) f(x-a) dx = 0$ is taken into account by modifying the functional $U_P(f)$ into $U'_P(f) - \lambda g(f)$, with λ being the Lagrange multiplier. The solution $f_0(x-a)$ follows from the necessary condition that the functional $U'_P(f)$ be stationary regarding any function $\epsilon(x)$ and we obtain

$$f_0(x-a) = c \frac{p_0(x) - \lambda \bar{n}(x)}{p_0(x)} \,. \tag{9}$$

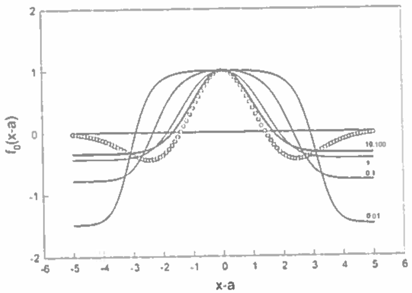

Fig. 1. The form of the function $f_0(x-a)$ for a Gaussian line shape with $\sigma = 1$ and a constant background is shown in the interval [-5, 5]. The curves are presented for different ratios of n_0/A, ranging from 100 down to 1/100 (solid lines). The approximation to these functions, used in [1], is shown in open circles.

The parameter λ is then tuned to satisfy $g(f_0) = 0$. The constant c is arbitrary, we chose it to be positive so the $f_0(0) = 1$. In Fig. 1 the form of $f_0(x-a)$ is presented for a Gaussian peak, given by Eqs. (2) and (3) and a constant background $\bar{n}(x) = n_0$. The function is intuitively an appealing one. It has negative almost constant sections in the region where only the background is present and a positive resonance which filters out the peak. The dependence on the peak-to-background ratio n_0/A is weak for $n_0/A > 1$. For the peaks with almost no background the function tends to be constant over the peak region, which is an expected result. Namely, the peak area is in the absence of the background simply the sum of the count in the region of the peak. For the relative uncertainty U_P of the peak area we obtain in this case the well known result $U_P = 1/\sqrt{P}$.

It is interesting to note that the optimal function $f_0(x-y)$ is rather weakly dependent on the shape of the background. A constant background, which we assumed in the previous example, is only an approximation. On the low-energy side of real peaks the background is in fact higher due to poor charge collection and almost forward Compton scattering of the photons within the source. We write, as suggested in [11], the background as a sum of a constant and a low energy component in the form

$$\bar{n}(x) = n_0(1 + b(1 - F(x,a))) . \tag{10}$$

The function

$$F(x,a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-(x-a)^2/2\right) dx$$

is the cumulative normal distribution and b is the amplitude of the low energy background component. For the ratio $n_0/A = 1$ and $b = 0.25$ the spectrum p_0 is shown in Fig. 2. Fig. 3 shows the function $f_0(x-a)$ for three different values of b and the peak-to-background ratio $n_0/A = 1$. We may observe a weak dependence on the parameter b even if we assume a much exaggerated value of $b = 1$. This suggests that the convolution method with the function $f(x-y)$, designed for the constant background, eliminates also the background found in actual gamma spectra.

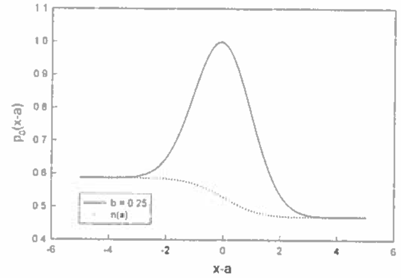

Fig. 2. Part of a spectrum with a background enhanced at the low energy part of the peak. The amplitude of the low energy component is $b = 0.25$, the peak-to-background ratio is 1.

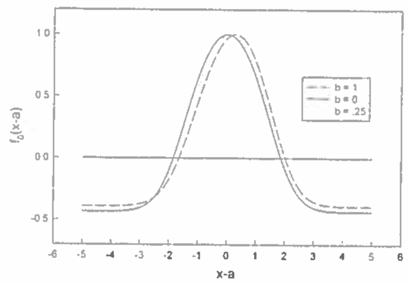

Fig. 3. The form of the functions $f_0(x-a)$ for a non-constant background with different amplitudes of the low energy components: $b = 0$ (solid line), $b = 0.25$ (dotted line) and $b = 1$ (dashed line). The peak-to-background ratio is 1.

4 Optimal peak position correlator

We next try to find the function $f(x-y)$ minimising the uncertainty in the position of the peak. This can be achieved by applying the convolution of the spectrum $p(x)$ and searching for the zero of the convolution function $I(y)$

$$I(y) = \int_{y-\beta}^{y+\beta} p(x) f(x-y) dx = 0 . \tag{11}$$

The solution y_0 of this equation for an ideal spectrum $p_0(x)$ is equal to a. Due to statistical variation $w(x)$ of the spectrum resulting in $p(x) = p_0(x) + w(x)$, as assumed in the case of the optimal peak area correlator, the actual solution y_m varies around a. The uncertainty of the peak position can be obtained by examining the fluctuation δa of the actual solution y_m around y_0, due to the presence of fluctuation $w(x)$. We put $y_m = y_0 + \delta a$ and in Eq. (11) we expand the function $f(x-y_m)$ in powers of δa and retain only the linear term

$$f(x-y_m) = f(x-y_0) + \frac{\partial f(x-y_m)}{\partial y_m} \delta a . \tag{12}$$

For the expression $\langle (y_m - y_0)^2 \rangle$ one then obtains

$$\langle (y_m - y_0)^2 \rangle = \frac{\int_{a-\beta}^{a+\beta} \langle w(x)^2 \rangle (f(x-a))^2 dx}{\left(\int_{a-\beta}^{a+\beta} p_0(x) (\frac{\partial f(x-a)}{\partial a}) dx \right)^2} . \tag{13}$$

In searching for the function $f_0(x-y)$, which minimises the above stated condition, we use the same procedure as in the case of the optimal peak area correlation function and find a simple result

$$f_0(x-y)_{y=a} = c \frac{dp_0}{dx} \frac{1}{p_0} . \tag{14}$$

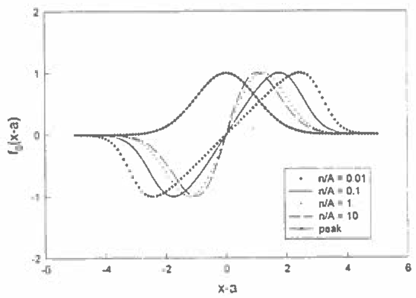

Fig. 4. The shapes of the optimal functions $f_0(x-y)$ which minimise the statistical variation of the deduced peak position for the peak-to-background ratios n_0/A ranging from 10 to 1/100. In the limit $n_0/A \to 0$ the curve reduces to a linear function.

It is easy to observe that this optimal function is in the limit of large ratio n_0/A simply the derivative on x of the optimal peak area correlator. In this limit the denominator $p_0(x)$ in Eq. (14) is a constant and therefore the function $f_0(x-a)$ coincides with the derivative of $p_0(x)$ up to an multiplicative constant. This can be observed in Fig. 4, where several optimal functions $f_0(x-y)|_{y=a}$ for different peak to background ratios are presented. In the limit of vanishing background, the function is a linear one with the zero value at $x=a$. With such a function the median of the peak is found, which is evidently the best estimate of the peak position [12] under such circumstances.

5 Optimal peak width correlator

What we learn from the considerations above is that the shape of the functions should be close to the shape of the peak we are analysing. The width of the functions should be therefore adjusted to the width of the peak considered. The knowledge of the width is, additionally, an important quantity for precise determination of the peak width systematics which can be used to search for the merged peaks in the later stages of the spectrum evaluation routines.

To find the width of the peak by the convolution method we search for the zero of the convolution function $I(y=a,\sigma)$. Te condition is therefore $I(y=a,\sigma)=0$, or

$$I(y=a,\sigma)=\int_{y-\beta}^{y+\beta} p(x) f(\frac{x-y}{\sigma})\,dx = 0\,, \tag{15}$$

from which we obtain the estimate σ_m for the width σ. In the case of an ideal spectrum the estimate coincides with the actual width of the peak. To obtain the uncertainty of the estimate, we follow the procedure used to determine the uncertainty of the peak position and arrive at

$$\langle(\sigma_m-\sigma)^2\rangle = \frac{\int_{a-\beta}^{a+\beta} (w(x)^2)(f(\frac{x-a}{\sigma}))^2 dx}{\left(\int_{a-\beta}^{a+\beta} p_0(x)(\frac{\partial f(\frac{x-a}{\sigma})}{\partial\sigma})dx\right)^2}\,. \tag{16}$$

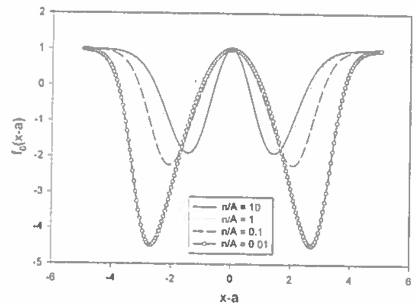

Fig. 5. The shapes of the optimal functions $f_0(x-y)$ which minimised the statistical variation of the peak width for the peak-to-background ratios n_0/A ranging from 10 to 1/100. In the limit $n_0/A \to 0$ the curve reduces to a quadratic function.

The search for the function with the minimal uncertainty yields the result

$$f_0(\frac{x-a}{\sigma}) = c\left(\frac{d}{dx}\left(\frac{x-a}{\sigma}p_0(x)\right)\right)/p_0(x)-\lambda\,, \tag{17}$$

where λ is chosen to fulfill Eq. (15).

Several shapes of the function $f_0((x-a)/\sigma)$ are shown in Fig. 5 for different signal-to-background ratios. In the limit of vanishing background the correlator reduces to a simple quadratic function.

6 Discussion

Although the obtained functions are relatively simple and intuitive, they are analytically not convenient enough to be used in an actual peak search routine. As shown in [1], for practical use an approximation to the functions f_0 has to be applied. For the peak position search the limiting function for the high background-to-peak ratio should be used, being simply the derivative of the peak shape function. In this step of the peak search procedure only a coarse approximation to the unknown widths of the optimal peak area convolution function is required. In the second step, the optimal function extracting the peak area is well represented by the convolution of the derivative of the peak with itself. The same function can be used to extract the peak width by searching for the maximum of the convoluted spectrum close to the peak position. For Gaussian shaped peaks such approximations allow the use of symbolic computation to achieve simple expressions for the uncertainties of the parameters obtained.

For peaks on low backgrounds the Gaussian shaped functions should perhaps be modified to better resemble the optimal ones. This may be important for efficient detection of small peaks on small background. In practice, however, this ability seems to of limited importance.

To conclude, we present optimal convolution functions in order to help choose the proper functions for an efficient approach to the peak search problem for complex spectra from high resolution Ge spectrometers. The method is based on a linear transformation of the spectrum which eliminates the background and allows for analytical expressions of the uncertainties of all of the peak parameters to be deduced when used with the Gaussian model of the peaks. The method is then easy to implement and appears to be robust and reliable [1].

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EFFECT OF INJECTION OF C-BAND AMPLIFIED SPONTANEOUS EMISSION ON TWO-STAGE L-BAND ERBIUM-DOPED FIBER AMPLIFIER

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Received 2 January 2003, in final form 3 February 2003, accepted 5 February 2003

An effect of injection of conventional-band amplified spontaneous emission (C-band ASE) on a two-stage long wavelength band erbium-doped fiber amplifier (L-band EDFA) is demonstrated. It uses two circulators and a broadband fiber Bragg grating (FBG) to route unused C-band backward ASE from the second stage back to the input end of the first stage of the amplifier. The amplifier gain is clamped at 15.5 dB and the saturation power increases from −13 dBm to −8 dBm with injection of the C-band ASE. The gain level can be controlled to be in the range from 15.5 to 16.8 dB by varying the variable optical attenuator (VOA) loss from 0 to 20 dB without much variation in noise figure. These results show that the injection of C-band ASE can be used to clamp the L-band gain in a two-stage L-band EDFA, which has higher gain compared to a single stage.

PACS: 42.60.Da, 42.81.–i, 42.81.Wg, 42.81.Uv

1 Introduction

Wavelength-division-multiplexing (WDM) techniques can very efficiently utilize the low loss transmission bandwidth of single mode fiber (SMF) to increase the transmission capacities of fiber systems. However, the transmission capacities of current 1.5 μm WDM systems are limited by the gain bandwidth of erbium-doped fiber amplifiers (EDFAs), which operate in the conventional wavelength band at 1529–1560 nm (C-band). Therefore the L-band (1568–1600 nm) is offered in addition to the conventional band (C-band) EDFA. Integration of L-band in parallel with C-band allows a gain bandwidth of about 70 nm to be achieved [1].

The excited erbium-doped fiber (EDF) will emit amplified spontaneous emission (ASE) in both forward and backward directions. The large amount of backward ASE at the input end of the EDFA system is totally unavoidable because ASE generation is quasi-random in direction. To date, there are many research efforts to enhance the amplification characteristics of the L-band EDFA by utilizing this unused C-band backward ASE [2–4]. In this letter, we demonstrate that the unused C-band backward ASE can be used to clamp a gain in the two-stage L-band EDFA. Gain clamping is very important for maintaining gain as the signal is added and dropped from the channels traveling through the L-band EDF.

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