ON WILSONIAN FLOWS IN GAUGE THEORIES[1]

J. M. Pawlowski[2]
Institut für Theoretische Physik III, Universität Erlangen, Staudtstraße 7, 91058 Erlangen, Germany

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An Exact Renormalisation Group (ERG) approach to non-Abelian gauge theories is discussed. We focus on the derivation of universal beta-functions and the choice of the initial effective action, the latter being a key input in the approach. To that end we establish the map between the renormalisation group (RG) scaling of the full theory and the anomalous scaling in an ERG approach. Then this map is used to sketch the derivation of the two loop β-function within a simple straightforward calculation. The implications for the choice of the initial effective action are discussed.

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Introduction

The investigation of the infra-red sector of non-Abelian gauge theories still offers challenges, both qualitatively and quantitatively. In the present contribution we consider a non-perturbative approach to these problems within the framework of the Exact Renormalisation Group (ERG) in its continuum formulation a la Polchinski [1]. So far, quite some effort has been devoted to the task of working out a formulation of gauge theories within this approach (for a review see ref. [2]). Most of these formulations are based on the following picture: an explicit infra-red cut-off leads to gauge *variant* flows. The gauge symmetry of the underlying theory is, during the flow, encoded in 'modified' Ward-Takahashi or BRST identities (mWI) [2–10]. These identities guarantee that *physical* Greens functions satisfy the usual gauge invariance requirements.

The background field approach to gauge theories, e.g. [11], allows for the definition of a gauge invariant effective action, a property, which can be maintained within ERG flows. Hence it is an attractive choice for ERG applications in non-Abelian gauge theories [2, 3, 9, 10, 12–19], Abelian gauge theories [20] and in gravity [21]. However, despite of gauge invariance of the effective action, one has to deal with mWI's [3,9,15,17,18]. Moreover, approximation schemes to ERG flows that exploit the advantages of the background field formulation require some further work concerning their reliability [17, 22].

Hence, for any attempt at reliable analytic or numerical calculations it is important to fully understand how basic properties of the underlying theory manifest themselves in an ERG approach. A prominent example are anomalous dimensions, in particular their universal parts.

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[2] E-mail address: jmp@theorie3.physik.uni-erlangen.de

A related issue concerns the choice of the effective action Γ_Λ at the initial scale Λ. This object is a key input of the formalism. In general, an accurate choice of Γ_Λ stabilises the flow. In other words, minimising the error (in the irrelevant vertices) in the choice of Γ_Λ also reduces the unphysical part of the flow. In order to make use of results obtained by other methods (e.g. perturbation theory, semi-classical approximations) we have to map them into the parameters of the initial effective action in the ERG approach.

Here we provide a summary of the issues described above. We aim at a presentation of the key ideas behind the calculations and of the general framework. The details will be given elsewhere. After a brief introduction to ERG's for non-Abelian gauge theories in the background field approach [3], we discuss the map from RG equation to ERG equation. Then we sketch the calculation of one loop and two loop β-functions with help of this map. Implications of our findings are discussed, in particular their relevance for the choice of the initial effective action.

Flow equation with background fields

The background field approach hinges on the splitting of the gauge field $A = a + \bar{A}$ into a background field $\bar{A}$ and a fluctuation field a. This a priori arbitrary splitting motivates the following gauge invariant definition of the effective action: we use the background gauge $D_\mu(\bar{A})a_\mu = 0$. This gauge constraint is invariant under simultaneous gauge transformations of A and $\bar{A}$. Within this gauge the classical action with gauge fixing and ghosts reads

$$S[\phi, \bar{A}] = \frac{1}{2}\int_x \mathrm{tr}\,(F^2)_{\mu\mu} + \frac{1}{2\xi}\int_x (\bar{D}\,a)^2 - \int_x \bar{c}\,\bar{D}D\,c, \tag{1}$$

where $\bar{D} = D(\bar{A})$ and the trace tr sums over the fundamental representation of the gauge group. We also introduced the short hands $\int_x = \int d^4x$ and the field $\phi = (a, c, \bar{c})$. For later use we also introduce $\phi^* = (a, \bar{c}, -c)$.

Strictly speaking, the derivation of the flow equation presented here only assumes the existence of a finite Schwinger functional of the full quantum theory. It does not rely on a path integral representation. However, in case the latter exists we have

$$\exp W[J, \bar{A}] = \int [\mathcal{D}\phi]_{\rm ren}\, \exp\left\{-S[\phi, \bar{A}] + \int_x J^*\phi\right\}, \tag{2}$$

where the subscript $_{\rm ren}$ refers to the necessary renormalisation of the path integral, $J = (J_a, \eta, \bar{\eta})$ and $J^* = (J_a, \bar{\eta}, -\eta)$. Then, $J^*\phi = J_a\,a + \bar{\eta}\,c + \bar{c}\,\eta$. The effective action $\Gamma[\phi, \bar{A}]$ is the Legendre transformation of $W[J, \bar{A}]$

$$\Gamma[\phi, \bar{A}] = \int d^4x\, J^*\phi - W[J, \bar{A}]. \tag{3}$$

Here ϕ comprises the expectation values of the gauge field fluctuation, the ghost and the anti-ghost respectively. The action $S[\phi, \bar{A}]$ is invariant under the transformation $(\boldsymbol{\delta} + \bar{\boldsymbol{\delta}})S = 0$, where $\boldsymbol{\delta}_\omega(\phi, \bar{A}) = \big([D(A), \omega]\ ,\ [C, \omega]\ ,\ [\bar{C}, \omega]\ ,\ 0\big)$ and $\bar{\boldsymbol{\delta}}_\omega(\phi, \bar{A}) = \big(-[D(\bar{A}), \omega]\ ,\ 0,\ 0\ ,\ [D(\bar{A}), \omega]\big)$. This entails that the effective action $\Gamma[\phi, \bar{A}]$ is invariant under a transformation with $(\boldsymbol{\delta} + \bar{\boldsymbol{\delta}})$:

$$(\boldsymbol{\delta} + \bar{\boldsymbol{\delta}})\Gamma[\phi, \bar{A}] = 0. \tag{4}$$

However, *physical* gauge invariance is encoded in Ward-Takahashi identities obtained from a gauge variation of the quantum fields ϕ under $\boldsymbol{\delta}$. It is now possible to define a gauge invariant effective action as $\Gamma[A] := \Gamma[\phi = 0, A]$, giving $\bar{A} = A$ the interpretation of the *physical* mean field $A = a + \bar{A}$ (thus taking $a = 0$) and setting the unphysical ghost fields to zero. Consequently gauge invariance of $\Gamma[A]$ mirrors physical gauge invariance.

This picture is the starting point for the background field formulation of the ERG approach for the effective action. The ERG is based on the introduction of an infra-red (IR) cut-off scale k to the theory: momentum degrees of freedom with momenta $p^2 \ll k^2$ are suppressed whereas the propagation of degrees of freedom with larger momenta is unchanged. Then, the flow equation describes the infinitesimal change of the infra-red regularised effective action with the cut-off scale k. The above picture is achieved by an appropriate cut-off term $\Delta S_k[\phi, \bar{A}]$. We restrict ourselves to flows that are one loop exact in the full field dependent propagator [3]. It has been shown that for one loop exact flows the cut-off terms have to be quadratic in the fields coupled to the currents [23]. Thus, a general cut-off term for gauge fields and ghosts reads

$$\Delta S_k[\phi, \bar{A}] = \frac{1}{2} \int d^4x \, \phi^* R_k[\bar{A}] \phi, \tag{5}$$

where $R_k = (R^a, R^c, R^c) \otimes 1\!\!1_\phi$. Eq. (5) leads to a modification of the propagators of ghosts and gauge field fluctuations as $\frac{1}{2}\phi^* R_k \, \phi = \frac{1}{2} a \, R^a \, a + \bar{c} \, R^c \, c$. The regulator $R_k[\bar{A}]$ depends on an infra-red (IR) scale k which interpolates from some ultra-violet (UV) scale $k = \Lambda$ to the infra-red limit $k = 0$. Eq. (5) is quadratic in the fields and therefore leads to a modification of the propagator. Regulators R_k should leave the UV modes unchanged but suppress the propagation of IR modes. General regulators, as functions of momentum, thus vanish for high momenta and behave like a mass or even diverge for small momenta. Moreover, they should vanish for $k \to 0$ and diverge for $k \to \Lambda$. The k-dependent Schwinger functional is defined with

$$\exp W_k[J, \bar{A}] = \frac{1}{\mathcal{N}_k} \exp\left(-\frac{1}{2} \int_x \frac{\delta}{\delta J} R_k^*[\bar{A}] \frac{\delta}{\delta J^*} \right) \exp W[J, \bar{A}] \,, \tag{6}$$

where $R^* = (R^a, -R^c, -R^c) \otimes 1\!\!1_\phi$ and $\mathcal{N}_k$ is a possibly $\bar{A}$-dependent normalisation. We emphasise that this is not precisely the same as adding the cut-off terms to the classical action in the exponent of the path integral in (2). In (6) the renormalisation of the full theory at $k = 0$ is lifted by the regulator dependent exponential to finite k, whereas adding the cut-off term to the classical action in (2) in general results in a k-dependent renormalisation. Upon Legendre transformation, (6) leads to the effective action

$$\Gamma_k[\phi, \bar{A}] = \int d^4x \, J^* \phi - W_k[J, \bar{A}] - \Delta S_k[\phi, \bar{A}]. \tag{7}$$

This object tends to the classical action for diverging cut-off $k \to \Lambda$, subject to an appropriate definition of $\mathcal{N}_k$. For $k \to 0$, when removing the regulator, Γ_k approaches the full effective action $\Gamma = \Gamma_{k=0}$. For $R_k[\bar{A}]$ that transform as tensors under gauge transformations, $\bar{\boldsymbol{\delta}}_\omega R_k = [R_k \,, \, \omega]$ one can define a gauge invariant effective action $\Gamma_k[A] := \Gamma_k[\phi = 0, A]$. This functional approaches the gauge invariant effective action of the underlying physical theory, $\Gamma[A]$, for $k \to$

[3]Multi-loop exact flows can be devised as well, but lack the accessibility of one loop exact flows.

0. Only in this limit gauge invariance of $\Gamma_k[A]$ is directly related to physical gauge invariance. For $k \neq 0$ an application of $\boldsymbol{\delta}$ leads to modified Ward-Takahashi identities for $\Gamma_k[\phi, A]$.

The flow equation describes how $\Gamma_k[\phi, \bar{A}]$ changes under an infinitesimal variation of the logarithmic scale $t = \ln k$. It is derived from (6) with $\partial_t W[J, \bar{A}] = 0$ and the commutator $[\partial_t\,,\, \frac{\delta}{\delta J} R_k^*[\phi] \frac{\delta}{\delta J^*}]$ and reads

$$\partial_t \Gamma_k[\phi, \bar{A}] = \frac{1}{2} \mathrm{Tr}\, G_k[\phi, \bar{A}]\, \partial_t R_k[\bar{A}] + \partial_t \ln \mathcal{N}_k[\bar{A}] \tag{8}$$

with

$$G_{k,ij}[\phi, \bar{A}] = \left[\frac{1}{\Gamma_k^{(2)}[\phi, \bar{A}] + R_k[\bar{A}]} \right]_{ij} \quad \text{and} \quad \Gamma_{k,ij}^{(2)}[\phi, \bar{A}](x, y) = \frac{\delta^2 \Gamma_k[\phi, \bar{A}]}{\delta \phi^{*j}(y) \delta \phi^i(x)}. \tag{9}$$

The trace Tr denotes a sum over momenta, indices and the different fields ϕ. The relative minus sign of fermionic loops is encoded in the definition of G_k in (9). The regulator $R_k[\bar{A}]$ not only dependson the IR scale k but also depends on the UV scale Λ. Hence, the flow of Γ_k w.r.t. $\partial_\lambda = \Lambda \partial_\Lambda + \partial_t$ entails the information about the full scale dependence introduced by the regulator $R_k[\bar{A}]$. It is obtained by just substituting ∂_λ for ∂_t in (8). The flow of the gauge invariant effective action, $\partial_t \Gamma_k[A]$, is described by (8) for $\phi = 0$. Since (8) depends on the propagator of the fluctuation field ϕ, the flow of $\Gamma_k[A]$ requires some knowledge about $\Gamma_k[\phi, A]$.

Flow equation versus RG equation

The solution of the flow equation provides us with the running of the k-dependent vertices of Γ_k. The anomalous coefficients of the underlying theory are related to the latter. The derivation of the map between them is done along the same lines as the standard discussion of the relation between the RG and Callan-Symanzik equations (see e.g. ref. [24]). Similar considerations in the context of the ERG have been put forward in refs. [25–27]. The RG equation for $\Gamma[\phi, \bar{A}]$ reads

$$\mu \frac{d}{d\mu} \Gamma[\phi, \bar{A}] = \left(\mu \partial_\mu + D^\phi \right) \Gamma[\phi, \bar{A}] = 0, \tag{10}$$

where μ is the usual RG-scale and

$$D^\phi = \gamma_g g \partial_g + \gamma_\xi \xi \partial_\xi + \gamma_{\phi_i} \int_x \phi_i \frac{\delta}{\delta \phi_i} + \gamma_A \int_x \bar{A} \frac{\delta}{\delta \bar{A}}. \tag{11}$$

In (11) a summation over i is understood: $\phi_1 = a$, $\phi_2 = c$, $\phi_3 = \bar{c}$. From (8) and (10) one derives similar equations for $(\partial_s + D^\phi)\Gamma_k$ and $(\mu\partial_\mu + D^\phi)\Gamma_k$. Here the derivative $\partial_s = \mu \partial_\mu + \partial_t + \Lambda \partial_\Lambda$ encodes the derivative w.r.t. all UV and IR scales. First we notice that Eq. (10) implies $\mu \frac{d}{d\mu} W[J, \bar{A}] = 0$ for $\mu \frac{d}{d\mu} J_\phi = -\gamma_\phi J_\phi$. With (6) we proceed along the same lines as in the derivation of the flow equation and are led to

$$\left(\partial_s + D^\phi \right) \left(\Gamma_k[\phi, \bar{A}] - \ln \mathcal{N}_k[\bar{A}] \right) \quad = \quad \frac{1}{2} \mathrm{Tr}\, G_k[\phi, \bar{A}] \left(\partial_s + D^\phi + 2\gamma_\phi \right) R_k[\bar{A}]. \tag{12}$$

The right hand side of (12) stems from the commutator $[\partial_s + D^\phi\,,\, \frac{\delta}{\delta J} R_k^*[\phi] \frac{\delta}{\delta J^*}]$. We also lift the RG equation (10) for the full effective action $\Gamma[\phi, \bar{A}]$ to one for $\Gamma_k[\phi, \bar{A}]$ by substituting

$\partial_s \to \mu\partial_\mu$ in (12). This means that for Γ_k the RG-scaling of the underlying theory is changed by the right hand side of (12) (with $\partial_s \to \mu\partial_\mu$).

A few comments are in order. It is obvious from the derivation that the anomalous dimensions $\gamma_\phi, \gamma_A, \gamma_g, \gamma_\xi$ in (12) are those of the full theory. Eq. (12) reduces to a Callan-Symanzik type equation for $R \propto k^2$. We also see that the right hand side of (12) (with $\partial_s \to \mu\partial_\mu$) vanishes smoothly in the IR limit $k \to 0$ for regulators R_k that decay fast enough for large momenta. There we are left with the RG-equation (10). In turn, this property is not guaranteed for regulators which decay too slowly for large momenta, e.g. $R \propto k^2$. Then, additional care is required concerning the limit $k \to 0$, for a more extensive discussion see [2, 10, 17].

An interesting class of regulators is provided by the R_k satisfying the condition

$$(\mu\partial_\mu + D^\phi + 2\gamma_\phi)R_k = 0. \tag{13}$$

With (13) the effective action Γ_k satisfies the same RG equation as the full effective action (10): $(\mu\partial_\mu + D^\phi)\Gamma_k = 0$. This follows directly from (12), when substituting ∂_s with $\mu\partial_\mu$. A quite general class of regulators R_k satisfying (13) is just proportional to (part of) the second derivative of Γ_k w.r.t. to the fluctuation fields: $R_k = \Gamma_k^{(2)}[\phi = 0, \bar{A}]\, r[x]$, where the dimensionless function $r[x]$ depends on an appropriately chosen covariant Laplacean (e.g. $x = D^2(\bar{A})$+spin parts).

Now we focus on the consequences of (background) gauge invariance for the anomalous dimensions introduced in (11). Gauge invariance of $\Gamma_k[A]$ implies the invariance of $g\bar{A}$ under *general* flows: $g\bar{A} \to Z_g Z_A^{1/2}\, g\bar{A}$, in particular

$$\gamma_g = -\gamma_A, \qquad \text{and} \qquad \partial_t(Z_g Z_A^{1/2}) = \partial_\lambda(Z_g Z_A^{1/2}) = 0. \tag{14}$$

Note that the first equation displayed in (14) already follows from the gauge invariance of $\Gamma[A]$, regardless, whether $\Gamma_k[A]$ has this property or not. The two equations in (14) are key relations to be exploited later. It is not surprising that this is so. Already in the usual perturbative approach with background fields it is precisely (14) which simplifies particular calculations tremendously.

Initial effective action and regulator dependence

We have stressed before that an important ingredient of the approach is the choice of an Ansatz for the effective action $\Gamma_k[\phi, \bar{A}]$. It should include the full information (in its coefficients) relevant for the issues under investigation. The strategy we pursue is the following: the calculation is done at the initial scale Λ, where all irrelevant vertices are suppressed. This facilitates the identification of trivial contributions in (12). Moreover the calculation of the remaining non-trivial terms in (12), namely the operator trace on the right hand side and of $\partial_s\Gamma_k$, is quite simple. We consider the following parametrisation for $\Gamma_{k,\rm rel}[\phi, \bar{A}]$:

$$\begin{aligned}\Gamma_{k,\rm rel}[\phi, \bar{A}] = S[Z_\phi^{1/2}\phi,\ Z_A^{1/2}\bar{A};\ Z_g g,\ Z_\xi \xi]\\ -\mathrm{Tr}\,\left(Z_D \bar{D}\otimes\bar{D} + Z_F F[g\bar{A}] + m_k^2\right) a\otimes a + O[\phi^3]\end{aligned} \tag{15}$$

In (15) the trace Tr denotes a sum over momenta, indices in the fundamental representation and the classical action $S[\phi, \bar{A}]$ is defined in (1). All other terms have (power counting) irrelevant couplings and vanish together with their t, λ-derivatives at one loop and $k = \Lambda$. Consequently, as we only need the one loop effective action $\Gamma_k[\phi, \bar{A}]$ as an input for the flow of

$\Gamma_k[A]$ at two loop, we safely can set these terms to zero for the present purpose[4]. Their contributions have to be considered only beyond two loop, where they can be deduced from the (flow of the) mWI. Moreover, no further terms with marginal couplings can be present in (15) as it follows from (4), that $\Gamma_k^{(2)}[\phi,\bar A]$ transforms as a tensor under a combined transformation $(\boldsymbol{\delta}+\bar{\boldsymbol{\delta}})\Gamma_k^{(2)}[\phi,\bar A]=[\Gamma_k^{(2)}[\phi,\bar A]\,,\,\omega]$. This property also facilitates in general the use of the mWI. It drastically reduces the number of relevant terms. The $Z|_{k=\Lambda}$'s, the mass m_Λ and R_k implicitly fix the renormalisation conditions $(Z_{k=0},m_0=0)$ of the full theory at $k=0$. Consequently the Z's and m_k depend on μ,k,Λ and on the regulator function R_k. A priori the λ-derivatives of the Z's have nothing to do with the anomalous dimensions γ generated by the implicit μ-dependence of g,ξ and the fields. This makes it generally difficult to extract the anomalous dimensions γ from $\partial_\lambda \ln Z$.

We have stated above that the initial effective action Γ_Λ depends on the regulator R_k via the coefficients Z and m_k. This information is displayed by the following equation

$$\mathrm{Tr}\left(\delta R_k\frac{\delta}{\delta R_k}\right)\Gamma_k[\phi,\bar A]=\frac{1}{2}\mathrm{Tr}\,G_k[\phi,\bar A]\,\delta R_k[\bar A], \tag{16}$$

valid for any scale k. In (16) δR_k is a change of the regulator up to our disposal. An important example is given by $\delta R_k=(\frac{\delta}{\delta\bar A}R_k)\,\delta\bar A$. For this choice (16) describes the background field dependence introduced by the regulator itself [10,17]. If the s-derivative of (16) is non-vanishing, we have introduced an implicit scale dependence of field ($\bar A$) dependent terms via the regulator. This spoils the identification of the t-running with the μ-running. When choosing regulators that diverge in the infra-red, this is even present at one loop, see [10, 17]. Generally, this happens if the cut-off term is more than quadratic in the fields, see also [28][5]. It was argued in [10, 17] that one can use $\mathcal{N}_k$ to cancel this unwanted dependence. Here we just restrict ourselves to a class of regulators where no implicit λ-scaling is introduced at two loop and $k=\Lambda$. We note, however, that for general k such terms are unavoidable.

UV renormalisation and consequences

It is left to identify all terms on the left hand side of (12). To that end we resort to a loop expansion of the effective action. This leads to

$$\left(\partial_s+D^\phi\right)\Gamma_k[A]=2\gamma_{\bar A}S_A[A]+\left(\partial_s+D^\phi\right)\Delta\Gamma[A], \tag{17}$$

where $\Delta\Gamma$ comprises renormalised vacuum loops and S_A is the classical Yang-Mills action. We choose regulators that do not introduce implicit λ-scaling at two loop. On the technical level, apart from other restrictions, this requires taking $\xi=1$ following from an evaluation of (16). Then, about $k=\Lambda$, Γ_k is a function of ratios of k,Λ and μ up to terms suppressed by powers of (Λ^2-k^2). At one loop this implies that $\partial_\lambda\ln Z=\gamma$. Hence, for this class of regulators we can read-off the anomalous dimensions γ of the full theory from the ERG-scaling $\partial_\lambda\ln Z$. We conclude that $\left(\partial_s+D^\phi\right)\Delta\Gamma=\partial_s\Delta\Gamma_2+D^\phi\Delta\Gamma_1=D^\phi\Delta\Gamma_1$ at $k=\Lambda$ and two loop. It is here where the choice of regulators satisfying (13) pays off. For other choices the contributions do

[4]Some care is required concerning the momentum dependent terms $F(g\bar A)f_1(p^2)+F^2(g\bar A)f_2(p^2)$ in $\Gamma_\Lambda^{(2)}$.

[5]When working on gauge invariant regularisations, this becomes an important issue, see [29].

not decouple that neatly and some further work is required[6]. It follows for $\xi = 1, m_\Lambda = 0$[7] and $k \to \Lambda$ that

$$\left(\partial_s + D^\phi\right) \Gamma_k[A] = 2\gamma_A S_A[A] + \text{3-loop terms.} \tag{18}$$

Then, the two loop flow follows from the one loop results for $\partial_\lambda \ln Z_{\phi_i} = 2\gamma_{\phi_i}$, $\partial_\lambda \ln Z_g = \gamma_g$, $\partial_\lambda \ln Z_\xi = \gamma_\xi$ and Z_F. So far we have achieved a result for $\xi = 1$ and $m_\Lambda = 0$. This result extends to $m_\Lambda \neq 0$, since contributions proportional to m_Λ are the same on both sides of (12). Taking the ξ-derivative of the flow (12) confirms the ξ-independence of γ_A. For details we refer to ref. [17].

Results and outlook

Using the relations described in the previous section we extract the correct one loop result for the β-function directly from the ERG scaling of the coupling. However, for regulators that diverge at vanishing momenta we have to consider $\partial_s \Gamma_\Lambda \neq 0$ even at one loop [10, 17]. Its contribution to the left hand side of (12) can be deduced from (16). It just combines with the right hand side to the correct regulator independent result for γ_g. Strictly speaking, the check of one loop universality within the ERG approach with background fields is not complete without this statement. Indeed, for regulators with singular IR behaviour the ERG scaling does not match the RG scaling: $2\gamma_g \neq -\partial_\lambda \ln Z_A$. In terms of the map between RG scaling and ERG scaling the difference can be tracked down very easily.

The two loop β-function of gauge theories has not yet been derived in the ERG approach. For scalar theories this was done in ref. [30], see also [26, 27, 31, 32]). Within the present approach the ERG scaling matches the RG scaling for a specific class of regulators. This leads to the identity (18) for the left hand side of (12). The computation of the operator traces on the right hand side of (12) requires the correct renormalisation related to the quantum fluctuations. This part of the renormalisation plays an important rôle in the flow equation approach as opposed to the usual perturbative background field approach. The remaining operator trace is calculated straightforwardly and we arrive at the correct value for γ_A at two loop. With (14) this gives the two loop β-function. Universality is encoded in the invariance under changing the Z's, which implicitly fix the renormalisation conditions at $k = 0$. Note also that in general γ_A cannot be deduced from the ERG-scaling $\partial_\lambda \ln Z_A$ alone as this scaling is not universal. The actual calculation is very simple and boils down to the evaluation of heat kernel traces as in the one loop case the only difference being a one loop improvement of the operators. Here the careful discussion of initial conditions and the map between ERG-scaling and RG scaling fully pays off.

Obviously, even though we have restricted ourselves to a calculation of the β-function, the methods presented here allow for an analytic calculation of the effective action beyond the present approximation. Clearly beyond one loop (and away from the scaling region) derivatives w.r.t. the UV scale μ and the IR scale t do not necessarily match even for coefficients that are universal at two loop. This is not too surprising since the proofs of universality at two loop assume a mass-independent renormalisation. The introduction of an explicit IR scale contrasts this demand. Hence, the non-trivial map of (UV) vertices of the underlying theory to vertices in

[6]Alternatively, for $\xi = 1$ and $m_\Lambda = 0$ we can trade the explicit μ-scaling for a Callan-Symanzik scaling.

[7]$m_\Lambda = 0$ imposes a further constraint on the class of regulators R_k.

Γ_k is chiefly important, if one aims at improving the stability of the flow by means of an accurate choice of the initial effective action.

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