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RENORMALIZATION GROUP AND SPECIAL RELATIVITY IN THE FOCK SPACE[1]

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Hamiltonian formulation of local quantum field theory in the Fock space requires renormalization and even if a method acceptable in quantum mechanics were found, one could ask how special relativity could be obtained from an effective theory with a small range of momenta while Lorentz boosts change momenta by arbitrarily large amounts. This talk explains the principles of the renormalization group procedure for Hamiltonians of effective particles in quantum field theory, and describes how this procedure leads to the Poincare algebra in the Fock space.

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1 Introduction

The problem addressed in this note can be stated as follows. Lorentz transformation in Minkowski space can change a particle momentum by an arbitrary large amount. How can one then construct a relativistic theory of effective particles with a finite renormalization group (RG) cutoff on their momenta? The answer described here is provided by a new RG procedure for particles in the Fock space [1], based on the similarity RG idea [2].

The key elements of the procedure are designed using canonical quantum field theory (QFT), where one derives all 10 Poincaré generators expressed in terms of creation and annihilation operators for effective particles [3]. To start with, the diverging (in all QFTs of interest) bare Poincaré algebra is regularized by limiting momentum transfers in the interaction terms, without limiting the individual particle momenta. Lorentz boost symmetry is maintained by using light-front quantization [4], based on the front form of Hamiltonian dynamics [5], where the boost generators do not involve interactions. Rotational symmetry is dynamical instead and is violated when one introduces cutoffs on the particle momenta. Therefore, the cutoffs are carefully imposed only on the relative momenta of interacting particles and RG is used to evolve these cutoffs down to finite values in a unitary fashion that allows the Poincaré algebra to be satisfied once some counterterms (calculable in the same scheme) are introduced in the regularized bare theory.

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The main point is to rewrite a generator, say a Hamiltonian H expressed in terms of operators a and $a^\dagger$ for bare particles, in terms of operators a_λ and $a_\lambda^\dagger$ for the effective particles, and to work in the Fock space basis built using $a_\lambda^\dagger$ instead of $a^\dagger$. λ is the RG parameter. It has dimension of mass, plays the role of a form-factor width in momentum space in interaction vertices, and ranges from infinity for point-like virtual particles in the case of bare theory down to any finite value in an effective theory labeled by λ, which is written in terms of the effective particles of spatial extension $\sim 1/\lambda$. The new RG structure is generated through a unitary transformation,

$$a_\lambda^\dagger = U_\lambda^\dagger a^\dagger U_\lambda \ . \tag{1}$$

The transformation U_λ is precisely designed so that coefficients of the products of operators a_λ and $a_\lambda^\dagger$ in the Hamiltonian H do contain the vertex form factors, f_λ, which depend on the momentum transfers. To fully understand the construction one has to consult Ref. [3]. Although there is not enough room here for the explanation, the required material is available in Ref. [3] and literature quoted there. Eventually, when the form factors limit momentum transfers from above by λ, an effective particle dynamics with a finite RG parameter is obtained.

2 Example of an effective particle interaction term

In order to see the characteristic features of an effective theory, the best thing to do is to analyze a simple example. Consider a generic scalar field theory that contains a term $g\phi^3$. g denotes a bare coupling constant in a standard fashion, i.e. as in the Lagrangian term $g\phi^3$ of a textbook case, and ϕ is the field strength. The canonical bare Hamiltonian would contain in this case a divergent interaction term of the form

$$H_I = 3g \int [123] \delta(1+2-3) a_1^\dagger a_2^\dagger a_3 \ . \tag{2}$$

The integration measure $[123]$ stands for Lorentz invariant integrations over three four-momentum variables, k_1, k_2, and k_3, with $k_i^2 = m^2$ for $i = 1, 2, 3$, and m denotes the bare mass parameter. The δ-function argument is written in an abbreviated form that, in fact, means $\vec{k}_1 + \vec{k}_2 - \vec{k}_3$, the arrow denoting three-vectors, and the same convention is applied to the labels of the creation and annihilation operators.

In this case, the renormalized Hamiltonian would contain a term (here we only write a result obtained in 1st order perturbation theory for solving the RG equations for U_λ)

$$H_{I\lambda} = 3g \int [123] \delta(1+2-3) f_\lambda(1,2) a_{1\lambda}^\dagger a_{2\lambda}^\dagger a_{3\lambda} \ . \tag{3}$$

The form factor is given by [3]

$$f_\lambda(1,2) = f(M_{12}^2/\lambda^2) \ , \tag{4}$$

where

$$M_{12}^2 = (k_1 + k_2)^2 \tag{5}$$

is the interaction-free invariant mass of the two created effective particles, and $f(x) = \exp -x^2$.

It is now seen that the effective interaction term cannot change momenta of the interacting particles by more than about λ. All generators of the effective Poincaré algebra are limited that way. Since the unitary RG rotation U_λ is designed to guarantee that the effective Hamiltonian is limited by such form factors to all orders of perturbation theory (see the original papers), the effective particle dynamics can be well contained within the range of relative momenta limited by the finite RG parameter λ. At the same time, the generators with finite λ are made insensitive to the regularization that spoiled the Poincaré algebra commutation relations in the bare QFT. Once the regularization dependence is removed, the commutation relations are restored. Thus, symmetries of special relativity are recovered with a finite RG cutoff λ. This is the place where one can start making contact with physics, which should not depend on the motion of an observer.

3 Conclusion

The key result is that eigenstates of the effective Hamiltonian H_λ have wave functions in the effective particle Fock space basis limited to the range of relative momenta on the order of λ (if the interactions are not extremely strong). The renormalized generators of the Poincaré algebra can be exponentiated and finite Lorentz transformations can be explicitly performed on all the states order by order in perturbation theory.

An example of how it is done in detail up to second-order terms is described in Ref [6]. Thus, the new RG can be employed in formulating a theory of effective particles in agreement with principles of quantum mechanics and special relativity, providing explicit expressions for quantum states and their relativistic transformations in the Fock space.

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