

**LARGE ORDER ASYMPTOTICS AND CONVERGENT PERTURBATION THEORY  
FOR CRITICAL INDICES OF THE  $\phi^4$  MODEL IN  $4 - \epsilon$  EXPANSION<sup>1</sup>****J. Honkonen<sup>2\*</sup>, M. Komarova<sup>3†</sup>, M. Nalimov<sup>4†</sup>***\*Department of Physical Sciences, University of Helsinki, and  
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Large order asymptotic behaviour of renormalization constants in the minimal subtraction scheme for the  $\phi^4$  ( $4 - \epsilon$ ) theory is discussed. Well-known results of the asymptotic  $4 - \epsilon$  expansion of critical indices are shown to be far from the large order asymptotic value. A convergent series for the model  $\phi^4$  ( $4 - \epsilon$ ) is then considered. Radius of convergence of the series for Green functions and for renormalisation group functions is studied. The results of the convergent expansion of critical indices in the  $4 - \epsilon$  scheme are revalued using the knowledge of large order asymptotics. Specific features of this procedure are discussed.

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**1 Introduction**

Calculation of critical indices is usually based on a certain asymptotic expansion. To obtain reliable results a resummation procedure is necessary. To this end the Borel-Leroy transform in fixed dimension [1, 2] or in  $\epsilon$  expansion [2–4], the simple Padé-Borel method [5] as well as self-similar exponential approximants [6] have been used. But divergent series can produce an arbitrary result. Thus some additional information about the series is needed.

When a convergent series is considered, additional information can also improve the result. The knowledge about location and character of singularity of the function investigated or about the large order asymptotic behaviour of the expansion may be used to accelerate convergence of the series. In this report we will discuss a resummation of the  $4 - \epsilon$  expansion of critical indices for the  $\phi^4$  theory with help of the large order asymptotics.

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## 2 Large order asymptotics of renormalization constants

Large order asymptotic behaviour of the  $O(n)$  symmetric  $\phi^4$  theory is well known. The  $N$ -th order term of the standard expansion for an arbitrary function  $F$  in the theory with the renormalized action

$$S_R(\phi, g) = \frac{1}{2} Z_\phi^2 \partial_i \phi_\alpha \partial_i \phi_\alpha + \frac{1}{2} Z_\phi^2 Z_\tau \tau \phi^2 + \frac{1}{4!} Z_\phi^4 Z_g g \mu^\epsilon (\phi^2)^2 \quad (1)$$

behaves at large  $N$  as ([7–9])

$$F^{(N)} \approx N^N e^{-N} a^N N^{b_F} c_F. \quad (2)$$

The notation  $F^{(N)}$  for the  $N$ -th order coefficient of the expansion of the function  $F$  will be used henceforth. Working within the minimal subtraction (MS) scheme in the  $4 - \epsilon$  expansion we have corrected the result (2). Namely, analysing the limits  $N \rightarrow \infty$  and  $\epsilon \rightarrow 0$  in more detail, we have obtained a more accurate estimate for the amplitudes  $c_F$  of the renormalization constants [10]. According to the results of Ref. [7] the asymptotic expression (2) for the  $\beta$  function coincides practically with the exact result for  $N = 3$ , but the asymptotics of [7] for  $N = 4, 5$  are larger than the exact value. Contrary to these conclusions, we have shown that the asymptotics (2) for the renormalization constants (and for the critical indices) are much smaller than the exact ones [11]. For example,  $[Z_g^{(5)}]_{asympt} \approx .01[Z_g^{(5)}]$  for  $n = 1$ , where  $[Z_i]$  is the residue of  $Z_i$  regarded a function of complex  $\epsilon$ . We have predicted that only starting from 10 - 15-th order the perturbation expansion could be near to the asymptotic value. Due to this fact we can state that the Borel transform of the 5 known terms of the  $\epsilon$  expansion has no theoretical ground.

Nevertheless, we propose extrapolation expressions for the unknown terms in the expansions of renormalization constants  $Z$  in the following form [10]:

$$\begin{aligned} [\bar{Z}_g^{(N)}]_{asympt} &= [Z_g^{(N)}]_{asympt} \left( 1 + \frac{\bar{c}_g}{N} \right), \quad \bar{c}_g = \frac{5[Z_g^{(5)}]}{[Z_g^{(5)}]_{asympt}} - 5 \\ [\bar{Z}_\phi^{(N)}]_{asympt} &= [Z_\phi^{(N)}]_{asympt} \left( 1 + \frac{\bar{c}_\phi}{N} \right), \quad \bar{c}_\phi = \frac{5[Z_\phi^{(5)}]}{[Z_\phi^{(5)}]_{asympt}} - 5. \end{aligned} \quad (3)$$

These expressions contain an additional correction of the  $1/N$  type normalized by direct comparison of the asymptotic value (2) for  $N = 5$  with the exact one. In (3),

$$[Z_g^{(N)}]_{asympt} = (-1)^N C_g N^N N^{\frac{7+n}{2}} e^{-N}, \quad [Z_\phi^{(N)}]_{asympt} = (-1)^N C_\phi N^N N^{\frac{3+n}{2}} e^{-N} \quad (4)$$

$$\begin{aligned} C_g &= -\frac{864\pi^{2+n/2} D_{et}}{n(n+2)\Gamma(n/2)} \exp \left( \frac{n+8}{6} \left[ \Psi(1) - \ln(\pi) - 2 \right] \right), \\ C_\phi &= -\frac{6\pi^{2+n/2} D_{et}}{n\Gamma(n/2)} \exp \left( \frac{n+8}{6} \left[ \Psi(1) - \ln(\pi) - 2 \right] \right), \end{aligned} \quad (5)$$

$$D_{et} = 2^{(7-4n)/2} 3^{5n/2} \pi^{-(5+n)/2} 5^{-5/2} e^{-4(n+2)/3-7(n+8)/18-R/12+(n+8)(\Psi(1)+2-\ln(\pi))/6},$$

$$R := - \sum_{l=2}^{\infty} \sum_{p=3}^{\infty} \frac{(l+1)(l+2)(2l+3)}{p(l+1)^p(l+2)^p} \left( 6^p + 2^p(n-1) \right) \quad (6)$$

Here,  $\Psi$  is the logarithmic derivative of the  $\Gamma$  function. Using similar expressions normalized with help of the 4-th exact term of the renormalization constant expansions, we could find the 5-th order term and estimate the accuracy of such calculation as at least 80%. Besides, we hope that the expressions (3) can give us a reliable estimate for more than 5 orders of the expansions of  $Z$ 's.

### 3 Convergent expansion for critical exponents

We will use the knowledge of large order asymptotics to improve initially convergent series for critical exponents. In Ref. [12] a new approach was suggested to calculate critical indices. It was based on the modified expansion for the  $O(n)$ -symmetric  $\phi^4$  model [13] which leads to *convergent* series. The standard action  $S = S_1 + S_2$  with  $S_1 = \int dx (\partial\phi)^2/2$ ,  $S_2 = g \int dx \phi^4/4!$  was cast in the form  $S = S_0 + S_I$ , where  $S_0 = S_1 + aS_1^2$  and  $S_I = \zeta(S_2 - aS_1^2)$ . Here,  $a$  is an arbitrary constant and  $\zeta$  is a new expansion parameter. The model coincides with the initial one at the 'physical' value of  $\zeta$ :  $\zeta_{ph} = 1$ . It was stated in Ref. [13] that the  $\zeta$  expansion is convergent for  $\zeta \leq \zeta_c = 1$ , when

$$a \geq \frac{g}{64\pi^2} = a_{min}. \quad (7)$$

In the framework of the  $\zeta$  expansion the renormalized  $2k$  point Green function can be written as an integral with respect to an additional variable  $\sigma$  as:

$$\begin{aligned} G_{2k}^R &= \frac{1}{\sqrt{\pi}} \int d\sigma e^{-\sigma^2} \int D\phi \frac{\phi(\mathbf{x}_1) \dots \phi(\mathbf{x}_{2k})}{(1 + 2i\sigma\sqrt{a(1-\zeta)})^k} \tilde{Z}_\phi^{-2k} \\ &\times \exp \left( -\frac{1}{2} \partial\phi\partial\phi - \frac{\zeta g \tilde{Z}_g \mu^\epsilon \phi^4}{4!(1 + 2i\sigma\sqrt{a(1-\zeta)})^2} \right). \end{aligned} \quad (8)$$

Here,  $g$  is the renormalized charge,  $\mu$  is the renormalization mass,  $Z_i(g)$ , and  $\tilde{Z}_i = Z_i(g\zeta/(1 + 2i\sigma\sqrt{a(1-\zeta)})^2)$  are the renormalization constants in the usual perturbation theory. This representation allows to construct the Feynman graphs in the usual manner. Due to the integration with respect to  $\sigma$  every diagram acquires an additional factor [13]:

$$\begin{aligned} U_j(a) &= 1/\sqrt{\pi} \int d\sigma \exp(-\sigma^2)/(1 + 2i\sigma\sqrt{a})^j \\ &= \int_0^\infty dt t^{j-1} \exp(-t - at^2)/(j-1)! \end{aligned} \quad (9)$$

The basic renormalisation group (RG) equation for the Green functions (8) was derived in Ref. [12] in the form

$$\mathcal{D}_{RG} G_{2k}^R \equiv [\mu\partial_\mu + \beta\partial_g + \beta_2\partial_g^2 + \dots + \gamma] G_{2k}^R = 0, \quad (10)$$

where  $\gamma$  and all  $\beta$  functions depend on the parameters  $g$ ,  $\zeta$ ,  $a$  and  $k$ . Using the MS scheme one can write for the RG functions in  $D = 4 - \epsilon$  dimensional space ( $n = 1$ ,  $k = 1$ ) the expressions

$$\gamma = -\frac{2}{U_1(a(1-\zeta))}g\partial_g \sum_{l=0}^2 g^l \zeta^l [Z_\phi^{(l)}] U_{2l+1}(a(1-\zeta)), \quad (11)$$

$$\begin{aligned} \beta &= -\epsilon g + \frac{1}{U_3(a(1-\zeta))}g\partial_g \sum_{l=0}^2 g^l \zeta^l \left( [Z_g^{(l)}] - 2[Z_\phi^{(l)}] \right) U_{2l+3}(a(1-\zeta)) - \gamma g \\ &\equiv -\epsilon g + \bar{\beta}(g)g \end{aligned} \quad (12)$$

It was shown that Eq. (10) governs the large-scale asymptotic behavior of the model. Critical exponents are related to the anomalous dimensions in the usual way [9]:  $\eta = \gamma(g_*)/k$ , and  $1/\nu = 2 - \gamma_\tau(g_*) + k\eta$  [ $\gamma_\tau$  is determined by equation (10) for the Green functions with the insertion of the composite operator  $\phi^2$ ]. Here  $g_*$  is the fixed point determined by the usual equation  $\beta(g_*) = 0$ , or

$$\epsilon = \bar{\beta}(g_*). \quad (13)$$

In Ref. [12] it was demonstrated that it is possible to solve Eq. (13) iteratively to calculate  $g_*$  in the form of a double expansion in  $\epsilon$  and  $\zeta$ . As a result, the exponents  $\eta$  and  $\nu$  were calculated for  $n = 1$ ,  $k = 1$ ,  $a = 0.134$  at the physical values  $\zeta = \epsilon = 1$  (see the table). The columns correspond to expansion order taken into account. Exponents marked by  $\epsilon$  are the results of the usual  $\epsilon$  expansion, which are quoted for comparison.

Exponent	1	2	3	4	5
$\eta$	0	0.02553	0.04342	0.039745	0.039741
$\eta_\epsilon$	0	0.01852	0.03721	0.02888	0.05454
$\nu$	0.621	0.671	0.663	0.674	0.651
$\nu_\epsilon$	0.583	0.627	0.607	0.678	0.461

Estimates based on the Borel transform of  $\epsilon$  expansion are  $\eta = 0.0360 \pm 0.0050$  [2],  $\eta = 0.035 \pm 0.002$  [3],  $\nu = 0.6290 \pm 0.0025$  [2],  $\nu = 0.628 \pm 0.001$  [3]. A typical lattice result is  $\nu = 0.6305 \pm 0.0015$  [9]. Thus, in spite of convergence of the  $\zeta$  expansions, the accuracy of the results of [12] is lower than in case of estimates based on Borel transforms. This is why we will improve the convergent expansion using the large order asymptotics.

#### 4 Large order asymptotics for the convergent perturbation expansion

In Ref. [13] it was stated that instanton analysis could not be used for the investigation of the convergent  $\zeta$ -expansion of Green functions. Using more sophisticated examination, the radius of convergence was estimated as  $\zeta_c = 1$ .

Contrary to this, we confirm the adequacy of instanton approach here. Indeed, let us determine the  $N$ -th order of  $\zeta$  expansion by

$$G_{2k}^{(N)} = \oint \frac{d\zeta}{\zeta^{N+1}} G_{2k},$$

where  $G_{2k}$  is given by (8). A subsequent steepest descent approach in the variables  $\sigma$ ,  $\zeta$  and  $\phi$  leads to an instanton and stationary point (for  $n = 1$ )

$$\phi_s = \sqrt{N}\phi_0 \left( \frac{2\sigma_0 \sqrt{a(1-\zeta_s)}}{\sqrt{\zeta_s g}} - \frac{i}{6\sqrt{\zeta_s g N}} + O\left(\frac{1}{N}\right) \right),$$

where

$$\phi_0 = \frac{4\sqrt{3}}{y} \frac{1}{1 + (\mathbf{x} - \mathbf{x}_0)^2/y^2},$$

with arbitrary  $\mathbf{x}_0$ ,  $y$ , and

$$\zeta_s = \bar{\zeta}_c \left( 1 + \frac{7}{6\sqrt{aN}} + O\left(\frac{1}{N}\right) \right), \quad \bar{\zeta}_c = \frac{1}{1 - g/(64\pi^2 a)}$$

$$\sigma_s = \sqrt{N}\sigma_0, \quad \sigma_0 = \sqrt{\bar{\zeta}_c} \sqrt{\frac{g}{64\pi^2 a}} + \frac{\bar{\zeta}_c}{\sqrt{N}} \frac{568a\pi^2 + 3g}{96\pi a\sqrt{g}} + O\left(\frac{1}{N}\right).$$

The instanton analysis results in the following behavior for the  $N$ -th order of the  $\zeta$ -expansion of the  $2k$  point Green function:

$$G_{2k}^{(N)} \sim N^\alpha \bar{\zeta}_c^N e^{-\sqrt{\frac{N}{a}}} \quad (14)$$

with some  $\alpha$ .

Contrary to [13] we have obtained a radius of convergence  $\bar{\zeta}_c$  which tends to infinity as  $a \rightarrow a_{min}$ . We consider the unrenormalized Green function in  $D = 4$  dimensional space only, but this is not essential for the treatment of the radius of convergence. The same result (14) can be obtained directly by the method proposed in [13]. In such a case the instanton  $\phi_c \sim N^{1/4}$ . Thus, the instanton does exist.

For usual divergent series of  $\epsilon$  expansions the large order asymptotics of Green functions characterise unambiguously the asymptotics of RG functions. In the case of convergent series the situation is more difficult. Namely, the denominators of the expressions (11), (12):  $U_i(a(1-\zeta))$  ( $i = 1, 3$ ) have the radius of convergence equal to 1. Therefore, the large order asymptotics of the  $\zeta$  expansion of these functions

$$U_i^{(N)}(a(1-\zeta)) \approx N^{i/2-1} e^{-\sqrt{N/a} + 1/(8a)} a^{1-i} / \Gamma(i)$$

have a non-trivial influence on the asymptotic behaviour of  $\beta$  and  $\gamma$  functions.

## 5 Character of singularity of the convergent expansion for critical indices

In addition to  $\zeta_c$  problem we have some difficulties with double  $\zeta$ ,  $\epsilon$  expansion of indices as we try to find the fixed point  $g_*$  of (13). The iteration solution  $g_*(\epsilon, \zeta)$  of the equation (13) has a

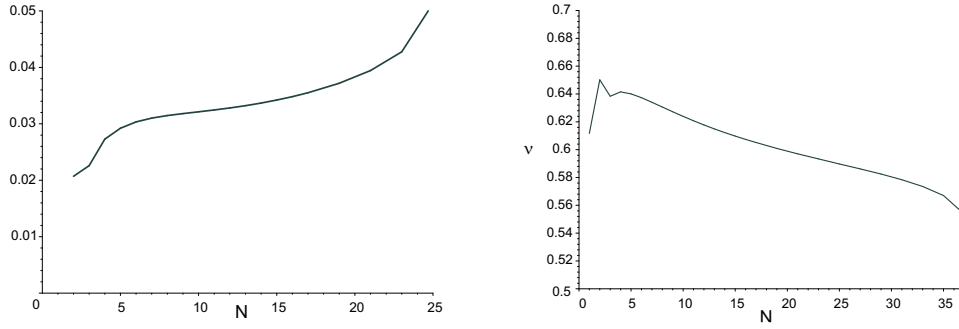


Fig. 1. The value of the index  $\eta$  and the index  $\nu$  as a function of perturbation order number  $N$  taken into account.

new singularity at the point  $\epsilon_m$  bounded by the nearest to zero extremum of the  $\bar{\beta}$  in the complex plane  $g$ . Namely, the point  $g_m$  given by

$$\epsilon_m = \bar{\beta}(g_m, \zeta), \quad \partial_g \bar{\beta}(g_m) = 0 \tag{15}$$

results in the singularity  $g_* \sim \sqrt{\epsilon - \epsilon_m}$ .

To calculate  $\epsilon_m$  the second equation (15) has to be solved in a form of a  $\zeta$  expansion. Unfortunately, our calculation shows that the five known terms of the expansion of  $\bar{\beta}$  are not sufficient for a reliable determination of  $g_m$ . If the extrapolation expressions (3) are used within the Padé approximant approach, then there is a whole set of conjugate points  $g_m$ . It is very difficult to find a suitable 'mapping' to exclude these singularities. Moreover, the corresponding minimal value of  $\epsilon_m$  turns out to be very small ( $\sim .02$ ) that decreases the radius of convergence in  $\epsilon$  of our double expansion. Thus the method to be used is to consider the variables  $g, \zeta$  instead of  $\epsilon, \zeta$ . A similar approach leads to a good result in the Kraichnan model, where a convergent series is dealt with as well [14].

Calculating  $g_*$  directly from the equation (13) for  $\epsilon = 1$  we were able to obtain five orders of the  $\zeta$  expansion for critical indices (for  $n = 1, \zeta_{ph} = 1$ ):

Exponent	1	2	3	4	5
$\eta$	0	.021	0.023	0.027	0.029
$g_*/(16\pi^2)$	0.96	0.86	0.65	0.61	0.57
$\nu$	0.612	0.650	0.638	0.641	0.640
$g_*/(16\pi^2)$	1.38	1.17	0.84	0.75	0.68

The exponent  $\eta$  was calculated for the value  $a = .14$ , the exponent  $\nu$  for  $a = .17$  which lead to the best rate of series convergence. Note that  $g_*$  obtained ensures the convergence of the  $\zeta$  series in both cases. However, using the extrapolation expressions (3) for unknown terms of the expansions of the renormalisation constants  $Z$ , we obtained the results shown in Fig. 1 demonstrating the failure of the usual extrapolation procedure in the calculation of critical indices.

To improve the convergence of the  $\zeta$  series we can investigate the large order asymptotics of the functions (11), (12). Denominators of these expressions have essential singularities of the type  $U_1(a(1-\zeta))$ ,  $U_3(a(1-\zeta))$  that must be extracted. It is convenient to introduce  $\delta Z_\phi^{(N)} \equiv [Z_\phi^{(N)}] - [\bar{Z}_\phi^{(N)}]_{asympt}$ . Let us rewrite the asymptotic expression (4) of the renormalization constant  $Z_\phi$  in the equivalent form

$$[\bar{Z}_\phi^{(N)}]_{asympt} = C_\phi (-1)^N N^2 \frac{(2N)!}{4^N \sqrt{2} N!} \left(1 + \frac{c_\phi}{N}\right).$$

Here, the constant  $c_\phi$  can be found in a way similar to  $\bar{c}_\phi$  in (3) by comparison of  $[\bar{Z}_\phi^{(5)}]_{asympt}$  with the exact expression  $[Z_\phi^{(5)}]$ .

Then substituting  $[\bar{Z}_\phi^{(5)}]_{asympt}$  into (11) and re-expanding it in  $\zeta$  one can write  $\gamma$  in the form

$$\begin{aligned} \gamma = & -2 \sum_{j=1}^5 \frac{j(g\zeta)^j}{(16\pi^2)^j} \frac{U_{2j+1}(a(1-\zeta))\delta Z_\phi^{(j)}}{U_1(a(1-\zeta))} - \frac{\sqrt{2}C_\phi}{U_1(a(1-\zeta))} \times \\ & \times \left( -\frac{(g\zeta)^3}{(64\pi^2)^3} 6!U_7 + \frac{(g\zeta)^2}{(64\pi^2)^2} 4!(3+c_\phi)U_5 - \frac{(g\zeta)}{(64\pi^2)} 2!(1+c_\phi)U_3 \right). \end{aligned} \quad (16)$$

Here, the omitted for brevity argument of the  $U_i$  functions is  $a(1-\zeta) + g\zeta/64\pi^2$ . Thus, we have investigated not only the location of  $\zeta_c$  and  $\bar{\zeta}_c$ , but also the character of the singularities of the  $\zeta$  expansion of the  $\gamma$  function.

In analogy with (16) we obtain for the  $\beta$  function

$$\begin{aligned} \beta = & \sum_{j=1}^5 j \frac{(g\zeta)^j}{(16\pi^2)^j} \frac{U_{2j+3}(a(1-\zeta))(\delta Z_g^{(j)} - 2\delta Z_\phi^{(j)})}{U_3(a(1-\zeta))} + \frac{C_g}{2^{5/2}U_3(a(1-\zeta))} \times \\ & \times \left( -\frac{(g\zeta)^3}{(64\pi^2)^3} 8!U_9 + \frac{(g\zeta)^2}{(64\pi^2)^2} 6!(3+c_g)U_7 - \frac{(g\zeta)}{(64\pi^2)} 4!(1+c_g)U_5 \right) - \gamma \end{aligned} \quad (17)$$

The expressions (16), (17) are physically meaningful at  $\zeta = 1$ . Solving numerically Eq. (13) with  $\beta$  from (17) and substituting the value  $g_* = 0.382$  obtained in  $\gamma$  (16) we obtain  $\eta = 0.0236$ . Taking into account the  $\epsilon$  expansion up to fourth order one obtains similarly  $\eta_4 = 0.0241$ .

An analogous procedure for the index  $\nu$  leads to  $\nu = 0.580$ , ( $\nu_4 = 0.624$ ). These results are the best we can obtain from the  $\epsilon$  expansion in the convergent scheme using all available information about the large order behaviour.

It is worthwhile noting that the accuracy of our results is similar to the accuracy of the approximation (3) determined by the rate with which asymptotics of renormalization constants tend to the exact value. Thus, we conclude that the accuracy of the  $\epsilon$  expansion resummation with help of the Borel transformation is lower than commonly quoted.

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