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INFLUENCE OF PHASE DAMPING IN THE PRESENCE OF STARK SHIFT ON NONCLASSICAL PROPERTIES OF THE TWO-MODE JCM

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In the present communication, we study the influence of phase damping in the presence of Stark shift for the multi-quanta JCM, using a master equation describing phase damping under the Markovian approximation. An analytic solution of the master equation for the multi-quant two-mode JCM Hamiltonian with the phase damping in the presence of Stark shift is obtained. We use this solution to investigate the influence of phase damping and Stark shift on nonclassical properties of the system, for the resonant and the off-resonant cases. We compare the behaviour of the system in the case of having a coherent superposition state and a statistical mixture of coherent states as an initial field. Our results show that the Stark shift plays an important role in the evolution of the population inversion in the JCM.

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1 Intoduction

The motion of the centre-of-mass (CM) of ultracold trapped ions has to be dealt with quantum mechanically [1]. Laser irradiation is used to monitor the ion's internal and external degrees [2–6]. Models have been constructed to describe a two-level ion undergoing quantized vibrational motion within a harmonic trapping potential and interacting with a classical light field [2,3,7,8]. It has been pointed out that the dynamics of a trapped ion can be described by a Hamiltonian similar to a Jaynes-Cummings Model [9] or its generalizations under certain regimes [4,5,10–12]. An equivalent Hamiltonian also describes a two mode micromaser as realized at the ENS in Paris [13].

Without damping, it is exactly solvable under the rotating-wave approximation. Nevertheless damping is always be present in any experimental realization of the model, the solution in the presence of damping is interest and importance from a practical point of view. It was found that the collapses and revivals of the atomic inversion oscillations predicted by the JCM are in agreement with the experiments done with Rydberg atoms in a microwave cavity [14–16].

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In these experiments, damping of the cavity mode is not negligbly small. Thus, for a detailed comparison with experiments, the effects of damping must be taken into account.

In the past few years, a number of authors have treated the JCM with damping by the use of analytic approximations [17–19] and numerical calculations [20–23]. In particular, Agarwal and Puri [24] presented an analytic solution for the initial state when the light field is a vacuum state. Daeubler et al. [25] found an analytic expression for the atomic inversion and the intensity of the field in the JCM with damping when the reservoir is at zero temperature. However, the damping treated in all the above studies is modelled by coupling to an external reservoir including energy dissipation. As is well known, in a dissipative quantum system, the system loses energy by creating a bath quantum. In this kind of damping the interaction Hamiltonian between bath and system does not commute with the system Hamiltonian. In general this leads to a thermalization of the system with a certain time constant. In addition it has be shown that such damping has a severe effect on quantum coherences of the system involving macroscopic superposition states [26–28], which damp out on a much faster time scale depending on their classical separation.

There are, however, other kinds of environmental coupling to the system, which do not involve energy exchange. In this so called phase damping [29] the interaction Hamiltonian commutes with that of the system and in the dynamics only the phase of the system state is changed in the interaction. Similar to standard energy damping the off diagonal elements of the density matrix in energy basis decay at a given rate. This phase damping can well describe some unaccounted decay of coherences in a single mode micromaser [30]. Recently, it has also been shown that phase damping seriously reduces the fidelity of the received qubit in quantum computers due to the decoherence induced by phase damping [31]. However, it is not clear whether the fast decay of macroscopic coherences is also present in this case.

It is worthwhile mentioning that the master equation obtained in this article can describe the phase damping not only in the JCM but also in other quantum systems. In particular, this kind of phase damping may play an important role in the decoherence problem [31] in quantum computers, and in the study of transport phenomena in mesoscopic system, since elastic scatterings play a key role in mesoscopic system [32]. Indeed, if we reinterpret the annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ of the center-of-mass (CM) phonons in ion trap quantum computers [33], then it is possible to apply the formalism developed in this article directly to investigation of the intrinsic decoherence [34] in ion trap quantum computers.

The phase damping in the JCM with one quantized field mode has been studied [35]. There has been considerable interest in the properties of the so-called superposition states of light (SS) involving superpositions of coherent states with strongly differing amplitude [36–41]. One particularly interesting case is the superposition of two coherent states of fixed amplitude but opposite phase [37–41]. Due to the quantum interference, the properties of such a superposition are very different from the properties of the constituent states (coherent states), as well as from the incoherent superposition or statistical mixture (SM) of coherent states.

In this model, when the two atomic levels are coupled with comparable strength to the intermediate relay level, the Stark shift becomes significant and can not be ignored [42–45]. The authors [43–45] studied the influence of the Stark shift term on the atomic inversion and dipole squeezing in the two-photon processes. They found that the dynamic Stark shift plays an important role in atomic inversion, but the influence of the Stark shift on the atomic inversion does not show if the two-levels are coupled equally strongly with the relay level under the condition of a strong initial field, and they also showed that the dipole squeezing shows a weak phase dependence in the absence of the Stark shift, but a strong phase dependence when it is present. Ashraf and Zubairy [46] included this power-dependent effect in their study of the equal-frequency two-photon micromaser. Gou [47] discussed how to eliminate the Stark shift through the use of a correlated two-mode field state in unequal-frequency absorption. Nasreen and Razmi [45,48] discussed the effect of the dynamic Stark shift on atomic emission and cavity field spectra in the two-photon JCM and have shown that the Stark shift leads to asymmetric vacuum field Rabi splitting.

The purpose of this work is to study the phase damping for the multi-quanta two-mode JCM in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM). We will introduce a master equation to describe phase damping, and obtain an analytic solution of the master equation for an effective two-mode Hamiltonian. We show that phase damping suppresses nonclassical effects of the cavity field in the JCM, also it is shown that the Stark shift leads to increasing of the values of the atomic revivals of the population inversion in SM and SS when the two levels of the particle are equally strongly coupled with the intermediate relay level, while it leads to decreasing of the values of the atomic revivals of the population inversion when the two levels have unequal Stark shift. This paper is organized as follows: In section 2, we present the master equation to describe the phase damping of a quantum system under the Markovian approximation. In section 3, we obtain an exact solution of the multi-quanta Jaynes-Cummings model with phase damping in the presence of Stark shift and give the explicit expression of this solution in the two-dimensional basis of the particle. Section 4 is devoted to an investigation of the influence of the phase damping and Stark shift on nonclassical properties of the field in the JCM either in the resonant or the off-resonant cases. Finally, some concluding remarks are provided while calculations are relegated to an appendix.

2 Master equation describing phase damping

We consider a system described by the Hamiltonian H and interacting with the heat-bath environment (the reservoir) which consists of an infinite set of harmonic oscillators. we assume that the system interacting with the environment can be described by the total Hamiltonian

$$\hat{H}_T = \hat{H} + \sum_i \left(\frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 x_i^2 \right) + \hbar \hat{H} \sum_i C_i x_i + \hbar^2 \hat{H}^2 \sum_i \frac{|C_i^2|}{2m_i \omega_i^2}, \tag{1}$$

where the second term is the Hamiltonian of the reservoir, the third represents the interaction between the system and the reservoir with the coupling constant $\hbar C_i$, and the last is the renormalization term [49]. Obviously, the system Hamiltonian $\hat{H}$ commutes with the interaction Hamiltonian in (1). Hence the damping described by the Hamiltonian (1) is a kind of phase damping. The coupling between the system and the reservoir which we adopt here is also of importance in the back-action-evading and quantum-nondemolition schemes [50], and in the study of decoherence of the quantum system [51–53].

From Louisell's approach [54], under the Markovian approximation we can obtain the following master equation describing the phase damping in the interaction picture (see Appendix A):

$$\hat{\dot{S}}(t) = -\gamma' [\hat{H}, [\hat{H}, \hat{S}(t_0)]] + \Delta\omega [\hat{H}, \hat{S}(t_0)] \hat{H} - \Delta\omega \hat{H} [\hat{H}, \hat{S}(t_0)], \tag{2}$$

where $\hat{S}(t)$ is the reduced density operator of the system in the interaction picture, γ' and $\Delta\omega$ are constants which depends on the temperature and the spectral density of the reservoir, and are given by

$$\gamma' = \frac{kT}{\hbar} \lim_{\omega \to 0} \frac{J(\omega)|C(\omega)|^2}{\omega}, \tag{3}$$

$$\Delta\omega = i\hbar \int_0^\infty d\omega \frac{J(\omega)|C(\omega)|^2}{2m(\omega)\omega^2}, \tag{4}$$

where $J(w)$ is the spectral density of the reservoir. In the derivation of the master equation (2), we have neglected the Lamb shift term.

It is easy to convert the master equation in the interaction picture (2) to that in the Schrödinger picture with this form:

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}(t)] - \gamma'[\hat{H}, [\hat{H}, \hat{\rho}(t)]] - \Delta\omega(\hat{H}[\hat{H}, \hat{\rho}(t)] - [\hat{H}, \hat{\rho}(t)]\hat{H}), \tag{5}$$

which can be expressed as the following compact form:

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}(t)] - \gamma[\hat{H}, [\hat{H}, \hat{\rho}(t)]], \tag{6}$$

where we have let $\gamma = \gamma' + \Delta\omega$. This is the desired master equation for the reduced density operator of the system in the Schrödinger picture under the Markovian approximation. This equation has been solved for the resonant multi-photon JCM [35]. In what follows we shall consider the exact solution of this equation for the two-mode JCM in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM).

Obviously, the master equation (6) has the same form as the Milburn's equation [55] under the diffusion approximation, but they come from completely different physical mechanisms. The former originates from the phase damping, while the latter is form the uncontinuous unitary evolution.

3 Analytic Solution for the nondegenerate bimodal multiquanta JCM with phase damping

In this section, we shall consider a Hamiltonian model that consists of two modes interacting with a three-level particle (atom or trapped ion) via Raman transition. We consider the nondegenerate case in which pairs of photons with two different frequencies are created or annihilated. The quantized radiation field is considered in the rotating wave approximation frame taking into account the effect of Stark shift. The atomic levels have identical parities such that each dipole is coupled with different modes of the field and to the set of intermediate states. If we assume that the intermediate states do not admit dipole transitions between themselves and the interaction field modes are far off-resonance from those intermediate states, then the particle can be seen as an effective two-level system by means of adiabatic elimination of the intermediate state [56,57].

The Hamiltonian for the system, in the rotating wave approximation, is written as:

$$\begin{aligned}\hat{H} &= \frac{\omega_\circ}{2}\hat{\sigma}_z + \sum_{j=1}^{2}\omega_j \hat{a}_j^\dagger \hat{a}_j + \hat{a}_1^\dagger \hat{a}_1 \beta_1 \mid g >< g \mid + \hat{a}_2^\dagger \hat{a}_2 \beta_2 \mid e >< e \mid \\ &+ \lambda(\hat{a}_1^{\dagger k_1}\hat{a}_2^{k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_+) \\ &= \omega_1[\hat{n}_1 + \frac{k_1}{2}(I+\hat{\sigma}_z)] + \omega_2[\hat{n}_2 + \frac{k_2}{2}(I-\hat{\sigma}_z)] - \frac{1}{2}(k_1\omega_1 + k_2\omega_2)I + \frac{\Delta}{2}\hat{\sigma}_z \\ &+ \beta_1\hat{n}_1 \mid g >< g \mid + \beta_2 \hat{n}_2 \mid e >< e \mid + \lambda(\hat{a}_1^{\dagger k_1}\hat{a}_2^{k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_+), \quad (\hbar = 1) \quad (7)\end{aligned}$$

where the detuning parameter $\Delta = \omega_\circ - k_1\omega_1 + k_2\omega_2$, $\hat{a}_j$ $(\hat{a}_j^\dagger)$ and $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ are the annihilation (creation) and number operators for the j-th mode, λ is the particle-field coupling constant, ω_1 and ω_2 are the field frequencies for the two modes, $\omega_\circ$ is the transition frequency of the particle (atom or trapped ion), $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the 'spin flip' operators which satisfy the relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$.

Now, we look for the exact solution for the density operator $\hat{\rho}(t)$ of the master equation (6) taking into account the Hamiltonian (7).

For convenience, we introduce three auxiliary superoperators [58–62] $\hat{J}$, $\hat{S}$ and $\hat{L}$ defined by

$$\exp(\hat{J}\tau)\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{(2\tau\gamma)^k}{k!}\hat{H}^k\hat{\rho}(t)\hat{H}^k \quad (8)$$

$$\exp(\hat{S}\tau)\hat{\rho}(t) = \exp(-i\hat{H}\tau)\hat{\rho}(t)\exp(i\hat{H}\tau) \quad (9)$$

$$\exp(\hat{L}\tau)\hat{\rho}(t) = \exp[-\gamma\tau\hat{H}^2]\hat{\rho}(t)\exp[-\gamma\tau\hat{H}^2]\,, \quad (10)$$

where the Hamiltonian $\hat{H}$ is given by (7).

It is straightforward to obtain the formal solution of the master equation (6) as follows:

$$\hat{\rho}(t) = \exp(\hat{J}t)\exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0)\,, \quad (11)$$

where $\hat{\rho}(0)$ is the density operator of the initial particle-field system.

We assume that the initial two modes of the field inside the cavity are in a superposition states and the particle in its excited state $\mid e >$, so that:

$$\begin{aligned}\hat{\rho}(0) &= \frac{1}{A}[\mid \alpha_1,\alpha_2 >< \alpha_1,\alpha_2 \mid + r^2 \mid -\alpha_1,-\alpha_2 >< -\alpha_1,-\alpha_2 \mid \\ &+ r(\mid \alpha_1,\alpha_2 >< -\alpha_1,-\alpha_2 \mid + \mid -\alpha_1,-\alpha_2 >< \alpha_1,\alpha_2 \mid)] \otimes \mid e >< e \mid, \quad (12)\end{aligned}$$

where $A = [1 + r^2 + 2r\exp(-2(\alpha_1^2 + \alpha_2^2))]$, with α_j $(j = 1, 2)$ are real. The parameter r can assume the values -1, 0 and 1, which corresponds to an odd coherent state, a coherent state and an even coherent state respectively. As we know, because the interference term in (12) have a rapid decay to a SM when we include dissipation, so we want to see how different would be the behaviour of the system if the input states are statistical mixture of the states $\mid \alpha_1,\alpha_2 >$ and $\mid -\alpha_1,-\alpha_2 >$, i.e.,

$$\hat{\rho}(0) = \frac{1}{2}[\mid \alpha_1,\alpha_2 >< \alpha_1,\alpha_2 \mid + \mid -\alpha_1,-\alpha_2 >< -\alpha_1,-\alpha_2 \mid] \otimes \mid e >< e \mid, \quad (13)$$

with $| \alpha_1, \alpha_2 >=| \alpha_1 > \otimes | \alpha_2 >$ defined by

$$| \alpha_1, \alpha_2 >= \sum_{n_1,n_2=0}^{\infty} Q_{n_1} Q_{n_2} | n_1, n_2 >= \sum_{n_1,n_2=0}^{\infty} Q_{n_1} Q_{n_2} | n_1 > \otimes | n_2 >, \qquad (14)$$

where $Q_{n_j} = e^{-\alpha_j^2/2} \frac{\alpha_j^{n_j}}{\sqrt{n_j!}}$, $(j = 1, 2)$.

In a two-dimensional basis for the particle the Hamiltonian (7) can be expressed as a sum of $(\hat{H}_\circ)$, which is diagonal in this basis and $(\hat{H}_I)$, which is not. It is easy to prove that $(\hat{H}_\circ)$ and $(\hat{H}_I)$ commute, i.e.

$$[\hat{H}_\circ, \hat{H}_I] = 0. \qquad (15)$$

Thus the representation now takes the form

$$\hat{H}_\circ = \begin{bmatrix} \hat{W}(\hat{n}_1 + k_1, \hat{n}_2) + \hat{\delta}_+(\hat{n}_1 + k_1, \hat{n}_2) & 0 \\ 0 & \hat{W}(\hat{n}_1, \hat{n}_2 + k_2) + \hat{\delta}_+(\hat{n}_1, \hat{n}_2 + k_2) \end{bmatrix} \qquad (16)$$

$$\hat{H}_I = \lambda \begin{bmatrix} [\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1 + k_1, n_2)] & \hat{a}_1^{k_1} \hat{a}_2^{\dagger k_2} \\ \hat{a}_1^{\dagger k_1} \hat{a}_2^{k_2} & -[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1, n_2 + k_2)] \end{bmatrix} \qquad (17)$$

with

$$\hat{W}(n_1, n_2 + k_2) = \omega_1 \hat{n}_1 + \omega_2(\hat{n}_2 + k_2), \quad \hat{\delta}_\pm(n_1, n_2 + k_2) = \frac{1}{2}[\beta_2(\hat{n_2} + k_2) \pm \beta_1 \hat{n_1}]. \qquad (18)$$

Similarly, the square of the Hamiltonian (7) can also be expressed as a sum of two matrices in the form

$$\hat{H}^2 = \hat{A} + \hat{B} \qquad [\hat{A}, \hat{B}] = 0, \qquad (19)$$

where $\hat{A}$ is diagonal in the form

$$\hat{A} = \begin{bmatrix} \hat{\Theta}^2(n_1 + k_1, n_2) & 0 \\ 0 & \hat{\Theta}^2(n_1, n_2 + k_2) \end{bmatrix} \qquad (20)$$

and $\hat{B}$ is given by

$$\hat{B} = 2\lambda \begin{bmatrix} \hat{\eta}(n_1 + k_1, n_2)\hat{\zeta}(n_1 + k_1, n_2) & \hat{a}_1^{k_1} \hat{a}_2^{\dagger k_2} \hat{\zeta}(n_1, n_2 + k_2) \\ \hat{\zeta}(n_1, n_2 + k_2)\hat{a}_1^{\dagger k_1} \hat{a}_2^{k_2} & -\hat{\eta}(n_1, n_2 + k_2)\hat{\zeta}(n_1, n_2 + k_2) \end{bmatrix} \qquad (21)$$

with

$$\hat{\eta}(n_1, n_2 + k_2) = [\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1, n_2 + k_2)],$$
$$\hat{\zeta}(n_1, n_2 + k_2) = [\hat{W}(n_1, n_2 + k_2) + \hat{\delta}_+(n_1, n_2 + k_2)] \qquad (22)$$

$$\hat{\mu}^2(n_1, n_2 + k_2) = \hat{\eta}^2(n_1, n_2 + k_2) + \hat{\nu}^2(n_1, n_2 + k_2) \qquad (23)$$

$$\hat{\nu}^2(n_1, n_2+k_2) = \hat{a}_1^{\dagger k_1}\hat{a}_1^{k_1}\hat{a}_2^{k_2}\hat{a}_2^{\dagger k_2} = \frac{\hat{n}_1!}{(\hat{n}_1-k_1)!}\frac{(\hat{n}_2+k_2)!}{\hat{n}_2!} \tag{24}$$

and

$$\hat{\Theta}^2(n_1, n_2+k_2) = \hat{\zeta}^2(n_1, n_2+k_2) + \lambda^2\hat{\mu}^2(n_1, n_2+k_2). \tag{25}$$

For convenience, we introduce the auxiliary operator $\hat{\rho}_2(t)$ defined by

$$\begin{aligned}\hat{\rho}_2(t) &= \exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0) \\ &= \exp(-i\hat{H}_I t)\exp(-\gamma t\hat{B})\hat{\rho}_1(t)\exp(-\gamma t\hat{B})\exp(i\hat{H}_I t).\end{aligned} \tag{26}$$

The auxiliary operator $\hat{\rho}_1(t)$ for the initial condition (13) defined by:

$$\hat{\rho}_1(t) = \begin{bmatrix} \mid \hat{\Psi}^+(t) >< \hat{\Psi}^+(t) \mid + \mid \hat{\Psi}^-(t) >< \hat{\Psi}^-(t) \mid & 0 \\ 0 & 0 \end{bmatrix}, \tag{27}$$

where

$$\mid \hat{\Psi}^{\pm}(t) >= \frac{1}{\sqrt{2}}\exp\Big[-\gamma t\hat{\Theta}^2(n_1+k_1, n_2)\Big]\exp[-i\hat{\zeta}(n_1+k_1, n_2)t] \mid \pm\alpha_1, \pm\alpha_2 > . \tag{28}$$

While for the initial (12) the operator $\hat{\rho}_1(t)$ defined by:

$$\hat{\rho}_1(t) = \begin{bmatrix} [\mid \hat{\Psi}^+(t) > +r \mid \hat{\Psi}^-(t) >][< \hat{\Psi}^+(t) \mid +r < \hat{\Psi}^-(t) \mid] & 0 \\ 0 & 0 \end{bmatrix} \tag{29}$$

with

$$\mid \hat{\Psi}^{\pm}(t) >= \frac{1}{\sqrt{A}}\exp\Big[-\gamma t\hat{\Theta}^2(n_1+k_1, n_2)\Big]\exp[-i\hat{\zeta}(n_1+k_1, n_2)t] \mid \pm\alpha_1, \pm\alpha_2 > . \tag{30}$$

The powers of the operator $\hat{B}$ can be written as

$$\hat{B}^{2k} = \begin{bmatrix} [2\lambda\hat{\zeta}(n_1+k_1, n_2)\hat{\mu}(n_1+k_1, n_2)]^{2k} & 0 \\ 0 & [2\lambda\hat{\zeta}(n_1, n_2+k_2)\hat{\mu}(n_1, n_2+k_2)]^{2k} \end{bmatrix} \tag{31}$$

$$\hat{B}^{2k+1} = \begin{bmatrix} \hat{\eta}_1\frac{[2\lambda\hat{\zeta}_1\hat{\mu}_1]^{2k+1}}{\hat{\mu}_1} & \hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2} \\ \frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & -\hat{\eta}_2\frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2} \end{bmatrix} \tag{32}$$

then we can write the operator $\exp[-\gamma t\hat{B}]$ in the form

$$\exp[-\gamma t\hat{B}] = \begin{bmatrix} \hat{X}_1(t) - \hat{\eta}_1\frac{\hat{Y}_1(t)}{\hat{\mu}_1} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{Y}_2(t)}{\hat{\mu}_2} \\ -\frac{\hat{Y}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{X}_2(t) + \hat{\eta}_2\frac{\hat{Y}_2(t)}{\hat{\mu}_2} \end{bmatrix}, \tag{33}$$

where

$$\hat{X}_2(t) = \cosh[2\lambda t\gamma\hat{\zeta}_2\hat{\mu}_2], \qquad \hat{Y}_2(t) = \sinh[2\lambda t\gamma\hat{\zeta}_2\hat{\mu}_2]. \tag{34}$$

Similarly, we can write the operator $\exp[-i\hat{H}_I t]$ in the two-dimensional basis for the particle as

$$\exp[-i\hat{H}_I t] = \begin{bmatrix} \hat{C}_1(t) - i\hat{\eta}_1 \frac{\hat{S}_1(t)}{\hat{\mu}_1} & -i\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{S}_2(t)}{\hat{\mu}_2} \\ i\frac{\hat{S}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{C}_2(t) + i\hat{\eta}_2\frac{\hat{S}_2(t)}{\hat{\mu}_2} \end{bmatrix} \tag{35}$$

with

$$\hat{C}_2(t) = \cos[\lambda t\hat{\mu}(n_1, n_2 + k_2)] \quad and \quad \hat{S}_2(t) = \sin[\lambda t\hat{\mu}(n_1, n_2 + k_2)]. \tag{36}$$

Then,

$$\exp[-i\hat{H}_I t]\exp\Big(-\gamma t\hat{B}\Big) = \begin{bmatrix} \hat{R}_1(t) - \eta_1\frac{\hat{V}_1(t)}{\hat{\mu}_1} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{V}_2(t)}{\hat{\mu}_2} \\ -\frac{\hat{V}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{R}_2(t) + \eta_2\frac{\hat{V}_2(t)}{\hat{\mu}_2} \end{bmatrix}, \tag{37}$$

where

$$\hat{R}_2(t) = \hat{C}_2(t)\hat{X}_2(t) + i\hat{S}_2(t)\hat{Y}_2(t), \qquad \hat{V}_2(t) = \hat{C}_2(t)\hat{Y}_2(t) + i\hat{S}_2(t)\hat{X}_2(t). \tag{38}$$

Note that in the above equations (32–38), we have used the subscript 1 instead of $(n_1 + k_1, n_2)$ and 2 instead of $(n_1, n_2 + k_2)$.

Now, we can obtain an explicit expression for the operator $\hat{\rho}_2(t)$ for the two initial (13) and (12) as follows:

Substituting (27) and (37) into (26), we obtain an explicit expression for the operator $\hat{\rho}_2(t)$ for the initial (13) as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\Psi}_{11}^{+}(t) + \hat{\Psi}_{11}^{-}(t) & \hat{\Psi}_{12}^{+}(t) + \hat{\Psi}_{12}^{-}(t) \\ \hat{\Psi}_{21}^{+}(t) + \hat{\Psi}_{21}^{-}(t) & \hat{\Psi}_{22}^{+}(t) + \hat{\Psi}_{22}^{-}(t) \end{bmatrix}, \tag{39}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}^{\pm}(t) = \mid \hat{\Psi}_i^{\pm}(t) >< \hat{\Psi}_j^{\pm}(t) \mid (i, j = 1, 2) \tag{40}$$

with

$$\mid \hat{\Psi}_1^{\pm}(t) >= \left[\hat{R}(n_1 + k_1, n_2, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2 + k, t)}\right] \mid \hat{\Psi}^{\pm}(t) > \tag{41}$$

$$\mid \hat{\Psi}_2^{\pm}(t) >= -\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2}\frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2 + k, t)}] \mid \hat{\Psi}^{\pm}(t) >, \tag{42}$$

where $\mid \hat{\Psi}^{\pm}(t) >$ is given by (28). Also by Substituting (29) and (37) into (26), we obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{bmatrix}, \tag{43}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}(t) =\mid \hat{\Psi}_i(t) >< \hat{\Psi}_j(t) \mid (i,j=1,2) \tag{44}$$

with

$$\mid \hat{\Psi}_1(t) >= \left[\hat{R}(n_1+k_1,n_2,t) - \hat{\eta}_1 \frac{\hat{V}(n_1+k_1,n_2,t)}{\hat{\mu}(n_1+k_1,n_2)}\right][\mid \hat{\Psi}^+(t) > +r \mid \hat{\Psi}^-(t) >] \tag{45}$$

$$\mid \hat{\Psi}_2(t) >= -\hat{a}_1^{\dagger k_1}\hat{a}_2^{k_2}\frac{\hat{V}(n_1+k_1,n_2,t)}{\hat{\mu}(n_1+k_1,n_2)}][\mid \hat{\Psi}^+(t) > +r \mid \hat{\Psi}^-(t) >], \tag{46}$$

where $\mid \hat{\Psi}^{\pm}(t) >$ is given by (30). By using the definition of the superoperator $\hat{J}$, it is straightforward to obtain the action of the operator $\exp(\hat{J}t)$ on the density operator $\hat{\rho}_2(t)$ as follows [58–62]:

$$\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{(2\gamma t)^k}{k!}\hat{H}^k\hat{\rho}_2(t)\hat{H}^k = \begin{bmatrix} \hat{\rho}_{11}(t) & \hat{\rho}_{12}(t) \\ \hat{\rho}_{21}(t) & \hat{\rho}_{22}(t) \end{bmatrix}, \tag{47}$$

with

$$\hat{\rho}_{ij}(t) = \sum_{k=0}^{\infty}\frac{(2\gamma\tau)^k}{k!}\hat{M}_{ij}^{(k)}(t), \tag{48}$$

where the Hamiltonian $\hat{H}$ is given by (7) and the operator $\hat{\rho}_2$ is given by (39) and (43) for SM and SS, respectively. Making use of this solution, we can evaluate mean values of operators of interest. In what follows, we will use it to study the influence of the phase damping on dynamics of the particle (atom or trapped ion) and the cavity field in the JCM.

4 Influence of the phase damping on nonclassical properties of the system

In this section, we investigate the influence of the phase damping on nonclassical properties of the particle and the field in the multiquanta two-mode JCM in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM).

4.1 Population inversion

It is well known that in the JCM the quantum coherences which are built up during the interaction between the field and the particle significantly affect the dynamics of the particle [11,63–66]. The existence of the quantum coherences is the reason why one can observe collapses and revivals of the population inversion of the particle. Now we evaluate the population inversion in the multiquanta JCM. The population inversion is defined as the expectation value of the operator $\hat{\sigma}_z$, i.e.

$$< \hat{\sigma}_z(t) >= Tr[\hat{\rho}(t)\hat{\sigma}_z]. \tag{49}$$

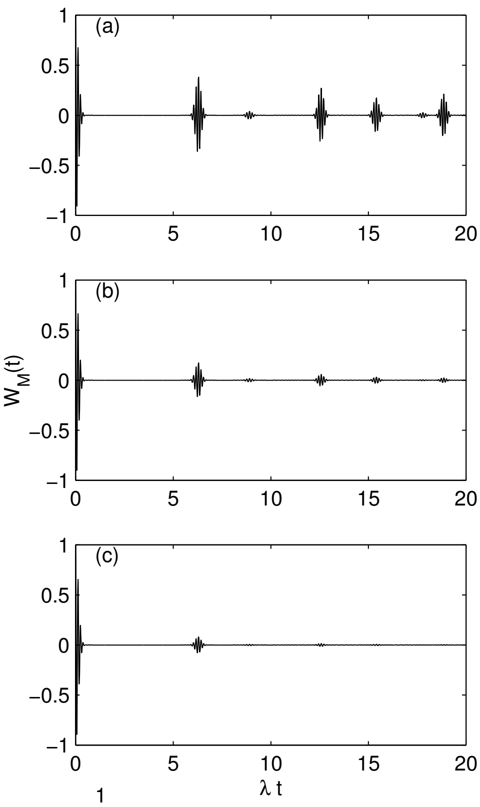

Fig. 1. Population inversion $<\sigma_z(t)>$ as a function of scaled time λt of the particle initially prepared in the excited state and the field initially prepared in a statistical mixture of coherent states $|\alpha_1, \alpha_2>$ and $|-\alpha_1, -\alpha_2>$ $(\bar{n_1} = \bar{n_2} = 25)$ for various values of the parameter γ: (a) $\gamma = 10^{-6}$, (b) $\gamma = 5 \times 10^{-5}$ and (c) $\gamma = 10^{-4}$.

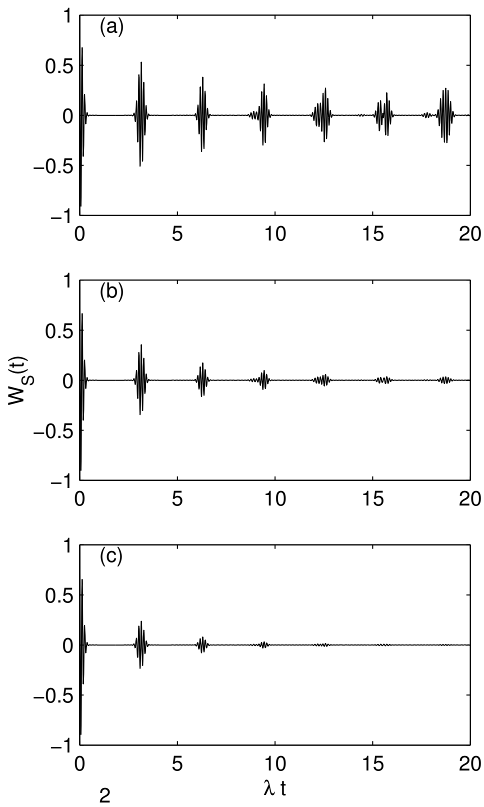

Fig. 2. The same as in Fig. 1 but with the field initially prepared in a superposition state (even coherent state).

By using equation (47), we can express equation (49) in the following form:

$$< \hat{\sigma}_z(t) >= \\ = \sum_{n_1,n_2,k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \Big\{ < n_1, n_2 \mid \hat{M}_{11}^{(k)}(t) \mid n_1, n_2 > - < n_1, n_2 \mid \hat{M}_{22}^{(k)}(t) \mid n_1, n_2 > \Big\}. \quad (50)$$

If the field is initially prepared in a SM of states $|\ \alpha_1, \alpha_2 >$ and $|\ -\alpha_1, -\alpha_2 >$ (13), the

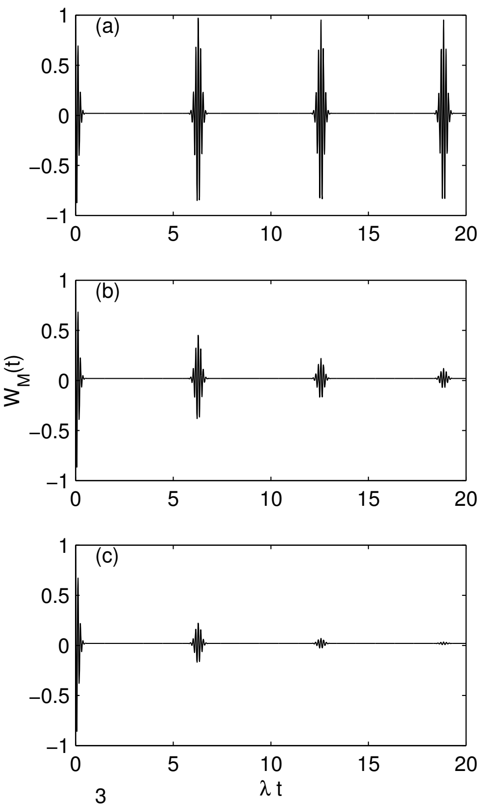

Fig. 3. The same as in Fig. 1 but in the presence of Stark shift $\beta_1/\lambda = \beta_2/\lambda = 1.0$.

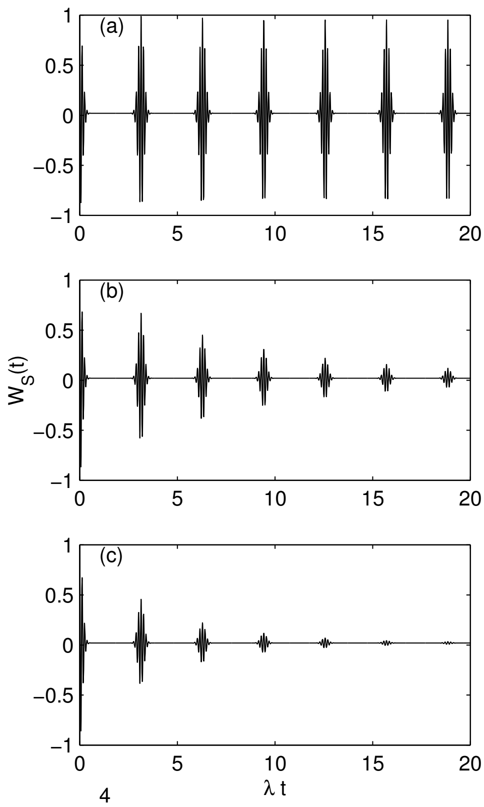

Fig. 4. The same as in Fig. 2 but in the presence of Stark shift $\beta_1/\lambda = \beta_2/\lambda = 1.0$.

population inversion will be:

$$\begin{aligned} W_M(t) &= \sum_{n_1,n_2=0}^{\infty} \frac{|\, Q_{n_1} \,|^2 |\, Q_{n_2} \,|^2}{\mu^2(n_1+k_1,n_2)} \Big\{ \eta^2(n_1+k_1,n_2) + \nu^2(n_1+k_1,n_2) \\ &\quad \times \exp\Big[-4\lambda^2\gamma t \mu^2(n_1+k_1,n_2)\Big] \cos 2\lambda t \mu(n_1+k_1,n_2) \Big\}. \end{aligned} \tag{51}$$

As we expected, the population inversion is the same as if the input field was a coherent state. However, if the field is initially prepared in a superposition of coherent states SS (12), the popu-

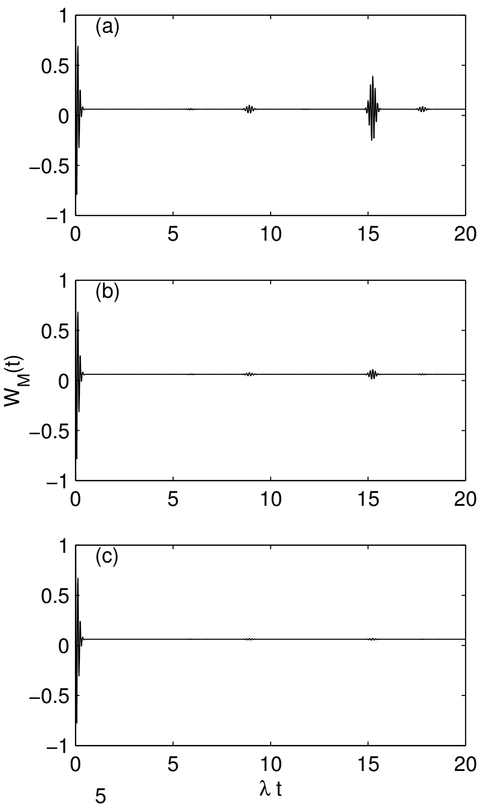

Fig. 5. The same as in Fig. 1 but with the presence of Stark shift $\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$.

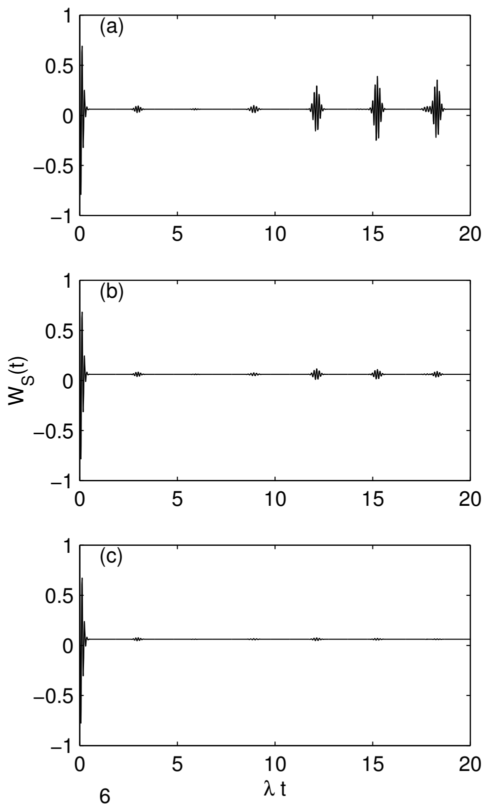

Fig. 6. The same as in Fig. 2 but with the presence of Stark shift $\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$.

lation inversion will be:

$$W_S(t) = \sum_{n_1,n_2=0}^{\infty} \frac{|\, Q_{n_1}\,|^2|\, Q_{n_2}\,|^2}{\mu^2(n_1+k_1,n_2)} \frac{[1+r(-1)^{n_1+n_2}]^2}{[1+r^2+2r\exp(-2(\alpha_1^2+\alpha_2^2))]} \Big\{ \eta^2(n_1+k_1,n_2)$$
$$+\nu^2(n_1+k_1,n_2)\exp\Big[-4\lambda^2\gamma t \mu^2(n_1+k_1,n_2)\Big]\cos 2\lambda t \mu(n_1+k_1,n_2)\Big\}. \quad (52)$$

Now, we discuss the general behaviour of the population inversion for the multi-quanta JCM, when the particle (atom or trapped ion) initially starts in the excited state and the field initially in a mixture state SM or a superposition of coherent states SS (even coherent state ($r = 1$)).

The numerical result are shown in Figs. (1–6), for various values of the parameter γ, and different values of the Stark shift parameter and fixed initial mean numbers of quanta $\bar{n}_1$ and $\bar{n}_2$ for two quanta ($k_1 = k_2 = 1$).

In Fig. 1 and 2, we plot the the population inversion (in the absence of Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 0$)) for three values of the parameter γ with the fixed initial mean numbers of quanta $\bar{n}_1$ and $\bar{n}_2$ in the case of the exact resonance, i.e., the detuning parameter $\frac{\Delta}{2\lambda} = 0$, when the field initially in a mixture state SM and in a superposition of coherent states SS (even coherent state ($r = 1$)), respectively. We see that for a superposition of coherent states SS (even coherent state ($r = 1$)) (see Fig. 2), the revival time will be approximately half of the revival time for the SM. This an effect due to the interference between the two coherent states in the superposition, and can be understood looking at the photon number distribution of the initial fields.

In Figs. (3, 4) and (5, 6), we plotted the population inversion in the presence of Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 1.0$) and ($\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$), respectively.

In the case described in Figs. (3, 4), the Stark shift parameter is ($\beta_1/\lambda = \beta_2/\lambda = 1.0$), which corresponds to the case in which the two levels of the particle are equally strongly coupled with the intermediate relay level. From these figures, we see the Stark shift leads to increasing of the values of the atomic revivals of the population inversion (see Figs. 3, 4).

In Figs. (5, 6), we show the cases in which the two levels have unequal Stark shift ($\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$). We see the Stark shift leads to decreasing of the values of the atomic revivals of the population inversion (see Figs. 5, 6).

Also, these figures show that with the increasing of the parameter γ, i.e., with a more rapid suppression of quantum coherence we can observe rapid deterioration of revivals of the population inversion. Which means that the dacay of quantum coherence is due to the very specific time evolution described by the master equation (6), i.e., due to the phase damping.

Obviously, when phase damping vanishes (without damping ($\gamma = 0$)), the population inversion reduces to the well known expression for the population inversion in the off-resonant non-degenerate multi-photon JCM governed by the von Neumann equation.

4.2 Oscillations of the number distribution

It is known that oscillations of the number distribution of the quanta in the JCM is a kind of the nonclassical effects of the cavity field. To see the influence of phase damping on this kind of nonclassical effects, we discuss statistics in the field modes. The reduced density operator of the cavity field can be obtained by taking the trace of the total density operator $\hat{\rho}(t)$ over the atomic states, that is ,$\hat{\rho}_F = Tr_A \hat{\rho}(t)$. Then the probability distribution function for finding n_j quanta in the j-th mode is calculated from the formula

$$P(n_1, n_2 + k_2; t) = \\ = \sum_{n_1, n_2, k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \left\{ < n_1, n_2 \mid \hat{M}_{11}^{(k)}(t) \mid n_1, n_2 > + < n_1, n_2 \mid \hat{M}_{22}^{(k)}(t) \mid n_1, n_2 > \right\}. \quad (53)$$

If the field is initially prepared in a SM of states (13), we find that

$$
\begin{aligned}
&P_M(n_1, n_2+k_2, t) =< n_1, n_2 \mid \hat{\rho}_F(t) \mid n_1, n_2 > \\
&= \frac{1}{2} \mid Q_{n_1} \mid^2 \mid Q_{n_2} \mid^2 \left\{ 1 + \frac{\eta^2(n_1+k_1, n_2)}{\mu^2(n_1+k_1, n_2)} \right. \\
&+ \frac{\nu^2(n_1+k_1, n_2)}{\mu^2(n_1+k_1, n_2)} \exp\left[-4\lambda^2\gamma t \mu^2(n_1+k_1, n_2) \right] \\
&\left. \times \cos 2\lambda t \mu(n_1+k_1, n_2) \right\} + \frac{1}{2} \mid Q_{n_1-k_1} \mid^2 \mid Q_{n_2-k_2} \mid^2 \left\{ 1 - \frac{\eta^2(n_1, n_2+k_2)}{\mu^2(n_1, n_2+k_2)} \right. \\
&\left. - \frac{\nu^2(n_1, n_2+k_2)}{\mu^2(n_1, n_2+k_2)} \exp\left[-4\lambda^2\gamma t \mu^2(n_1, n_2+k_2) \right] \cos 2\lambda t \mu(n_1, n_2+k_2) \right\}. \qquad (54)
\end{aligned}
$$

If the field is initially prepared in a superposition of coherent states (12), we find:

$$
\begin{aligned}
&P_S(n_1, n_2+k_2; t) = \frac{1}{2} \mid Q_{n_1} \mid^2 \mid Q_{n_2} \mid^2 \frac{[1 + r(-1)^{n_1+n_2}]^2}{[1 + r^2 + 2r\exp(-2(\alpha_1^2 + \alpha_2^2))]} \\
&\times \left\{ 1 + \frac{\eta(n_1+k_1, n_2)^2}{\mu^2(n_1+k_1, n_2)} + \frac{\nu^2(n_1+k_1, n_2)}{\mu^2(n_1+k_1, n_2)} \right. \\
&\left. \times \exp\left[-4\lambda^2\gamma t \mu^2(n_1+k_1, n_2) \right] \cos 2\lambda t \mu(n_1+k_1, n_2) \right\} \\
&+ \frac{1}{2} \mid Q_{n_1-k_1} \mid^2 \mid Q_{n_2+k_2} \mid^2 \frac{[1 + r(-1)^{n_1-k_1+n_2-k_2}]^2}{[1 + r^2 + 2r\exp(-2(\alpha_1^2 + \alpha_2^2))]} \\
&\times \left\{ 1 - \frac{(\eta_2)^2}{\mu^2(n_1, n_2+k_2)} - \frac{\nu^2(n_1, n_2+k_2)}{\mu^2(n_1, n_2+k_2)} \right. \\
&\left. \times \exp\left[-4\lambda^2\gamma t \mu^2(n_1, n_2+k_2) \right] \cos 2\lambda t \mu(n_1, n_2+k_2) \right\}, \qquad (55)
\end{aligned}
$$

where $\mid Q_{n_1} \mid^2$ and $\mid Q_{n_2} \mid^2$ are the initial values for the distribution function given by equation (16). We can use the time-dependent number distribution that is obtained in equation (54) to evaluate some quantities relevant to the field statistics. For example, the mean number of quanta in the i-th mode are found to be for the two cases:

$$
\begin{aligned}
< n_i(t) >_M = \bar{n}_i + \frac{k_i}{2} \sum_{n_1, n_2=0}^{\infty} \mid Q_{n_1} \mid^2 \mid Q_{n_2} \mid^2 \frac{\nu^2(n_1+k_1, n_2)}{\mu^2(n_1+k_1, n_2)} \\
\times \left(1 - \exp\left[-4\lambda^2\gamma t \mu^2(n_1+k_1, n_2) \right] \cos 2\lambda t \mu(n_1+k_1, n_2) \right) \qquad (56)
\end{aligned}
$$

and

$$< n_i(t) >_S = \bar{n}_i + \frac{k_i}{2} \sum_{n_1,n_2=0}^{\infty} | Q_{n_1} |^2 | Q_{n_2} |^2 \frac{[1+r(-1)^{n_1+n_2}]^2}{[1+r^2+2r\exp(-2(\alpha_1^2+\alpha_2^2))]}$$
$$\times \left\{ \frac{\nu^2(n_1+k_1,n_2)}{\mu^2(n_1+k_1,n_2)} \left(1 - \exp\left[-4\lambda^2\gamma t\mu^2(n_1+k_1,n_2) \right] \right.\right.$$
$$\left.\left. \times \cos 2\lambda t\mu(n_1+k_1,n_2) \right) \right\}. \quad (57)$$

From the above expressions, we see that the phase damping term in the master equation (6) leads to the appearance of the decay factors $\exp[-4\lambda^2\gamma t\mu^2(n_1+k_1,n_2)]$ in (54–57), which are responsible for the weakening of the oscillatory behaviour of the mean number of quanta in the field, where it weakened with the increase of the phase damping parameter γ.

Obviously, when the phase damping vanishes, i.e, $\gamma = 0$, eqs. (54–57) reduce to the usual expressions of the oscillations of the number distribution and the intensity of the cavity field governed by the Schrödinger dynamics.

4.3 Conclusion

In this paper we have studied the phase damping for two-mode JCM in the presence of Stark shift, by introducing a master equation (6) describing phase damping under the Markovian approximation. We have considered an effective Hamiltonian (7) describing the interaction between a two-level particle (atom or trapped ion) and a two-mode field through multiquanta. An analytic solution for the master equation for the multiquanta model with detuning has been obtained. The density operator is then used to study the influence of the phase damping and Stark shift on non-classical properties in the JCM, such as population inversion and photon number statistics. It is shown that phase damping suppresses nonclassical effects of the cavity field in the JCM, also we see the Stark shift leads to increasing of the values of the atomic revivals of the population inversion in SM and SS when the two levels of the particle are equally strongly coupled with the intermediate relay level, while it leads to decreasing of the values of the atomic revivals of the population inversion when the two levels have unequal Stark shift.

Appendix A

In this appendix, we present the derivation of the master equation (2) in the interaction picture. For convenience, we rewrite the total Hamiltonian (1) in the form

$$\hat{H}_T = \hat{H} + \hat{H}_R + \hat{V}, \quad (A.1)$$

where

$$\hat{H}_R = \sum_i \hbar\omega_i \hat{b}_i^\dagger \hat{b}_i, \quad \hat{V} = \hat{H} \sum_{i=1}^{3} \hat{F}_i, \quad (A.2)$$

with

$$\hat{F}_1 = \hbar \sum_i C_i \hat{b}_i, \quad \hat{F}_2 = \hbar \sum_i C_i \hat{b}_i^\dagger, \quad \hat{F}_3 = \hbar^2 \hat{H} \sum_i \frac{|C_i^2|}{2m_i\omega_i^2}, \quad (A.3)$$

where $\hat{b}_i$ and $\hat{b}_i^\dagger$ are the boson annihilation and creation operators for the reservoir.

The total density operator for the system and the reservoir satisfies the Liouville-von Neuman equation in the interaction picture [54],

$$\frac{d\hat{\chi}}{dt} = -\frac{i}{\hbar}[\hat{V}(t), \hat{\chi}], \tag{A.4}$$

which can be expressed as the following equivalent integral equation:

$$\hat{\chi}(t) = \hat{\chi}(0) + \frac{1}{i\hbar}\int_{t_0}^{t}[\hat{V}(t'), \hat{\chi}(0)]dt' + \left(\frac{1}{i\hbar}\right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt''[\hat{V}(t'), [\hat{V}(t''), \hat{\chi}(t'')]]. \tag{A.5}$$

Tracing both sides of the above equation over the reservoir, we obtain the change in the reduced density operator $s = TR_R\hat{\chi}$ for the system

$$\begin{aligned}\hat{S}(t) - \hat{S}(t_0) = &\frac{1}{i\hbar}\int_{t_0}^{t} dt' Tr_R[\hat{H}\sum_i \hat{F}_i(t'), \hat{S}(t_0)\hat{f}(t_0)] \\ &+ (\frac{1}{i\hbar})^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' Tr_R[\hat{V}(t'), [\hat{V}(t''), \hat{\chi}(t'')]],\end{aligned} \tag{A.6}$$

where we assumed that the system and reservoir are uncoupled, and $\hat{\chi}(0)$ can be factorized as $\hat{\chi}(0) = \hat{S}(t_0)\hat{f}(0)$ at the initial time.

We assume that the reservoir is initially in thermal equilibrium at a temperature T. Then it is straightforward to see that

$$< \hat{F}_1 >_R = < \hat{F}_2 >_R = 0, \quad < \hat{F}_3 >_R = \Delta\omega\hat{H}, \tag{A.7}$$

where

$$\Delta\omega = i\hbar \int_0^\infty d\omega \frac{\hat{J}(\omega)|\hat{C}(\omega)|^2}{\omega}, \tag{A.8}$$

where $\hat{J}(\omega)$ is the spectral density of the reservoir.

The total density operator [54,55] can be expressed as

$$\hat{\chi}(t) = \hat{S}(t)\hat{f}(t) + \hat{\chi}_c(t), \tag{A.9}$$

where $\hat{\chi}_c(t)$ represents the correlation between the system and the reservoir described by the density operator $\hat{f}(t)$ at time t. The reservoir assumption allows as to take $\hat{f}(t) = \hat{f}(0)$. Then (A.6) becomes

$$\begin{aligned}\hat{S}(t) - \hat{S}(t_0) = &-(t - t_0)\Delta\omega(\hat{H}[\hat{H}, \hat{S}(t_0)] - [\hat{H}, \hat{S}(t_0)]\hat{H}) + (\frac{1}{i\hbar})^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \\ &\left(Tr_R[\hat{V}(t'), [\hat{V}(t''), \hat{S}(t'')\hat{f}(0)]] + Tr_R[\hat{V}(t'), [\hat{V}(t''), \hat{\chi}_c(t'')]]\right),\end{aligned} \tag{A.10}$$

where the second term is a higher-order than the first one and can be neglected for the weak damping case [67]. After dropping the second term in (A.10), differentiating both sides of (A.10) with respect to time, we have the equation

$$\dot{\hat{S}}(t) = -\Delta\omega(\hat{H}[\hat{H},\hat{S}(t_0)] - [\hat{H},\hat{S}(t_0)]\hat{H}) + \left(\frac{1}{i\hbar}\right)^2 \int_{t_0}^{t} dt' Tr_R[\hat{V}(t),[\hat{V}(t'),\hat{S}(t')\hat{f}(0)]], \tag{A.11}$$

where the t' integration is over the correlation function of the reservoir, which are characterized by a time which is short but finite.

Substituting the interaction term $\hat{V} = \sum_i \hat{H}\hat{F}_i$ with a change of variable $t' = t - \tau$ into (A.11), neglecting the terms with the order of $\hbar^4$, under the Markovian approximation $\hat{S}(t') = \hat{S}(t)$, we can rewrite (A.11) as

$$\dot{\hat{S}}(t) = -\Delta\omega(\hat{H}[\hat{H},\hat{S}(t_0)] - [\hat{H},\hat{S}(t_0)]\hat{H}) - [\hat{H},[\hat{H},\hat{S}(t_0)]] \sum_{i,j=1}^{2} \int_0^\infty d\tau \hat{W}_{ij}(\tau) - [\hat{H},\hat{S}(t_0)]\hat{H} \sum_{i,j=1}^{2} \int_0^\infty d\tau [\hat{W}_{ij}(\tau) - \hat{W}'_{ij}(\tau)], \tag{A.12}$$

where the correlation function of the reservoir are defined by

$$\hat{W}_{ij}(\tau) = Tr_R F_i(t) F_j(t-\tau) f(0) =< F_i(t) F_j(t-\tau) >_R, \tag{A.13}$$

$$\hat{W}'_{ij}(\tau) = Tr_R F_i(t-\tau) F_j(t) f(0) =< F_i(t-\tau) F_j(t) >_R, \tag{A.14}$$

where all operators are in the interaction picture. In derivation of (A.12), we used $W_{i3}(\tau) = W_{3i}(\tau) = W'_{i3}(\tau) = W'_{3i}(\tau) = 0$ for $i = 1$ and 2. The correlation functions needed in (A.12) were calculated in Ref. [54].

It is not difficult to see that under the higher-temperature approximation, in which the temperature is assumed high enough to so that the Markovion approximation is valid, (A.12) reduces to

$$\dot{S}(t) = -\gamma'[\hat{H},[\hat{H},[\hat{S}(t_0)]] + \Delta\omega[\hat{H},\hat{S}(t_0)]\hat{H} - \Delta\omega\hat{H}[\hat{H},\hat{S}(t_0)]\,, \tag{A.15}$$

which is the master equation in the interaction picture, where we have neglected the Lamb shift term, and γ' is a constant which depends on the temperature and the spectral density of the reservoir, which given by (3).

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