ON THE CRITICAL BEHAVIOUR OF THREE-DIMENSIONAL MAGNETIC SYSTEMS WITH EXTENDED IMPURITIES[1]

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We investigate the critical properties of $d = 3$-dimensional magnetic systems with quenched defects, correlated in ε_d dimensions (which can be considered as the dimensionality of the defects) and randomly distributed in the remaining $d - \varepsilon_d$ dimensions. Our renormalization group (RG) calculations are performed in the minimal subtraction scheme. We analyze the 2-loop RG functions for different fixed values of the parameter ε_d. To this end, we apply the Chisholm-Borel resummation technique and report the numerical values of the critical exponents for the new universality class.

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The effect of weak quenched point-like disorder on the critical behaviour of magnetic systems is predicted by the Harris criterion [1]: only if the critical exponent α_p of the pure (undiluted) system is positive, i.e. the heat capacity diverges at the critical point, disorder changes the critical exponents. Only the pure Ising model is characterized by a value of $\alpha_p > 0$ and thus is affected by point-like weak disorder at criticality.

Systems with so-called "extended" (macroscopic) defects are not covered by the original Harris criterion and have attracted much interest [2–11]. Dorogovtsev [2] proposed the model of a d-dimensional m-component spin system with quenched random nonmagnetic impurities, that are strongly correlated in ε_d dimensions and randomly distributed over the remaining $d - \varepsilon_d$ dimensions. Such a system is no longer isotropic; the idea of two different correlation lengths naturally arises since the system is expected to behave differently along the directions "parallel" to the ε_d-dimensional impurity and along the "perpendicular" directions. The case $\varepsilon_d = 0$ is associated with point-like defects, and extended parallel linear (planar) defects are related to the

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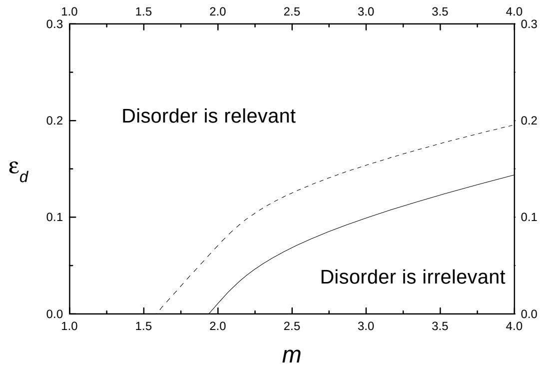

Fig. 1. The phase diagram of the critical behaviour of 3-dimensional m-component magnetic systems in different regions of the m, ε_d-plane. The solid line is obtained by substituting into (1) the 6-loop results [12] for the critical exponents of pure m-vector magnet. The dashed line shows our present results, obtained in 2-loop approximation.

cases ε_d = 1(2). This model is valid for low densities of defects. A double expansion in both $\varepsilon = 4 - d$, ε_d was suggested and renormalization group (RG) functions were calculated to order ε, ε_d [2]. These calculations were extended to the second order in Ref. [3]. It was argued, that the Harris criterion is modified in the presence of extended impurities: the randomness is relevant, if

$$\varepsilon_d > -\alpha_p/\nu_p, \tag{1}$$

where α_p, ν_p are the exponents of the pure system. Taking the best known estimates from the 6-loop $d = 3$ RG expansion [12], one finds that disorder with extended defects is relevant for $d = 3$ over a wider range of m than point defect disorder with a lower marginal value of ε_d as shown in figure 1.

Although Ref. [3] reports the RG functions with two loop accuracy a numerical analysis is provided only to order ε, ε_d. For the special case of $d = 3$ and $\varepsilon_d = 1$, corresponding to linear defects the critical exponents were calculated to the second order of an expansion in ε and ε_d, and Padé-like approximants were used to give numerical estimations [4]. In our present approach we explore a wider region of the phase diagram shown in figure 1 and apply resummation schemes that may be expected to give more reliable numerical results.

Another class of extended-defect systems are those with cubic anisotropy [5]. Here, one considers ε_d as a non-negative real number to include cases where complex random defect systems can be reduced to extended defect systems, i.e. ε_d is treated as an effective fractal dimension.

A related model with long-range-correlated quenched disorder, which is characterized by a correlation function with a power law decay $g(r) \sim r^{-a}$ with distance r, has been proposed in [6]. The influence of this type of disorder on the properties of various systems was investigated in recent numerical [7] and analytic [8] studies. A model that combines features of both extended defects and long range correlation was studied in [10].

Although systems with extended quenched defects are subject of a number of studies, the critical behaviour of 3-dimensional magnets of this type has not been completely clarified. In particular, the RG series are known to be asymptotic at best and the application of appropriate resummation techniques is needed to extract reliable numerical estimates from the series [13].

For our present study of the critical behaviour of the 3-dimensional m-component magnetic systems with ε_d-dimensional extended defects, we make use of the RG functions in 2-loop approximation as derived in Refs. [3, 4]. The model of the m-vector magnet in this context is described by the Hamiltonian [2]:

$$\mathcal{H} = \int d^d x \left[\frac{1}{2}((\mu_0^2 + V(x))\vec{\phi}^2(x) + (\nabla_\perp \vec{\phi}(x))^2 + \alpha_0 (\nabla_{||}\vec{\phi}(x))^2 + \frac{u_0}{4!}(\vec{\phi}^2(x))^2 \right], \quad (2)$$

here, $\vec{\phi}$ is an m-component vector field: $\vec{\phi} = \{\phi^1 \cdots \phi^m\}$, μ_0 and u_0 are the bare mass and the coupling of the magnetic model, and $V(x)$ represents the impurity potential. The impurity-probability distribution is defined to yield:

$$\langle\langle V(x) \rangle\rangle = 0, \quad \langle\langle V(x)V(y) \rangle\rangle = -v_0 \delta^{d-\varepsilon_d}(x_\perp - y_\perp). \quad (3)$$

Here, $\langle\langle ... \rangle\rangle$ denotes the average over the potential distribution; the positive constant $-v_0$ is proportional to both the concentration of impurities and the strength of their potential. The impurities are envisaged as ε_d-dimensional objects, each extending throughout the system along the coordinate directions symbolized as $x_{||}$, whereas in the remaining $d - \varepsilon_d$ dimensions denoted by $x_\perp$ they are randomly distributed. One assumes, that the linear size of the defects $X_{||}$ is much greater than the spin-correlation length and that the linear separation $X_\perp$ between the defects, corresponds to a defect concentration well below the percolation limit. The anisotropy constant α_0 takes into account that the extended defects make the space coordinate anisotropic. As noted in [2] the correlation of the order parameter in two points $\mathbf{x}$ and $\mathbf{x}'$ depends on the direction of the vector $\mathbf{x} - \mathbf{x}'$, due to the anisotropy. As a result, parallel and transverse components of the pair correlation function and the correlation length, and corresponding exponents $\eta_{||}, \eta_\perp, \nu_{||}$, and $\nu_\perp$ have to be introduced. The magnetic susceptibility of the system, and hence the critical exponent γ, is isotropic. The anisotropic exponents obey new scaling relations; we will turn to this fact below.

Applying the replica method to average the free energy over the random potential distribution, leads to the effective Hamiltonian [2]:

$$\begin{aligned} \mathcal{H} &= \sum_{\alpha=1}^{n} \int \mathrm{d}^d x \left[\frac{1}{2}[\mu_0^2 \vec{\phi}_\alpha^2(x) + (\nabla_\perp \vec{\phi}_\alpha(x))^2 + \alpha_0(\nabla_{||}\vec{\phi}_\alpha(x))^2] + \frac{u_0}{4!}(\vec{\phi}_\alpha^2(x))^2 \right] \\ &+ \frac{v_0}{2} \sum_{\alpha,\beta=1}^{n} \int \mathrm{d}^d x \int \mathrm{d}^d y \, \delta^{d-\varepsilon_d}(x_\perp - y_\perp) \vec{\phi}_\alpha^2(x) \vec{\phi}_\beta^2(y). \end{aligned} \quad (4)$$

Here, Greek indices denote symmetric replicas and the replica limit $n \to 0$ is implied.

To describe the long-distance properties of the model (4) near the second order phase transition, the field-theoretical RG method is used. The change of the couplings $u_0, v_0 \to u, v$ under renormalization defines a flow in parametric space, governed by corresponding β-functions. The fixed points u^*, v^* of this flow are given by the solutions of the system of equations: $\beta_u(u^*, v^*) = 0, \beta_v(u^*, v^*) = 0$. The stable fixed point is defined as the fixed point, where the stability matrix $B_{ij} = \partial\beta_{u_i}/\partial u_j$, possesses eigenvalues with positive real parts. The accessible stable fixed point corresponds to the critical point of the system. At this point, the perpendicular

	$m=1$		$m=2$		$m=3$	
ε_d	u^*	v^*	u^*	v^*	u^*	v^*
0	1.5772	-0.2416	1.1415	0	1.0016	0
0.1	1.7640	-0.4187	1.1688	-0.0372	1.0016	0
0.2	1.9169	-0.5635	1.2713	-0.1857	1.0016	0
0.3	2.0478	-0.6859	1.3509	-0.3117	1.0467	-0.1028
0.4	2.1633	-0.7919	1.4145	-0.4195	1.0908	-0.2186
0.5	2.2671	-0.8853	1.4665	-0.5125	1.1238	-0.3188
0.6	2.3619	-0.9688	1.5096	-0.5932	1.1486	-0.4053
0.7	2.4493	-1.0445	1.5457	-0.6635	1.1672	-0.4799
0.8	2.5306	-1.1131	1.5763	-0.7249	1.1812	-0.5440
0.9	2.6067	-1.1762	1.6025	-0.7788	1.1917	-0.5991
1.0	2.6783	-1.2343	1.6249	-0.8261	1.1995	-0.6461
1.1	2.7466	-1.2896	1.6443	-0.8675	1.2053	-0.6858

Tab. 1. Coordinates of stable fixed points of RG equations for magnetic systems with extended defects obtained by Chisholm-Borel resummation in 3d scheme.

components of the critical exponents $\eta_\perp, \nu_\perp$ and the parallel component $\nu_{||}$ are defined by appropriate RG functions (see [2], [3] for details). Other critical exponents can be obtained from the scaling relations [2]:

$$\gamma = (2-\eta_\perp)\nu_\perp = (2-\eta_{||})\nu_{||}; \quad \alpha = 2-(d-\varepsilon_d)\nu_\perp - \varepsilon_d\nu_{||}. \tag{5}$$

In order to derive the quantitative characteristics of the critical behaviour of magnetic systems with extended impurities, we analyze the 2-loop RG functions, obtained in [3, 4] in the minimal subtraction scheme. The minimally subtracted RG flow with one parameter ε ($\varepsilon_d = 0$) has only trivial (engeneering) dimension dependence and can be evaluated for fixed $\varepsilon = 1$ [14]. We propose to extend this approach to the RG flow of the present model, i.e. to treat it directly at $d = 3$ $(\varepsilon = 1)$ and for different fixed values of the (fractal) defect dimensionality ε_d. The expansions for the RG functions have the form of divergent series with zero radius of convergence, familiar to the theory of critical phenomena [13]. We obtain our results by a two-variable Chisholm-Borel resummation technique [15, 16]. It has shown its efficiency in the analysis of the RG expansions for models with point-like disorder [16, 17]. We present the values of the stable fixed point coordinates and the critical exponents of the 3-dimensional m-component magnetic systems with extended impurities at $m = 1; 2; 3$ in Tables 1 and 2.

Let us draw some conclusions from the results shown. As mentioned above, the case $\varepsilon_d = 0$ describes point-like quenched disorder and the well known results are reproduced: for $m = 1$ this type of disorder is relevant according to the Harris criterion and the corresponding random fixed point $(u \neq 0, v \neq 0)$ is stable. Increasing the parameter ε_d leads to a shift of the stable fixed point value. For $m = 2; 3$ the situation differs. Here, point-like disorder is irrelevant, and the pure fixed point is stable, whereas the random fixed point lies in an unphysical region and is unstable. With the increase of ε_d at some marginal value $\varepsilon_d^{\rm marg}$ the random fixed point moves to the physical region and becomes stable. This corresponds to crossover to a new universality class. Note, that the relation $\nu_{||} > \nu_\perp$ holds for every ε_d and m. The extended defects cut interacting paths of

	$m=1$			$m=2$			$m=3$		
ε_d	$\nu_{\vert\vert}$	$\nu_{\perp}$	γ	$\nu_{\vert\vert}$	$\nu_{\perp}$	γ	$\nu_{\vert\vert}$	$\nu_{\perp}$	γ
0	–	0.665	1.308	–	0.684	1.344	–	0.720	1.411
0.1	0.714	0.680	1.338	0.691	0.688	1.352	0.720	0.720	1.411
0.2	0.741	0.692	1.362	0.719	0.705	1.386	0.720	0.720	1.411
0.3	0.765	0.702	1.384	0.744	0.718	1.414	0.740	0.732	1.438
0.4	0.786	0.712	1.402	0.766	0.730	1.439	0.763	0.746	1.467
0.5	0.805	0.720	1.419	0.785	0.739	1.460	0.784	0.757	1.493
0.6	0.822	0.727	1.434	0.802	0.747	1.479	0.801	0.766	1.515
0.7	0.838	0.733	1.448	0.818	0.754	1.495	0.817	0.773	1.533
0.8	0.853	0.739	1.460	0.831	0.760	1.509	0.831	0.779	1.549
0.9	0.867	0.745	1.472	0.843	0.765	1.522	0.842	0.783	1.562
1.0	0.880	0.750	1.483	0.854	0.769	1.532	0.852	0.787	1.573
1.1	0.892	0.754	1.493	0.863	0.773	1.542	0.860	0.789	1.581
1.0 [4]	0.84	0.67	1.34	0.60	0.56	1.13	0.66	0.61	1.24

Tab. 2. Critical exponents of magnetic systems with extended defects obtained by Chisholm-Borel resummation in 3d scheme. For comparison the last line shows the results of Ref. [4].

spins perpendicular to the extended-defect direction, so in the parallel direction the fluctuations are stronger and the correlation length more sharply diverges.

The marginal value of $\varepsilon_d^{\rm marg}$, at which the crossover to the random fixed point occurs is shown in Figure 1 by a dashed line. Note, that in the first order of ε, ε_d-expansion the disorder is relevant for every $m < 4$ and positive ε_d [3].

Another interesting question concerns existence of an upper critical value for the defect dimensionality ε_d. This question has not raised in previous works, where the double $\varepsilon, \varepsilon_d$-expansion was exploited. In our analysis, increasing ε_d leads to the appearance of poles in the Chisholm approximants for $\varepsilon_d > 1$ making impossible a definite answer about the presence and stability of the fixed points for high ε_d. However, interpreting ε_d as a fractal dimensionality of defects, it is clear that it can not exceed the dimension of the embedding space ($d = 3$). Moreover, physically one may expect that extended defects of large dimension (e.g., planar defects with $\varepsilon_d = 2$) will divide the system into non-interacting regions and thus prevent it from ferromagnetic ordering.

The numbers given in Table 2 provide numerical estimations for the critical exponents of the 3-dimensional m-component magnetic systems in the presence of extended defects with (fractal) dimensionality ε_d. Unfortunately, we have not come across any Monte Carlo investigation of the model [18]. The only numerical results for the exponents of the model (4) were obtained so far for $\varepsilon_d = 1$ using the Padé-analysis [4], they are shown in the last line of Table 2. We note however, that our present resummation technique has shown its efficiency and accuracy in studies of models with point-like structural disorder [17] and is generally known to provide more reliable data as compared to the simple Padé-analysis.

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