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INFLUENCE OF INTRINSIC DECOHERENCE IN THE PRESENCE OF STARK SHIFT ON THE MULTI-QUANTA PROCESSES

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The Milburn equation is solved exactly for multi-quanta Jaynes-Cummings model (JCM) in the presence of Stark shift. The influence of intrinsic decoherence and Stark shift on nonclassical properties (such as collapses and revivals of the population inversion and squeezing of the field modes) of the system is studied. The dynamical behaviour is adjusted in the presence of Stark shift. It is shown that the dynamic Stark shift plays an important role in nonclassical effects in multi-quanta processes. Also, even in the presence of Stark shift the revivals of the population inversion and squeezing of the field are destroyed in the decoherence process.

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1 Introduction

Due to the recent advances in cooling and trapping of ions [1] the motion of the center-of-mass (c.m.) of trapped ions has to be dealt with quantum mechanically. Laser irradiation [2–6] is used to control this motion coherently by coupling the ion's external and internal degrees of freedom. Models have been constructed to describe a two-level ion undergoing quantized vibrational motion within a harmonic trapping potential and interacting with a classical light field [2,3,7,8]. It has been pointed out that the dynamics of a trapped ion can be described by a Hamiltonian similar to a Jaynes-Cummings model [9] or its generalizations under certain regimes [4,5,10–12]. Within the framework of these Jaynes-Cummings-like models, various aspects of the dynamics of trapped ions have been studied. For example, quantum nondemolition measurement of vibrational quanta of trapped ions has been analyzed theoretically [13] and several schemes have been proposed [14] for the reconstruction of quantum-mechanical vibrational states of a trapped ion

One of these schemes has been successfully applied to the experimental reconstruction of the Wigner function of nonclassical states of the vibrational mode of a trapped ion [15]. It is to be noted that ion trap experiments suffer from decoherence due to classical noise in the laser

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beams and trapping potential. Such effect has been seen in recent experiments [16]. This kind of decoherence may be described using the intrinsic decoherence models [17–22].

The intrinsic decoherence approach has been proposed and investigated in the framework of several models [23–27]. In particular, Milburn [17,28] proposed a simple intrinsic decoherence models based on an assumption that on sufficiently short time steps the system evolves in a stochastic sequence of identical unitary transformations. This assumption modifies the von Neumann equation for the density operator of a quantum system through a simple modification of the usual Schrödinger evolution equation. The off-diagonal elements of the density operator in Milburn's model are intrinsically suppressed in the energy eigenstate basis, thereby intrinsic decoherence is realized without the usual dissipation associated with the normal decay. The decay is entirely of phase dependence only. Free evolution of a given quantum system has been discussed early [17] but investigations of interacting sub-systems followed [18–20]. The later were concerned with the Jaynes-Cummings model either with one-photon or multiphoton transitions. The Jaynes-Cummings model (JCM) [9] in quantum optics describes many pure quantum phenomena, such as collapses and revivals of the atomic inversion and oscillations of photon number distribution. It has been generally accepted that these nonclassical effects originate to form quantum coherences between the amplitudes. Therefore, it is an interesting topic to investigate the effects of the intrinsic decoherence on the nonclassical properties in the JCM, when we have single-mode of the interacting field affecting the interaction, and hence multi-quanta JCM. Such a model is discussed in this article when it is governed by the Milburn equation. On the other hand, there has been increased interest in the problem of decoherence in quantum mechanics because of its possible applications in quantum measurement processes and quantum computers [29].

Decoherence due to normal decay is often said to be the most efficient effect in physics, to a point where observation comes too late after the effect has reached completion [30]. The effect in action has been observed in quantum optics where the decoherence phenomena transforming a Schrödinger-cat into a statistical mixture was observed while unfolding [31]. It is well known that the Jaynes-Cummings model (JCM) in quantum optics [32] and cavity QED with cold trapped ions [11] can describe many pure quantum phenomena, called nonclassical properties, such as collapses and revivals of population inversion, oscillations of number distributions for quanta and squeezing of the cavity field.

In this model, when the two atomic levels are coupled with comparable strength to the intermediate relay level, the Stark shift becomes significant and can not be ignored [33–36]. The authors [34–36] studied the influence of the Stark shift term on the atomic inversion and dipole squeezing in the two-photon processes. They found that the dynamic Stark shift plays an important role in atomic inversion, but the influence of the Stark shift on the atomic inversion does not show if the two-levels are coupled equally strongly with the relay level under the condition of a strong initial field, and they also showed that the dipole squeezing shows a weak phase dependence in the absence of the Stark shift, but a strong phase dependence when it is present. Ashraf and Zubairy [37] included this power-dependent effect in their study of the equal-frequency two-photon micromaser. Gou [38] discussed how to eliminate the Stark shift through the use of a correlated two-mode field state in unequal frequency absorption. Nasreen and Razmi [36,39] discussed the effect of the dynamic Stark shift on atomic emission and cavity field spectra in the two-photon JCM and have shown that the Stark shift leads to asymmetric vacuum field Rabi splitting.

The purpose of this work is to study the multi-quanta JCM governed by Milburn equation in the presence of Stark shift. When the Stark shift is included, the dynamical behaviour is significantly affected. Our results show that the dynamic Stark shift plays an important role in nonclassical effects in multi-quanta processes. Also, it will be shown that the intrinsic decoherence in the particle (atom or trapped ion)-field interaction modifies the time evolution of the population inversion of the quanta and squeezing of the cavity field even in the presence of Stark shift.

This article is organized as follows. In section 2, We present an exact solution of the Milburn equation for the multi-quanta Jaynes-Cummings Hamiltonian in the presence of Stark shift and give the explicit expression of this solution in the two-dimensional basis of the particle. In section 3, We investigate the influence of the intrinsic decoherence and Stark shift on population inversion and squeezing of the radiation field in the JCM either in the resonant or the off-resonant cases. Finally, conclusion are presented in section 4.

2 Exact solution of the Milburn equation

We consider a quantum system described by the density operator $\rho(t)$. In standard quantum mechanics, dynamics of the system is governed by the evolution operator $\hat{U}(t) = \exp[-\frac{i}{\hbar}t\hat{H}]$, where $\hat{H}$ is the Hamiltonian describing the system. Milburn assumed [17] that on sufficiently short time steps the system does not evolves continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformation. Based on this assumption, he has derived the equation for the time evolution density operator $\rho(t)$ of the quantum system [17]

$$\frac{d}{dt}\hat{\rho}(t) = \gamma\left\{ \exp\left[-\frac{i}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right] - \hat{\rho}(t)\right\}, \quad (1)$$

where γ is the mean frequency of the unitary time step. This equation formally corresponds to the assumption that on sufficiently short time steps the system evolves with a probability $p(\tau) = \gamma\tau$. Obviously, the generalized Eq. (1) alters the Schrödinger dynamics. It reduces to the ordinary von Neuman equation for the density operator in the limit $\gamma \to +\infty$. Expanding Eq. (1) to first order in γ^{-1}, the following dynamical equation is obtained:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}]], \quad (2)$$

which is the Milburn equation that we shall study below. This equation has been solved for a harmonic oscillator and a precessing spin system [17] the simple JCM [18,19], the resonant multiphoton JCM [20] and the nondegenerate two-mode JCM [21,22]. In what follows we shall consider the exact solution of this equation for the multi-quanta JCM with a detuning parameter in the presence of the Stark shift.

The system considered here consists of a two-level particle (atom or trapped ion) interacting with a single-mode quantized field via m-quanta transition processes. The Hamiltonian in the rotating wave approximation (RWA) [40], is written as:

$$\hat{H} = \omega\hat{a}^\dagger\hat{a} + \frac{\omega_\circ}{2}\hat{\sigma}_z + +\hat{a}^\dagger\hat{a}(\beta_1 \mid g >< g \mid + \beta_2 \mid e >< e \mid) + \lambda(\hat{a}^{\dagger m}\hat{\sigma}_- + \hat{a}^m\hat{\sigma}_+) \quad (\hbar = 1), \quad (3)$$

where ω is the field frequency and $\omega_\circ$ is the transition frequency between the excited and ground states of the particle (atom or trapped ion), $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and the creation operators of the cavity field respectively, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transition to the intermediate relay level; λ is the effective coupling constant, $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the "spin flip" operators which satisfy the relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$, with the detuning parameter $\Delta = \omega_\circ - m\omega$.

Now, we look for the exact solution for the density operator $\hat{\rho}(t)$ of Eq. (2) taking into account the Hamiltonian (3). For convenience, we introduce three auxiliary superoperators [18–22] $\hat{J}$, $\hat{S}$ and $\hat{L}$ defined by

$$\exp(\hat{J}\tau)\hat{\rho}(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{\tau}{\gamma}\right)^k \hat{H}^k \hat{\rho}(t) \hat{H}^k \tag{4}$$

$$\exp(\hat{S}\tau)\hat{\rho}(t) = \exp(-i\hat{H}\tau)\hat{\rho}(t)\exp(i\hat{H}\tau) \tag{5}$$

$$\exp(\hat{L}\tau)\hat{\rho}(t) = \exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]\hat{\rho}(t)\exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]. \tag{6}$$

From Eqs. (4–6) it follows that

$$\hat{J}\hat{\rho} = \frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H}, \quad \hat{S}\hat{\rho} = -i[\hat{H}, \hat{\rho}], \quad \hat{L}\hat{\rho} = -\frac{1}{2\gamma}\{\hat{H}^2, \hat{\rho}\} = -\frac{1}{2\gamma}(\hat{H}^2\hat{\rho} + \hat{\rho}\hat{H}^2)\,. \tag{7}$$

By substituting Eq. (7) into Eq. (2), we can obtain the formal solution of the Milburn equation as follows:

$$\hat{\rho}(t) = \exp(\hat{J}t)\exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0)\,, \tag{8}$$

where $\hat{\rho}(0)$ is the density operator of the initial particle-field system.

We assume that initially the field is prepared in the coherent state $\mid \alpha >$:

$$\mid \alpha >= \sum_{n=0}^{\infty} \exp(-\frac{1}{2}|\alpha|^2)\frac{\alpha^n}{\sqrt{n!}} \mid n >= \sum_{n=0}^{\infty} Q_n \mid n > \tag{9}$$

and the particle (atom or trapped ion) is in its excited state $\mid e >$, so that

$$\hat{\rho}(0) = \mid \alpha >< \alpha \mid \otimes \mid e >< e \mid\,. \tag{10}$$

In a two-dimensional basis for the particle the Hamiltonian (3) can be expressed as a sum of $\hat{H}_\circ$, which is diagonal and $\hat{H}_I$, which is not

$$\hat{H} = \hat{H}_\circ + \hat{H}_I, \quad [\hat{H}_\circ, \hat{H}_I] = 0, \tag{11}$$

where

$$\hat{H}_\circ = \begin{bmatrix} \omega(\hat{n} + \frac{m}{2}) + \hat{\delta}_+(n+m) & 0 \\ 0 & \omega(\hat{n} - \frac{m}{2}) + \hat{\delta}_+(n) \end{bmatrix} \tag{12}$$

$$\hat{H}_I = \lambda \left[\begin{array}{cc} [\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n+m)] & \hat{a}^m \\ \hat{a}^{\dagger m} & -[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n)] \end{array} \right] \tag{13}$$

with

$$\hat{\delta}_\pm(n) = \frac{1}{2}[\beta_2(\hat{n}-m) \pm \beta_1\hat{n}], \quad \frac{1}{\lambda}\hat{\delta}_-(n) = [(\hat{n}-m) - R^2\hat{n}]/2R, R^2 = \beta_1/\beta_2, \lambda = \sqrt{\beta_1\beta_2}\,. \tag{14}$$

Similarly, the square of the Hamiltonian (3) can also be expressed as a sum of two matrices in the form

$$\hat{H}^2 = \hat{A} + \hat{B}, \quad [\hat{A}, \hat{B}] = 0\,, \tag{15}$$

where $\hat{A}$ is diagonal in the form

$$\hat{A} = \left[\begin{array}{cc} \hat{\Theta}^2(n+m) & 0 \\ 0 & \hat{\Theta}^2(n) \end{array} \right] \tag{16}$$

and $\hat{B}$ is given by

$$\hat{B} \quad = \quad 2\lambda \left[\begin{array}{cc} \hat{\eta}(n+m)\hat{\zeta}(n+m) & \hat{a}^m\hat{\zeta}(n) \\ \hat{\zeta}(n)\hat{a}^{\dagger m} & -\hat{\eta}(n)\hat{\zeta}(n) \end{array} \right] \tag{17}$$

with

$$\hat{\eta}(n) = [\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n)], \quad \hat{\zeta}(n) = [\omega(\hat{n} - \frac{m}{2}) + \hat{\delta}_+(n)] \tag{18}$$

$$\hat{\Theta}^2(n) = \hat{\zeta}^2(n) + \lambda^2\hat{F}^2(n). \tag{19}$$

$$\hat{F}^2(n) = \hat{\eta}^2(n) + \hat{\nu}^2(n), \quad \hat{\nu}^2(n) = \frac{\hat{n}!}{(\hat{n}-m)!}\,. \tag{20}$$

Taking into account the initial condition (10) we can write down the following expression

$$\begin{aligned} \hat{\rho}_2(t) \quad &= \exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0) \\ &= \exp(-i\hat{H}_I t)\exp\left(-\frac{t}{2\gamma}\hat{B}\right)\hat{\rho}_1(t)\exp\left(-\frac{t}{2\gamma}\hat{B}t\right)\exp(i\hat{H}_I t)\,, \end{aligned} \tag{21}$$

where the auxiliary operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t) = \left[\begin{array}{cc} \mid \hat{\Psi}(t) >< \hat{\Psi}(t) \mid & 0 \\ 0 & 0 \end{array} \right] \tag{22}$$

with

$$\mid \hat{\Psi}(t) >= \exp\left[-\frac{t}{2\gamma}\hat{\Theta}^2(n+m)\right]\exp[-i\hat{\zeta}(n+m)t] \mid \alpha > \ . \tag{23}$$

The powers of the operator $\hat{B}$ can be written as

$$\hat{B}^{2k} = \begin{bmatrix} [2\lambda\hat{\zeta}(n+m)\hat{F}(n+m)]^{2k} & 0 \\ 0 & [2\lambda\hat{\zeta}(n)\hat{F}(n)]^{2k} \end{bmatrix} \tag{24}$$

$$\hat{B}^{2k+1} = \begin{bmatrix} \hat{\eta}(n+m)\dfrac{[2\lambda\hat{\zeta}(n+m)\hat{F}(n+m)]^{2k+1}}{\hat{F}(n+m)} & \hat{a}^m\dfrac{[2\lambda\hat{\zeta}(n)\hat{F}(n)]^{2k+1}}{\hat{F}(n)} \\ \dfrac{[2\lambda\hat{\zeta}(n)\hat{F}(n)]^{2k+1}}{\hat{F}(n)}\hat{a}^{\dagger m} & -\hat{\eta}(n)\dfrac{[2\lambda\hat{\zeta}(n)\hat{F}(n)]^{2k+1}}{\hat{F}(n)} \end{bmatrix} \tag{25}$$

then we can write the operator $\exp[-\frac{t}{2\gamma}\hat{B}]$ in the form

$$\exp[-\frac{t}{2\gamma}\hat{B}] = \begin{bmatrix} \hat{X}(n+m,t) - \hat{\eta}(n+m)\dfrac{\hat{Y}(n+m,t)}{\hat{F}(n+m)} & -\hat{a}^m\dfrac{\hat{Y}(n,t)}{\hat{F}(n)} \\ -\dfrac{\hat{Y}(n,t)}{\hat{F}(n)}\hat{a}^{\dagger m} & \hat{X}(n,t) + \hat{\eta}(n)\dfrac{\hat{Y}(n,t)}{\hat{F}(n)} \end{bmatrix}, \tag{26}$$

where

$$\hat{X}(n,t) = \cosh\left[\frac{\lambda t}{\gamma}\hat{\zeta}(n)\hat{F}(n)\right], \qquad \hat{Y}(n,t) = \sinh\left[\frac{\lambda t}{\gamma}\hat{\zeta}(n)\hat{F}(n)\right]. \tag{27}$$

Similarly, we can write the operator $\exp[-i\hat{H}_I t]$ in the two-dimensional basis for the particle as

$$\exp[-i\hat{H}_I t] = \begin{bmatrix} \hat{C}(n+m,t) - i\hat{\eta}(n+m)\dfrac{\hat{S}(n+m)}{\hat{F}(n+m)} & -i\hat{a}^m\dfrac{\hat{S}(n,t)}{\hat{F}(n)} \\ i\dfrac{\hat{S}(n,t)}{\hat{F}(n)}\hat{a}^{\dagger m} & \hat{C}(n,t) + i\hat{\eta}(n)\dfrac{\hat{S}(n,t)}{\hat{F}(n)} \end{bmatrix} \tag{28}$$

with

$$\hat{C}(n,t) = \cos[\lambda t\hat{F}(n)] \quad and \quad \hat{S}(n,t) = \sin[\lambda t\hat{F}(n)]. \tag{29}$$

Then,

$$\exp[-i\hat{H}_I t]\exp\left(-\frac{t}{2\gamma}\hat{B}\right) = \begin{bmatrix} \hat{R}(n+m,t) - \hat{\eta}(n+m)\dfrac{\hat{V}(n+m,t)}{\hat{F}(n+m)} & -\hat{a}^m\dfrac{\hat{V}(n,t)}{\hat{F}(n)} \\ -\dfrac{\hat{V}(n,t)}{\hat{F}(n)}\hat{a}^{\dagger m} & \hat{R}(n,t) + \hat{\eta}(n)\dfrac{\hat{V}(n,t)}{\hat{F}(n)} \end{bmatrix}, \tag{30}$$

where

$$\hat{R}(n,t) = \hat{C}(n,t)\hat{X}(n,t) + i\hat{S}(n,t)\hat{Y}(n,t) \tag{31}$$

$$\hat{V}(n,t) = \hat{C}(n,t)\hat{Y}(n,t) + i\hat{S}(n,t)\hat{X}(n,t). \tag{32}$$

Substituting Eqs. (22) and (30) into Eq. (21), we obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{bmatrix}, \tag{33}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}(t) =| \hat{\Psi}_i(t) >< \hat{\Psi}_j(t) | \quad (i, j = 1, 2) \tag{34}$$

with

$$| \hat{\Psi}_1(t) >= \left[\hat{R}(n+m,t) - \hat{\eta}(n+m)\frac{\hat{V}(n+m,t)}{\hat{F}(n+m)}\right]| \hat{\Psi}(t) > \tag{35}$$

$$| \hat{\Psi}_2(t) >= -\hat{a}^{\dagger m}\frac{\hat{V}(n+m,t)}{\hat{F}(n+m)}] | \hat{\Psi}(t) >, \tag{36}$$

where $| \hat{\Psi}(t) >$ is given by Eq. (23). Taking into account the definition of the superoperator $\hat{J}$, it is straightforward to obtain the action of the operator $\exp(\hat{J}t)$ on the density operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}(t) = \sum_{k=0}^{\infty} \frac{1}{k!}(\frac{t}{\gamma})^k \hat{H}^k \hat{\rho}_2(t)\hat{H}^k, \tag{37}$$

where the Hamiltonian $\hat{H}$ and the operator $\hat{\rho}_2(t)$ are given by Eqs. (3) and (33), respectively. Equation (37) describes the exact solution of the Milburn equation (2) for the multi-quanta processes. Once the density operator is calculated all relevant statistical quantities can be computed.

3 Influence of the intrinsic decoherence on nonclassical properties of the system

In this section, we investigate the effect of the intrinsic decoherence and Stark shift on nonclassical properties of the multi-quanta JCM when the particle (atom or trapped ion) is taken to be prepared initially in the excited state.

3.1 Population inversion

It is well known that in the JCM the quantum coherences which are built up during the interaction between the field and the particle significantly affect the dynamics of the particle [11,32,40-43]. The existence of the quantum coherences is the reason why one can observe collapses and revivals of the population inversion of the particle. Now we evaluate the population inversion in the multi-quanta JCM. Using the exact solution $\hat{\rho}(t)$, we find that population inversion is given by

$$\begin{aligned} < \hat{\sigma}_z(t) > &= Tr[\hat{\rho}(t)\hat{\sigma}_z] \\ &= \sum_{n=0}^{\infty} \frac{|\, Q_n\,|^2}{F^2(n+m)} \Big\{ \eta^2(n+m) + \nu^2(n+m) \\ &\times \exp\left[-\frac{2\lambda^2 t}{\gamma} F^2(n+m) \right] \cos 2\lambda t F(n+m) \Big\}. \end{aligned} \tag{38}$$

We now discuss the numerical results of the population inversion $< \sigma_z(t) >$ given by equation (38) for the multi-quanta JCM, when the particle (atom or trapped ion) initially starts in the excited state.

The time evolution of the population inversion is presented in Figs. (1–4), for various values of the parameter λ/γ, and fixed initial mean number of quanta $\bar{n}$ for two-quanta precesses $(m = 2)$.

In Fig. 1 we plotted the population inversion $< \sigma_z(t) >$ for three values of the parameter λ/γ (namely 10^{-6}, 10^{-4}, 10^{-3}) with the fixed initial mean numbers of quanta $\bar{n} = 20$ in the absence of Stark shift $R = 0$ $(\beta_1 = \beta_2 = 0)$.

In Fig. 2, 3 and 4, we plotted the population inversion in the presence of Stark shift $R = 1.0$ $(\beta_1 = \beta_2)$, $R = 0.5$ and $R = 0.3$, respectively. From Fig. 1 (in the absence of Stark shift), we see that the population inversion evolves at a revival period π/λ.

In the case described in Fig. 2, the Stark shift parameter is $R = 1.0$ $(\beta_1 = \beta_2)$, which corresponds to the case in which the two levels of the particle are equally strongly coupled with the intermediate relay level. By comparing Fig. 1 and 2, we see that the population inversion is almost similar for both cases. In this case, the Stark shift does not effect the time evolution of the population inversion.

For the cases described in Figs. (2–4) (in the presence of Stark shift), Ref. [34] gives the period of the revivals, $t_R = 2\pi R/[\lambda(1 + R^2)]$.

When $R = 1.0$ (the Stark shifts of the two levels are equal), the population inversion evolves at a revival period $t_R = \pi/\lambda$, while t_R becomes small, the excited particle has the tendency to trap the excitation energy.

In Figs. 3 and 4, we show the cases in which the two levels have unequal Stark shift ($R < 1$, in Fig. 3 $R = 0.5$ while in Fig. 4 $R = 0.3$). We see the Stark shift leads to decreasing of the values of the atomic revivals of the population inversion and the evolution period of the population inversion decreases with reduced of the Stark shift parameter R.

Also, these Figures show that with the decrease of the parameter γ, i.e., with a more rapid suppression of quantum coherences we can observe rapid deterioration of revivals of the population inversion. Which means that the decay of quantum coherences is due to the very specific time evolution described by Eq. (2), i.e., due to the intrinsic decoherence.

3.2 Amplitude-Squared Squeezing Of The Field

Now, we study the amplitude-squared squeezing of the field of multi-quanta JCM governed by the Milburn equation in the presence of the Stark shift and discuss effects of the intrinsic decoherence

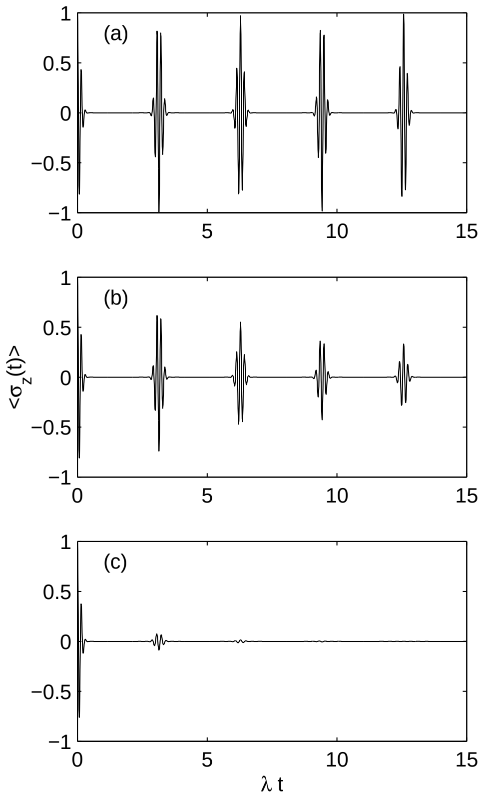

Fig. 1. The population inversion as function of the scaled time λt when the particle initially prepared in the excited state, the field in the coherent state with mean number $\bar{n} = 20$ in the absence of Stark shift $(R = 0)$ for various values of the parameter λ/γ: (a) $\lambda/\gamma = 10^{-6}$, (b) $\lambda/\gamma = 10^{-4}$ and (c) $\lambda/\gamma = 10^{-3}$.

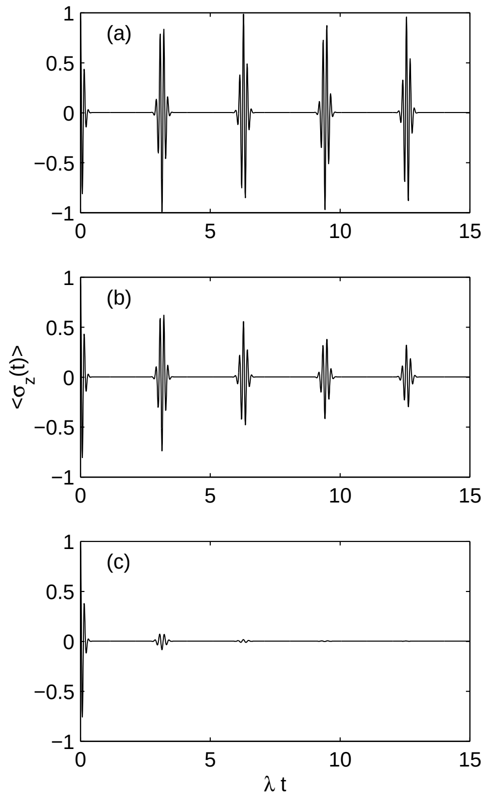

Fig. 2. The population inversion in the presence of Stark shift $R = 1.0$ $(\beta_1 = \beta_2)$.

and Stark shift on the squeezing. We define the operators of the real and imaginary parts of the square of the amplitude [44]

$$\hat{X}_1 = \frac{1}{2}[\hat{a}^2(t) + \hat{a}^{\dagger^2}(t)], \quad \hat{X}_2 = \frac{1}{2i}[\hat{a}^2(t) - \hat{a}^{\dagger^2}(t)]\,. \tag{39}$$

These operators satisfy the commutation relation,

$$[\hat{X}_1, \hat{X}_2] = i(2n(t) + 1), \tag{40}$$

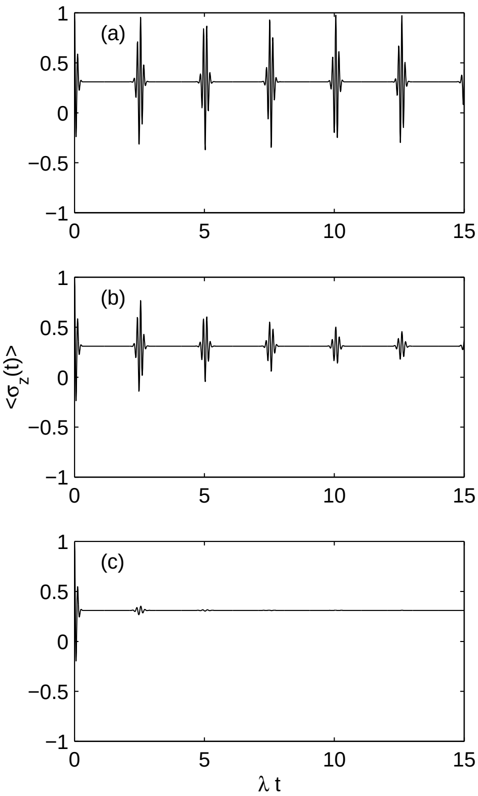

Fig. 3. The same as in Fig. 2, but with $R = 0.5$.

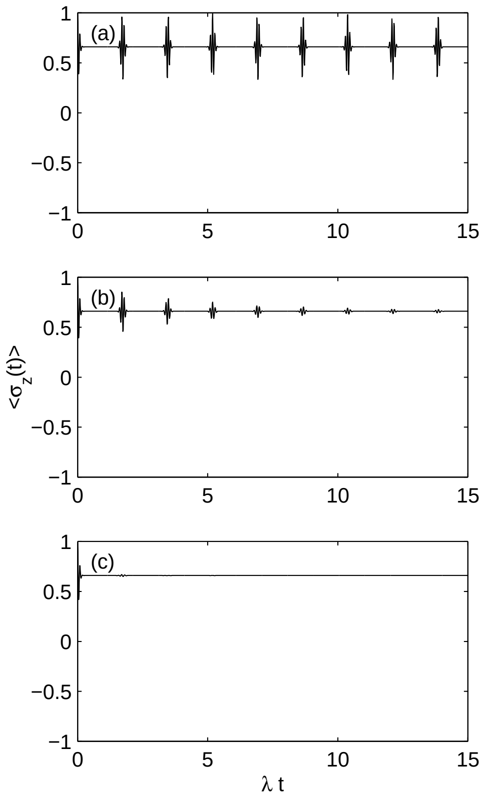

Fig. 4. The same as in Fig. 2, but with $R = 0.3$

which implies the uncertainty relation

$$(\Delta\hat{X}_1)^2(\Delta\hat{X}_2)^2 \geq \frac{1}{4}\ |<[\hat{X}_1, \hat{X}_2]>|^2\ . \tag{41}$$

The state of the field is said to be amplitude-squared squeezed whenever one of the two quadratures satisfies the relation:

$$(\Delta\hat{X}_i) < \frac{1}{2}(2n+1), (i = 1 or 2), \tag{42}$$

where $n = \hat{a}^\dagger\hat{a}$. On the other hand, the condition (42) can be rewritten as

$$S_i = (\Delta\hat{X}_i)^2 - \frac{1}{2}(<2n+1>), (i = 1, 2), \tag{43}$$

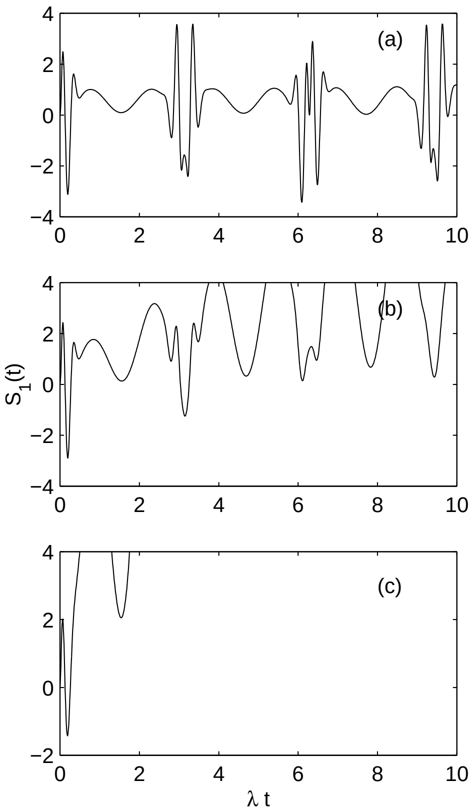

Fig. 5. The amplitude-squared squeezing as function of the scaled time λt with mean number $\bar{n} = 10$ in the absence of Stark shifts $(R = 0)$ for various values of the parameter λ/γ: (a) $\lambda/\gamma = 10^{-6}$, (b) $\lambda/\gamma = 10^{-3}$ and (c) $\lambda/\gamma = 10^{-2}$.

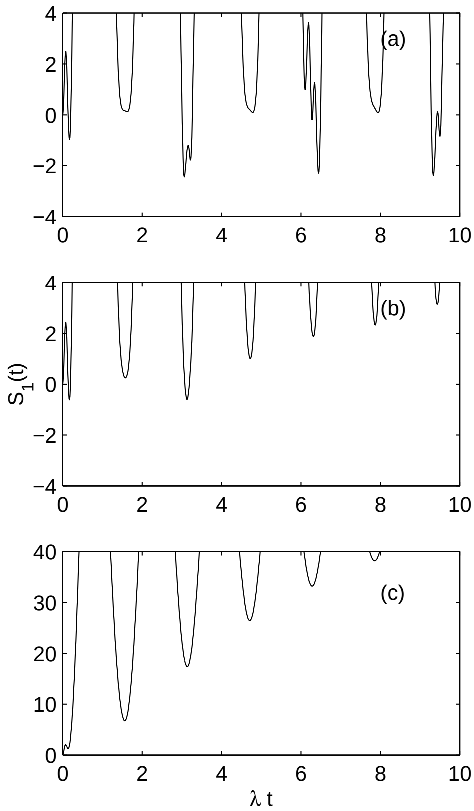

Fig. 6. The amplitude-squared squeezing in the presence of Stark shift $R = 1.0$ $(\beta_1 = \beta_2)$.

and squeezing occurs when S_1 or $S_2 < 0$. In terms of the photon annihilation and creation operators of the field, we get for the amplitude-squared squeezing factors S^j the expression

$$S_1 = \frac{1}{4}[2 < \hat{a}^{\dagger^2}\hat{a}^2 > + < \hat{a}^4 > + < \hat{a}^{\dagger^4} > -(< \hat{a}^2 > + < \hat{a}^{\dagger^2} >)^2], \tag{44}$$

$$S_2 = \frac{1}{4}[2 < \hat{a}^{\dagger^2}\hat{a}^2 > - < \hat{a}^4 > - < \hat{a}^{\dagger^4} > +(< \hat{a}^2 > - < \hat{a}^{\dagger^2} >)^2]. \tag{45}$$

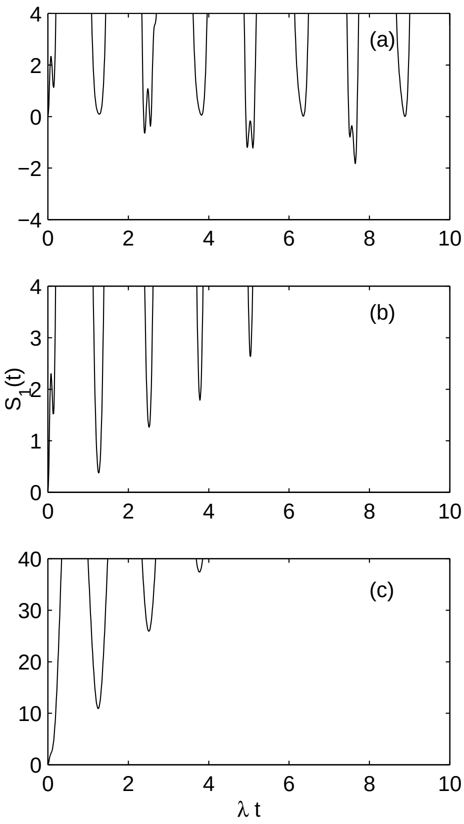

Fig. 7. The same as in Fig. 6 but with $R = 0.5$

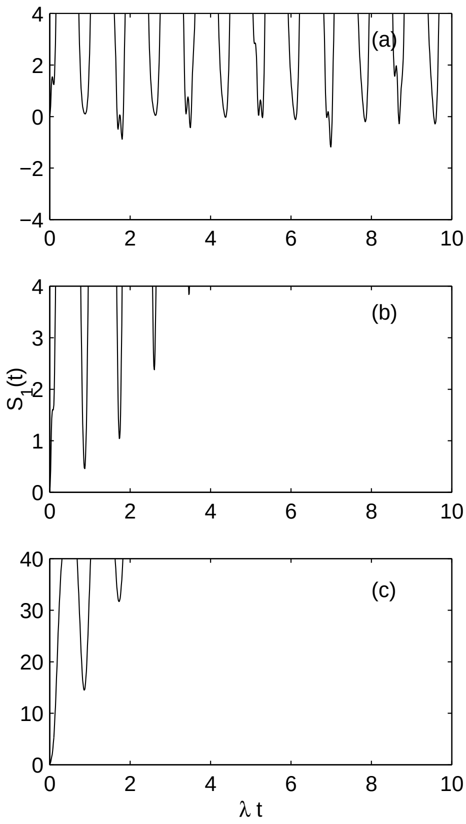

Fig. 8. The same as in Fig. 6 but with $R = 0.3$

By using the field density operators $\hat{\rho}_F(t)$, we obtain the expectation value in the general form for the field operators $\hat{a}^{\dagger^r}\hat{a}^s$ as follows:

$$
\begin{aligned}
< \hat{a}^{\dagger^r}\hat{a}^s > \quad = \quad & \frac{1}{4}|\alpha|^{r+s}\sum_{n=0}^{\infty} \mid Q_n \mid^2 \exp[i\lambda t[(r-s)(1+R^2)/2R]] \\
& \times\Big\{[(1+D_S)(1+D_r)+J]\exp(i\lambda t a_-(n))\exp(-\frac{t}{2\gamma}b_+(n)) \\
& +[(1+D_s)(1-D_r)-J]\exp(-i\lambda t a_+(n))\exp(-\frac{t}{2\gamma}c_-(n)) \\
& +[(1-D_s)(1+D_r)-J]\exp(i\lambda t a_+(n))\exp(-\frac{t}{2\gamma}c_+(n)) \\
& +[(1-D_s)(1-D_r)+J]\exp(-i\lambda t a_-(n))exp(-\frac{t}{2\gamma}b_-(n))\Big\}, \qquad (46)
\end{aligned}
$$

where

$$J = \frac{(n)!}{(n+m)!}\frac{\nu^2(n+s+m)}{F_{n+s+m}}\frac{\nu^2(n+r+m)}{F_{n+r+m}}$$

$$D_s = \frac{[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n+s+m)]}{F_{n+s+m}}$$

and D_r is obtained from D_s by $s \to r$ with

$$a_\pm(n) = F_{n+r+m} \pm F_{n+s+m}, \tag{47}$$

$$b_\pm(n) = \{(\omega(r-s) + \frac{\lambda}{2R}[(r-s)(1+R^2)] \pm \lambda a_-(n)\}^2, \tag{48}$$

$$c_\pm(n) = \{(\omega(r-s) + \frac{\lambda}{2R}[(r-s)(1+R^2)] \pm \lambda a_+(n)\}^2. \tag{49}$$

By using Eqs. (46–49), and specifying the exponents r, s we get the expression for the amplitude-squared squeezing of the multi-quanta Jaynes-Cummings model.

Now, we discuss the temporal behaviour of the $S_1(t)$, which gives information on amplitude-squared squeezing, when we take $\bar{n} = 10$ and investigate the influence of the decoherence parameter and Stark shifts on amplitude-squared squeezing.

Numerical results for Eq. (44) are presented in Figs. (5–8), we plotted $S_1(t)$ against λt for $\bar{n} = 10$ and different values of the decoherence parameter λ/γ (namely 10^{-6}, 10^{-3}, 10^{-2}) for two-quanta processes ($m = 2$).

In Fig. 5, we display amplitude-squared squeezing for three values of the parameter λ/γ (namely 10^{-6}, 10^{-3}, 10^{-2}) with the fixed initial mean numbers of quanta $\bar{n} = 10$ in the absence of Stark shift $R = 0$ ($\beta_1 = \beta_2 = 0$).

Figs. 6, 7 and 8 are the same as in Fig. 5, but with the Stark shift parameter $R = 1.0$, $R = 0.5$ and $R = 0.3$, respectively.

In two Figs. 5 and 6 (see Figs. 5 and 6), i.e, $R = 0$ (in the absence of the Stark shift) and $R = 1.0$ ($\beta_1 = \beta_2$) gives better amplitude-squared squeezing and it reoccurs at later times. Here it is observed that squeezing occurs during the revival periods in contrast to the two cases $(R = 0.5)$ and $(R = 0.3)$ (see Figs. 7 and 8).

Also in Figs. 7 and 8 i.e. $(R = 0.5)$ and $(R = 0.3)$, we observed that no squeezing appear in the short time $0 \leq \lambda t \leq 3$ and appear many times at later times and the amount of squeezing decreasing with decreasing R.

In view of these Figures, we note that with the decrease of the parameter γ i.e., with a more rapid decohering, we observe rapid decrease of the amount of squeezing for amplitude-squared squeezing even in the presence of the Stark shift.

4 Concluding remarks

In this paper, we have considered an effective Hamiltonian (3) describing the interaction between a two-level particle (atom or trapped ion) and a single mode field through multiquanta. We have found the exact solution of the Milburn equation (Eq. 2) for the multi-quanta JCM in the presence of Stark shifts . Using the exact solution (Eq. 37), we have discussed the effect of the intrinsic decoherence and Stark shift on population inversion and squeezing of the radiation field. Our results show that the dynamic Stark shift plays an important role in nonclassical properties in multi-quanta processes. It is shown that even in the presence of Stark shifts the intrinsic decoherence in the particle-field interaction suppress the nonclassical effects, where with the decrease of the parameter γ, i.e. with a more rapid decohering, we observed a rapid decrease of the amount of squeezing.

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