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STABILITY OF THE MIXED FIXED POINT OF THE mn-VECTOR MODEL[1]

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We study the conditions under which the critical behaviour of a three-dimensional mn-vector model is non-trivial, i. e. its universal characteristics do not belong to an $O(m)$ universality class. In the calculations we rely on the field-theoretical renormalization group approach in different regularization schemes. We use adjusted resummation and extended analysis of the series for renormalization-group functions which are known for the model in high orders of perturbation theory. As a result we build the regions in $m-n$ plane where non-trivial critical behaviour is realized.

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1 Introduction

The success of the renormalization group (RG) approach in comprehension of the essence of critical phenomena in condensed matter physics has been well appreciated since seventies. While developing, the approach required much efforts for elaborating principal methods via scrupulous study of basic models [1]. Nowadays, attention is paid presumably to advanced applications of the methods and description of the critical behaviour of realistic models. Speaking the RG language, they are characterized by a complicated symmetry of an order parameter. The problem of universality classes formation and crossovers among them is one of the most important in description of such models. To solve it, one has to find the conditions under which the complicated symmetry does impact the model at criticality, so the model is not reduced by fluctuations to a simpler one and exhibits a non-trivial critical behaviour.

In this report we address the conditions of non-trivial critical behaviour of the mn-vector model [2]. The model is introduced by a ϕ^4 Hamiltonian:

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$$\mathcal{H}[\phi(x)] = \int \mathrm{d}^d x \Big\{ \frac{1}{2} \sum_{\alpha=1}^{n} \Big[|\nabla \vec{\phi}^\alpha|^2 + m_0^2 |\vec{\phi}^\alpha|^2 \Big] + \frac{u_0}{4!} \sum_{\alpha=1}^{n} \Big(|\vec{\phi}^\alpha|^2 \Big)^2 + \frac{v_0}{4!} \left(\sum_{\alpha=1}^{n} |\vec{\phi}^\alpha|^2 \right)^2 \Big\}, \qquad (1)$$

where $\vec{\phi}^\alpha = (\phi^{\alpha,1}, \phi^{\alpha,2}, \dots, \phi^{\alpha,m})$ is a tensor field of the dimension n and m along the first and the second index; d is the space dimension; u_0 and v_0 are bare couplings; m_0^2 is a bare mass squared, which is a measure of the temperature distance to the critical point.

The long-distance properties of the model (1) are known to describe critical phenomena in systems of various microscopic nature. In particular, $m = 1$ with arbitrary n ($m = 1, \forall n$) corresponds to the cubic model [3] which was introduced to account for the reaction of the order parameter to the magnet's lattice structure. For $m = 2$, $n = 2$ and $n = 3$, the model describes structural phase transitions [4]. The essential common trait of these two cases is the restriction on signs of the couplings: $\forall\, u_0,\, v_0 \geq 0$. For the sake of convenience in our further discussion we will refer to the models (1) with arbitrary m, n but with the fixed sign $v_0 \geq 0$ as to the *"cubic-like"* models.

The model (1) is also exploited to study the critical properties of the weakly diluted quenched $O(m)$ model [5]. In this case it is considered in a zero limit of the replica index n. The microscopic base of the approach strictly defines $u_0 > 0,\, v_0 \leq 0$. Again for convenience, we will refer to the models (1) with arbitrary m, n but with the fixed signs $u_0 > 0,\, v_0 \leq 0$ as to the *"disordered-like"* models.

Finally, it is interesting to note that two ϕ^4 terms of the model (1) become of the same symmetry and thus should be considered as one term at a unified coupling $u_0 + v_0$ at some values of m, n. Except for the familiar case of $n = 1$ corresponding to the $O(m)$ model, this emerges for $m = 0$ (in the polymer limit) [6] and for $m \to \infty$ (the spherical model).

All of the mentioned particular cases of the mn model were subjects of separate extensive studies (see e. g. [7] and references therein). A possibility of crossover from an $O(m)$-symmetric universality class to a new one appeared to be the common feature of their critical behaviour. In particular, the cubic model ($m = 1$) triggers to a new universality class for $n > n_c$, while for the random $O(m)$ model ($n = 0$) this occurs for $m < m_c$. Here, n_c and m_c stand for the marginal dimensions of the cubic model and of the random $O(m)$ model.

In this study we want to reveal general restrictions for m, n considered as arbitrary positive numbers ensuring that the model (1) belongs to a new universality class. To this aim, we apply a field-theoretical RG technique to the most accurate available expansions for the RG functions of the model (1) and its special cases. In order to refine the analysis, we exploit adjusted resummation of the (asymptotic) series under consideration.

2 The treatment

In the field-theoretical RG method the change of the couplings u and v under renormalization is described by β-functions, while a fixed point (FP) $\{u^*,\, v^*\}$ corresponds to their simultaneous zero:

$$\left\{ \begin{array}{lllll} \beta_u(u^*,v^*) & = & u^*\,\varphi(u^*,v^*) & = & 0\,, \\ \beta_v(u^*,v^*) & = & v^*\,\psi(u^*,v^*) & = & 0\,. \end{array} \right. \tag{2}$$

From the set of solutions of the system (2), only the reachable stable FP corresponds to the critical point of a system. Such a FP is situated in a region with correct signs of couplings and possesses stability matrix eigenvalues with positive real parts.

The structure of the β-functions (2) yields a possibility of four FP's. The first Gaussian point $\{u^*=0, v^*=0\}$ at $d<4$ is always unstable and thus is out of physical interest. The points $\{u^*=0, v^*\neq 0\}$ and $\{u^*\neq 0, v^*=0\}$ correspond to the $O(mn)$ and $O(m)$ universality classes. We will be interested in the stability and accessibility of the mixed FP $\{u^*\neq 0, v^*\neq 0\}$ ensuring an appearance of a new non-trivial critical behaviour.

It was conjectured by Aharony [3] that the model (1) is described by a mixed FP when for $n>1$ the inequality holds: $n_c < mn < m_c n$, where n_c and m_c are suggested to coincide with the marginal dimensions of the cubic and of the random $O(m)$ models. The case $n<1$ has been investigated so far only within the bulk of studies of random models, i. e. for $n=0$ (see [7, 8] for a review). Here, the stability of the mixed FP is governed by m_c according to the Harris criterion [9].

The problem of n_c and m_c precise determination was a challenge of many studies. In particular, it has been shown with a rather high accuracy that n_c is though very close but definitely less than 3. The most accurate estimates come from the five-loop series [10] within the minimal subtraction RG scheme ($n_c = 2.87 \pm 0.05$) [11] and from the six-loop series within the massive scheme ($n_c = 2.89 \pm 0.04$) [11]. The outcome of estimating m_c states that it should be close to 2, however its numerical value remains ambiguous. Calculated so far only within the massive RG scheme, m_c was estimated to be greater than 2: $m_c = 2.01$ [12], $m_c = 2.07$ [13] and less than 2: $m_c = 1.942 \pm 0.026$ [14].

Let us first focus our attention on m_c. It can be reconstituted from any of three conditions: (i) the coordinate v^* of the mixed FP changes its sign; (ii) the $O(m)$ FP and mixed FP interchange their stability; (iii) the heat capacity critical exponent α of the $O(m)$ model equals to zero (as a consequence of the Harris criterion). It is straightforward to show that the conditions (i) and (ii) are equivalent, while the coincidence of (ii) and (iii) follows from the fact, that stability of the mixed point is governed by the sign of α [3, 15]. We will determine m_c from the condition (iii) which reduces the problem to study of the $O(m)$ model of one coupling.

We will analyze the expansions for m_c as they may be obtained in two different renormalization schemes [16]. Starting from the five-loop expansion for the exponent α of the $O(m)$ model [17] obtained within minimal subtraction scheme [18] we get m_c in the form of $\varepsilon = 4-d$-expansion:

$$m_c = 4 - 4\varepsilon + 4.707199\varepsilon^2 - 8.727517\varepsilon^3 + 20.878373\varepsilon^4. \tag{3}$$

In the massive scheme [19], the RG series for the $O(m)$ model at $d=3$ are known with record six-loop accuracy [20]. Exploiting the pseudo-ε expansion [21] we get:

$$m_c = 4 - 8/3\tau + 0.766489\tau^2 - 0.293632\tau^3 + 0.193141\tau^4 - 0.192714\tau^5. \quad (4)$$

Here, $\tau = 1$ is the pseudo-ε expansion formal parameter [21].

Since the series appearing in the RG method are known to be asymptotic at best, their analysis requires an application of special resummation procedures. Results of a detailed analysis of series (3) and (4) are presented elsewhere [16]. Here, we exploit the pseudo-ε expansion (4) both because it is obtained in a higher approximation and has better convergence properties (compare a mere summation of the first terms in ε and τ). First we present values of m_c as they are obtained from different $[M/N]$ Padé-approximants [22] in the form of a Padé-table:

$$\begin{bmatrix} 4 & 2.4 & 2.0839 & 1.9669 & 1.9398 & 1.9106 \\ 1.3333 & 1.9287 & \mathit{1.8799} & 1.9311 & \mathit{2.2425} & o \\ 2.0998 & 1.8875 & 1.9084 & 1.9085 & o & o \\ 1.8062 & 1.9227 & 1.9085 & o & o & o \\ 1.9993 & 1.9029 & o & o & o & o \\ 1.8066 & o & o & o & o & o \end{bmatrix}. \quad (5)$$

Here, the symbol o means that the approximant can not be constructed, unrealible results are written in *italic*. It is known in Padé-analysis that the main diagonal of the Padé-table and the nearest to it possess the best convergence. Note that, the six loop approximants $[3/2]$ and $[2/3]$ as well as the five loop approximant $[2/2]$ give practically the same value of m_c. Next we applied a Padé-Borel-Leroy resummation [23] to the series (4) leading to our final result $m_c = 1.912 \pm 0.004$ [16] in coherence with the estimate $m_c = 1.942 \pm 0.026$ [14].

The result for m_c is worth to be compared with the complementary results for n_c. Here, one must treat the complex model (1) of two couplings. In the minimal subtraction scheme n_c was written within the five-loop approximation as an ε-expansion [10]:

$$n_c = 4 - 2\varepsilon + 2.588476\varepsilon^2 - 5.874312\varepsilon^3 + 16.827039\varepsilon^4\,. \quad (6)$$

Based on the six-loop RG functions obtained for the model (1) at $m = 1,\, d = 3$ in massive scheme [24], we have obtained n_c as a pseudo-ε expansion [25]:

$$n_c = 4 - 4/3\tau + 0.290420\tau^2 - 0.189677\tau^3 + 0.199510\tau^4 - 0.224652\tau^5. \quad (7)$$

Similar as for m_c series (4), a simple Padé-analysis of the series (7) allows obtaining a consequence of convergent estimates of n_c along the main diagonal of the Padé-table. A refined analysis of the series (7) by means of the Padé-Borel-Leroy resummation yields $n_c = 2.862 \pm 0.005$ [25]. Within a variety of data (see [25] for a review) this result confirms the general conclusion $n_c < 3$.

Based on the above analysis we arrive at the diagram 1 (a) of the FPs stability, as it follows from the Aharony's conjecture. In the upper left region in Fig. 1 (a) both the $O(m)$ and $O(mn)$ FPs are stable, their accessibility depends on the initial couplings values. We stress that this result is valid for the "cubic-like" models $\forall\, u,\, v \geq 0$.

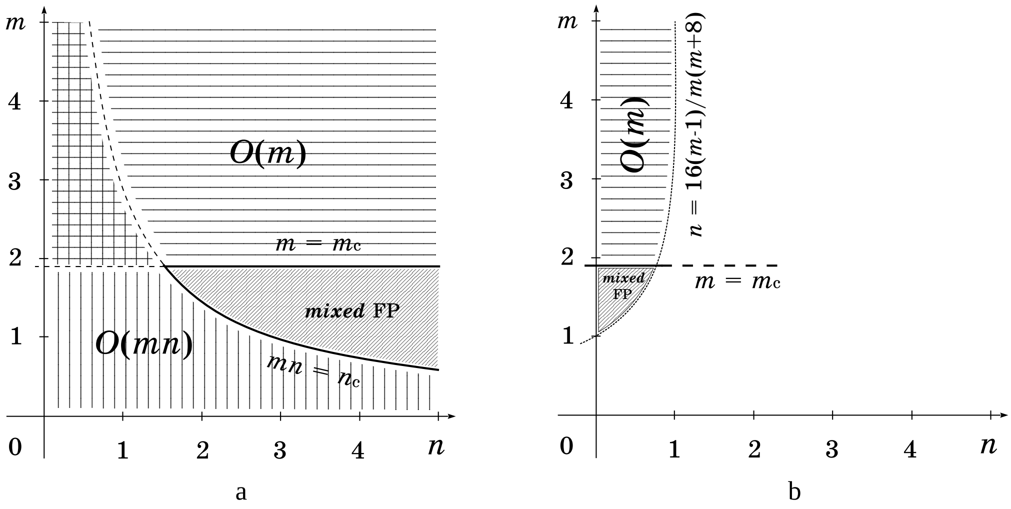

Fig. 1. The domains of the FPs stability of the "cubic-like" models $\{\forall\, u,\ v \geq 0\}$ (the left-hand figure) and those of the "disordered-like" models $\{u > 0,\ v \leq 0\}$ (the right-hand figure). See the text for the whole description.

Let us pass to the case of "disordered-like" models $u > 0, v \leq 0$. It is known, that the one-loop degeneracy of the β-functions (2) for $m = 1, n = 0$ leads in particular to the $\sqrt{\varepsilon}$-expansion of random Ising model exponents [5]. Note however, that the one-loop degeneracy is encountered for a more general relation between m and n:

$$n = \frac{16(m-1)}{m(m+8)}\,. \qquad (8)$$

The equation (8) is an exact one and governs the stability of the mixed fixed point of the "disordered-like" models as it is shown in the diagram 1 (b). The nature of the RG flow scenario in the blanc region in the Fig. 1 (b) is rather complicated. There exists a stable FP in this region, however, being unphysical it is to be eliminated from consideration. An extensive discussion of this subject will be presented elsewhere.

3 Conclusions

Traditionally, the RG approach is used as a tool for precise computation of universal characteristics of critical behaviour, i. e. critical exponents and amplitudes ratios. However, order parameters critical dimensions that govern crossovers among different universality classes are also universal and can be accurately determined within the approach. In this paper we study the conditions under which the mn vector model (1) at criticality does not belong to spherically symmetric universality class. We use adjusted resummation and extended analysis of the series for RG functions which are known for the model in high orders of perturbation theory [10, 24]. Though the nature of the exploited series is still unknown, our analysis shows that their treatment by means of Padé and Padé-Borel like analysis permits obtaining well convergent results. Along with the estimates $m_c = 1.912 \pm 0.004$, $n_c = 2.862 \pm 0.005$ which, according to the Aharony's conjecture determine the stability of the mixed FP for $n > 1$, we obtain the relation (8) supplementing it for $n < 1$. A comprehensive analysis of the calculations will be presented elsewhere.

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