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INTERNAL SPACE RENORMALIZATION GROUP METHODS FOR ATOMIC AND CONDENSED MATTER PHYSICS[1]

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The functional renormalization group method is used to take into account the vacuum polarization around localized bound states generated by external potential. The application to Atomic Physics leads to improved Hartree-Fock and Kohn-Sham equations in a systematic manner within the framework of the Density Functional Theory. Another application to Condensed Matter Physics consists of an algorithm to compute quenched averages with or without Coulomb interaction in a non-perturbative manner.

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1 Introduction

The renormalization group will be used in this talk as an algorithm to solve strongly coupled quantum field theories without any intention to gain insight into the scale dependence of the dynamics. We shall study the system of non-relativistic electrons propagating in the presence of static external potential. We consider non-relativistic systems because the comparison with experiment is more direct than in the relativistic domain. The common challenge in both regimes is to trace the polarization effects in the vacuum, i.e. the particle-hole or the particle-anti particle fluctuations in the non-relativistic or relativistic region, respectively.

We shall consider two different cases. First, the electrons will be placed in an external localized field and one finds a problem with inhomogeneous ground state, characteristic of Atomic Physics. Second, we assume the presence of a static random external impurity potential which obeys a Gaussian probability distribution and we introduce a scheme to compute the quenched averages of kinetic transport coefficients in a translation invariant manner. The common technics applied is a generalization of the functional renormalization group [1] where the role of the running cut-off is played by an arbitrary control parameter which generates differentiable changes in the dynamics.

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2 Renormalization in the internal space

The degrees of freedom are eliminated successively in the renormalization group method, for instance the Kadanoff-Wilson blocking strategy orders the modes according to their scales in the external space, i.e. in the the space-time (energy-momentum). One may construct a blocking procedure where the blocking proceeds in an order determined by a scale in the internal space, the space of the field amplitude. A well known example is the Callan-Symanzik equation where the fluctuations with larger and larger amplitudes are taken into account as the mass of the particles is lowered.

It is easy to generalize this scheme for any parameter in the dynamics by means of functional techniques [2]. In fact, let us suppose that the dynamics contains a control parameter λ and the generator functional $W_\lambda[j]$ for the connected Green functions of a scalar field ϕ is given by

$$e^{\frac{i}{\hbar}W_\lambda[j]} = \int \mathcal{D}[\phi] e^{\frac{i}{\hbar}(S_\lambda[\phi]+j\cdot\phi)} \tag{1}$$

where $f\cdot g = \int_x f_x g_x$, $\int_x = \int d^4x$ and $S_\lambda[\phi]$ is the action. The evolution equation

$$e^{\frac{i}{\hbar}W_\lambda[j]}\partial_\lambda W_\lambda[j] = \int \mathcal{D}[\phi]\partial_\lambda S_\lambda[\phi] e^{\frac{i}{\hbar}(S_\lambda[\phi]+j\cdot\phi)} = \partial_\lambda S_\lambda\left[\frac{\hbar}{i}\frac{\delta}{\delta j}\right] e^{\frac{i}{\hbar}W_\lambda[j]} \tag{2}$$

is valid without assuming the existence of any small parameter. The way this equation is obtanied is reminescent of the derivation of the Schwinger-Dyson equations except that the latter contains the complete action and the former the suppression controling part, $\partial_\lambda S_\lambda$, only. It is worthwhile noting that this relation appears to be such a functional generalization of the Hellman-Feynman theorem corresponding the Hamiltonian H_λ [3],

$$\partial_\lambda E_\lambda = \langle E_\lambda|\partial_\lambda H_\lambda|E_\lambda\rangle, \quad E_\lambda|E_\lambda\rangle = H_\lambda|E_\lambda\rangle, \tag{3}$$

for time dependent processes and general matrix elements which remains compatible with approximations one employs to solve the functional differential equation (2) in restricted functional spaces.

In the examples presented here the electric charge and the average strength of the impurity field fluctuations will be chosen as control parameters. The corresponding evolution equations will be written for a local functional, the effective action for the density and the current. The effective action of the physical system can be obtained by integrating these equations from an artificial perturbative initial condition imposed at weak Coulomb interaction and disorder into the physical regime.

Similar schemes could be constructed by means of the more traditional renormalization group method, based on the external space. But the present version has the following two advantages. First, it preserves gauge invariance, an important feature in the computation of the electric conductivity. Second, it avoids the artificial discontinuities during the evolution which may occur when the saddle point structure is changed by the running cut-off [4].

3 Density Functional Theory

The density functional theory [5,6] is a powerful method to describe bound states induced by external potential. The quantity of central importance is the density functional, $E_{\rm gr}[\rho]$, the ground state energy when the the density is constrained to be ρ. According to the the Hohenberg-Kohn theorems the ground state energy of the original problem is the minimum of the density functional, $E_{gr} = E_{\rm gr}[\rho_{\rm gr}]$. The shortcoming of the traditional approach is its phenomenological nature, the lack of a constructive definition of the density functional. We propose that the effective action $\Gamma[\rho]$ for the density ρ [7] provides a clear and generally applicable definition of the density functional. Furthermore, the internal space renormalization group method leads to a systematical approximation scheme which turns the phenomenological knowledge about correlations into an improvement of the approximation by choosing the ansatz for $\Gamma[\rho]$ in an appropriate manner [8].

The generator functional for the connected Green function is defined as

$$e^{\frac{i}{\hbar}W[\sigma]} = \int \mathcal{D}[u]\mathcal{D}[\psi]\mathcal{D}[\psi^\dagger] e^{\frac{i}{\hbar}\psi^\dagger\cdot(G^{-1}+eu+\sigma_\alpha j_\alpha)\cdot\psi+\frac{i}{2\hbar}\partial u\cdot\partial u}, \tag{4}$$

where ψ and u denote the electron and the temporal photon fields, respectively, $j_{\alpha,x}$ stands for the electric density ($\alpha = 0$) and current ($\alpha = 1,2,3$),

$$G^{-1}_{x,x'} = \delta_{x,x'}\left(\partial_t - \frac{\hbar^2}{2m}\Delta_{\mathbf{x}} - \mu + V^{ext}_{\mathbf{x}}\right) \tag{5}$$

is the inverse of the non-interacting electron propagator in the presence of an external potential V^{ext} and chemical potential μ. We shall consider the density Green functions and use $\alpha = 0$ only in this section. The Legendre transform of the $W[\sigma]$, the effective action $\Gamma[\rho]$ for the density ρ reaches its minimum at the ground state density, $\beta E_{gr} = \Gamma[\rho_{\rm gr}]$, and will serve as the density functional. The effective action is first obtained for imaginary time, at finite temperature, $T = 1/\beta$, and the projection onto the ground state is achieved by taking the zero temperature limit $\beta \to \infty$. After the rescaling $e \to \lambda e$ of the electric charge one can derive the evolution equation

$$\begin{aligned}\partial_\lambda \Gamma_\lambda[\rho] &= -\lambda e^2 \left\{ \rho\cdot\frac{1}{\Delta}\cdot\rho + {\rm T}r\left[\left(\frac{\delta^2\Gamma_\lambda[\rho]}{\delta\rho\delta\rho}\right)^{-1}\cdot\frac{1}{\Delta}\right]\right\} \\ &= \int_{\mathbf{x},\mathbf{y},t} \langle\psi^\dagger_{\mathbf{x},t}\psi_{\mathbf{x},t}\psi^\dagger_{\mathbf{y},t}\psi_{\mathbf{y},t}\rangle \partial_\lambda \frac{\lambda^2 e^2}{8\pi|\mathbf{x}-\mathbf{y}|},\end{aligned} \tag{6}$$

where $\int_{\mathbf{x}} = \int d\mathbf{x} = \int d^3x$. The second equation shows that the evolution equation simply follows the change of the Coulomb energy according to the Hellman-Feynman theorem.

The initial condition for the effective action is imposed at weak Coulomb interaction, $\lambda_0 \approx 0$, where we find in the leading order perturbation expansion for spinless electrons

$$\Gamma[\rho] = \frac{1}{2}(\rho-\rho^*)\cdot\tilde{\mathcal{G}}^{-1}\cdot(\rho-\rho^*) - \rho\cdot\frac{1}{2\Delta}\cdot\rho + \mathcal{C}[\rho] \tag{7}$$

with $\mathcal{C}[\rho] = -{\rm T}r\log G^{-1} + \frac{1}{2}{\rm T}r\log D^{-1} + \mathcal{O}(e^2) + \mathcal{O}(\rho^4)$. The leading order particle-hole propagator $\tilde{G}_{x,y} = -G_{x,y}G_{y,x}$ is used to obtain the one-loop improved version, $\tilde{\mathcal{G}} = [\tilde{G}^{-1} +$

$e^2/\Delta]^{-1}$, $D=[-\Delta+e^2\tilde{G}]^{-1}$ denotes the photon propagator and $\rho^*_x=-G_{x,x}$. By minimizing the effective action one recovers the Hartree-Fock energy functional in $\mathcal{O}(e^2)$. The non-classical nature of the exchange contribution to the interaction is reflected in the fact that it arises from the photon fluctuation determinant, the term $\mathrm{T}r\log D^{-1}$.

The evolution equation (6) should be projected into a restricted functional space in order to make it more manageable. For this end we introduce the multi-local truncation scheme for the effective action. The free k-local cluster approximation, f_k, corresponds to the functional form

$$\Gamma[\rho]=\sum_{j=0}^{k}\prod_{j=1}^{k}\sum_{n_j}\int dx_j\Gamma^{n_1,\ldots,n_k}_{x_1,\ldots,x_k}\rho^{n_1}_{x_1}\cdots\rho^{n_k}_{x_k} \tag{8}$$

with arbitrary $\Gamma^{n_1,\ldots,n_k}_{x_1,\ldots,x_k}$. The effective action in the constrained k-local cluster approximation, c_k, can be written in the same manner except that the functions $\Gamma^{n_1,\ldots,n_k}_{x_1,\ldots,x_k}$ are parameterized. The interactive electron field operator is assumed to be of the form $\psi_{t,\mathbf{x}}=\sum_n c_{n,t}\Psi_{n,\mathbf{x}}$, and the propagator will be written as

$$G_{x,x'}=\sum_{n=1}^{N}e^{-E_n(t-t')}\Psi_{n,\mathbf{x}}\Psi^*_{n,\mathbf{x}'}+\sum_{n,n'=1}^{\infty}g_{n,n'}(t-t')\Psi_{n,\mathbf{x}}\Psi^*_{n',\mathbf{x}'}, \tag{9}$$

for $t<t'$ leaving $\Psi_{n,\mathbf{x}}$, E_n and $g_{n,n'}(t)$ as parameters.

We introduce a local density-dependent self-energy, $\sigma(x,\rho_x)$, in the photon propagator and assume the form

$$\mathcal{C}[\rho]=-\mathrm{T}r\log G^{-1}+\frac{1}{2}\mathrm{T}r\log[D^{-1}+\sigma]+\int_x U(x,\rho_x)+\sum_{n,m}\rho^n\cdot\gamma^{(n,m)}\cdot\rho^m \tag{10}$$

which is a f_2 ansatz with infinitely many higher order constrained clusters arising from the photon fluctuation determinant. The corresponding evolution equation, considered at the minimum of the effective action,

$$\rho_{\mathrm{g}r}=\left(1+\tilde{G}\cdot\frac{\lambda^2e^2}{\Delta}\right)\cdot\left(\rho^*-\tilde{\mathcal{G}}\cdot\frac{\delta\mathcal{C}[\rho_{\mathrm{g}r}]}{\delta\rho}\right), \tag{11}$$

can be written in a variational form as

$$\frac{\delta\mathcal{H}[\Psi^*,\Psi,E,\rho]}{\delta\Psi^*_{n,\mathbf{x}}}_{|\rho=\rho_{\mathrm{g}r}}=\frac{\delta\mathcal{H}[\Psi^*,\Psi,E,\rho]}{\delta\Psi_{n,\mathbf{x}}}_{|\rho=\rho_{\mathrm{g}r}}=\frac{\partial\mathcal{H}[\Psi^*,\Psi,E,\rho]}{\partial E_n}_{|\rho=\rho_{\mathrm{g}r}}=0, \tag{12}$$

after the appropriate choice of the constant $\mathcal{C}[0]$. A generalized HF functional

$$\mathcal{H}[\Psi^*,\Psi,E,\rho]=\mathcal{H}^{\mathrm{f}r}[\Psi^*,\Psi,E]+\mathcal{H}^{\mathrm{C}}[\Psi^*,\Psi]+\mathcal{H}^{\mathrm{p}h}[\Psi^*,\Psi,\rho]+\mathcal{H}^{\mathrm{i}}[\Psi^*,\Psi,\rho], \tag{13}$$

was introduced here which is the sum of the one-particle, exchange, direct and interaction pieces,

$$\mathcal{H}^{\mathrm{f}r}[\Psi^*,\Psi,E] \quad = \quad \sum_{n=1}^{N}\left[E_n+\int_{\mathbf{x}}\Psi^*_{n,\mathbf{x}}\left(-E_n-\frac{\hbar^2}{2m}\Delta+V^{ext}_{\mathbf{x}}\right)\Psi_{n,\mathbf{x}}\right],$$

$$\begin{aligned}
\mathcal{H}^{ph}[\Psi^*,\Psi,\rho] &= \frac{1}{2\beta}\mathrm{T}r\log\left(-\Delta+\sigma(\rho)+\lambda^2e^2\tilde{G}\right),\\
\mathcal{H}^{\mathrm{C}}[\Psi^*,\Psi] &= \frac{\lambda^2e^2}{2}\sum_{m,n=1}^{N}\int_{\mathbf{x},\mathbf{y}}\Psi^*_{m,\mathbf{x}}\Psi_{m,\mathbf{x}}\frac{1}{4\pi|\mathbf{x}-\mathbf{y}|}\Psi^*_{n,\mathbf{y}}\Psi_{n,\mathbf{y}},\\
\mathcal{H}^{\mathrm{i}}[\Psi^*,\Psi,\rho] &= \frac{1}{\beta}\rho^*\cdot\frac{\delta\mathcal{C}[\rho]}{\delta\rho}\\
&\quad -\frac{1}{2\beta}\left(\frac{\delta\mathcal{C}[\rho]}{\delta\rho}-\rho^*\cdot\frac{\lambda^2e^2}{\Delta}\right)\cdot\tilde{G}\cdot\left(\frac{\delta\mathcal{C}[\rho]}{\delta\rho}-\frac{\lambda^2e^2}{\Delta}\cdot\rho^*\right). \qquad (14)
\end{aligned}$$

The variational form (12) has the following remarkable features. The generalized Hartree-Fock functional includes higher order radiative corrections which involve time-dependent, dynamical quantities. Furthermore, it shows that the evolution, imposed at the ground state where the effective action is the best approximated, automatically involves an optimization with respect to the choice of the quasi-particles, the wave functions Ψ_n in the electron field operator.

We shall consider two simple approximation schemes for the evolution equation (12): In a c_2 truncation we keep the $\mathcal{O}(e^2)$ and $\mathcal{O}(\rho^2)$ perturbative effective action and recover the usual Hartree-Fock equations for the single particle wave functions Ψ_n and energies E_n. Another f_1c_2 approximation scheme is where one keeps the local potential $U(\rho)$ arbitrary but sets $\gamma=0$. It is important to realize that the transformation

$$U(x,\rho)\to U(x,\rho)+\eta(x,\rho),\quad \sigma(x,\rho)\to\sigma(x,\rho)+\frac{2}{D_{x,x}}\eta(x,\rho) \qquad (15)$$

leaves the effective action invariant. As pointed out above the photon fluctuation determinant gives the exchange contribution and is density independent. But the transformation (15) can be used to trade a possible, non-perturbative density-dependent photon self-energy term σ into a local potential U. The result is a generalization of the Kohn-Sham scheme [6], namely Eqs. (12) applied for the functional

$$\mathcal{H}_{\mathrm{K}S}[\Psi^*,\Psi,E,\rho]=\mathcal{H}^{\mathrm{f}r}[\Psi^*,\Psi,E]+\mathcal{H}^{\mathrm{C}}_{\mathrm{K}S}[\Psi^*,\Psi,\rho]+\mathcal{H}^{\mathrm{i}}_{\mathrm{K}S}[\Psi^*,\Psi,\rho], \qquad (16)$$

where

$$\begin{aligned}
\mathcal{H}^{\mathrm{C}}_{\mathrm{K}S}[\Psi^*,\Psi,\rho] &= -n_s\lambda^2e^2\int_{\mathbf{x},\mathbf{y}}\sum_{n=1}^{N}\Psi^*_{n,\mathbf{x}}\Psi_{n,\mathbf{x}}\frac{1}{4\pi|\mathbf{x}-\mathbf{y}|}\rho_{\mathbf{y},0} \qquad (17)\\
\mathcal{H}^{\mathrm{i}}_{\mathrm{K}S}[\Psi^*,\Psi,\rho] &= -\frac{1}{2\beta}\rho\cdot\frac{\lambda^2e^2}{\Delta}\cdot\tilde{G}\cdot\frac{\lambda^2e^2}{\Delta}\cdot\rho. \qquad (18)
\end{aligned}$$

The ground state energy is determined by the differential equation

$$\partial_\lambda\int_x U(x,0) = -\lambda e^2\mathrm{T}r\,\tilde{G}\frac{1}{\Delta}-\frac{1}{2}\rho^*\cdot\frac{\lambda^2e^2}{\Delta}\cdot\tilde{G}\cdot\frac{2\lambda e^2}{\Delta}\cdot\tilde{G}\cdot\frac{\lambda^2e^2}{\Delta}\cdot\rho^* \qquad (19)$$

together with the initial condition $U=0$ and contains the exchange term.

4 Quenched averages

The impurities represent a challenge in Condensed Matter Physics since they appear to be static from the point of view of the measurements and are distributed randomly. The usual way to take them into account is to average the connected Green functions, the logarithm of the partition function over the impurity distributions [9]. This average can be obtained either by analytical continuation in the number of replicas [10], or by the introduction of fictious particles related to the real ones by super-transformations [11] or by using the Keldysh contour in computing loop-integrals [12]. The functional renormalization group idea offers an alternative algorithm to compute quenched averages which goes further than these methods in being fully non-perturbative and in allowing annealed interactions [13]. For the sake of simplicity we constrain the present discussion to the non-interacting case only.

Consider the generator functional for the connected Green functions of the density and current, introduced in Eq. (4), and write its quenched average as

$$\tilde{W}[\sigma] = \frac{\int \mathcal{D}[v] e^{-\frac{1}{2g}\int_{\mathbf{x}} v_{\mathbf{x}}^2} W[\sigma_0 + v, \sigma_1, \ldots]}{\int \mathcal{D}[v] e^{-\frac{1}{2g}\int_{\mathbf{x}} v_{\mathbf{x}}^2}}. \tag{20}$$

The Legendre transform of $\tilde{W}[\sigma]$, the effective action $\Gamma[\rho]$, satisfies the evolution equation

$$\partial_\lambda \Gamma[\rho] = -\frac{g}{2}\int_{\mathbf{z}} \left[\frac{\delta^2\Gamma[\rho]}{\delta\rho\delta\rho}\right]^{-1}_{(0,0,\mathbf{z}),(0,0,\mathbf{z})}, \tag{21}$$

where the parameter λ was introduced by making the rescaling $g \to g\lambda$ and the notation of multiple-index $(\alpha, t, \mathbf{x})$ is used.

The gradient expansion ansatz for the functionals $\Gamma[\rho]$ seems natural when the kinetic transport coefficient are sought,

$$\begin{aligned}
\Gamma[\rho] \;=\; & \int_z \Bigg\{ \frac{1}{2}\rho_{0,z}\left[\partial_\omega \Gamma^{tt} i\partial_0 - \frac{1}{2}\partial_\omega^2\Gamma^{tt}\partial_0^2 - \partial_{q^2}\Gamma^{tt}\Delta\right]\rho_{0,z} \\
& -\frac{i}{2}\rho_{0,z}(\Gamma^{ts} + \partial_\omega\Gamma^{ts} i\partial_0)\partial_k\rho_{k,z} - \frac{i}{2}\rho_{k,z}(\Gamma^{st} + \partial_\omega\Gamma^{st} i\partial_0)\partial_k\rho_{0,z} \\
& +\frac{1}{2}\rho_{j,z}\Bigg[\delta_{j,k}\left(\Gamma^{ss} + \partial_\omega\Gamma^{ss} i\partial_0 - \frac{1}{2}\partial_\omega^2\Gamma^{ss}\partial_0^2\right) \\
& -\Gamma_T(\delta^{j,k}\Delta - \partial_j\partial_k) - \Gamma_L\partial_j\partial_k\Bigg]\rho_{k,z} + \frac{\vec{\rho}^2}{2}\Gamma^{ss} - \tilde{U}(\rho_0)\Bigg\}.
\end{aligned} \tag{22}$$

The Γ coefficients are functions of the control parameter λ and the density ρ_0 and have been computed in the leading order of the perturbation expansion in g. Such an approximation is reliable for weak disorder, $\kappa = gm^2/2\pi\hbar^3 p_F < 1$. Some combinations of the Γ-functions appear in the Kubo formula, e.g. the electric conductivity, $\sigma_{j,k} = \delta_{j,k}e^2\partial_\omega\Gamma^{ss}/(\Gamma^{ss})^2$, the diffusion constant, $D = \partial_{q^2}\Gamma^{tt}/(\Gamma^{tt})^2$, and the quantities Γ^{ss} and $\Gamma^{tt} = -\partial_{\rho_0}^2\tilde{U}$ give the density and current susceptibilities, respectively

When the vertex corrections are ignored then the evolution equation can be written as

$$\begin{aligned}
\partial_\lambda \Gamma &= 0 \quad for \quad \Gamma = \partial_\omega \Gamma^{tt}, \partial_\omega \Gamma^{ss}, \\
\partial_\lambda \Gamma &= g \partial^2_{\rho_0} \Gamma \int_{\mathbf{p}} \frac{1}{2d_p^2} \quad for \quad \Gamma = \Gamma^{ss}, \Gamma_T, \Gamma_L, \\
\partial_\lambda \gamma &= g(\phi \partial_{\rho_0} + 1) \left(\partial^2_{\rho_0} \gamma \int_{\mathbf{p}} \frac{1}{2d_p^2} \right) \quad for \quad \gamma = \partial^2_\omega \gamma^{tt}, \partial_{q^2} \gamma^{tt}, \\
\partial_\lambda \gamma &= g(\phi \partial_{\rho_0} + 2) \left(\partial^2_{\rho_0} \gamma \int_{\mathbf{p}} \frac{1}{8d_p^2} \right), \quad for \quad \gamma = \gamma^{ts}, \partial_\omega \gamma^{ts}, \\
\partial_\lambda \Gamma^{tt} &= g \left(\partial^2_{\rho_0} \Gamma^{tt} \int_{\mathbf{p}} \frac{1}{2d_p^2} + \partial^2_{\rho_0} \partial_{q^2} \gamma^{tt} \int_{\mathbf{p}} \frac{\mathbf{p}^2}{2d_p^2} \right),
\end{aligned} \tag{23}$$

where $\partial^2_\omega \gamma^{tt} = (\rho_0 \partial_{\rho_0} + 1) \partial^2_\omega \Gamma^{tt}$, $\partial_{q^2} \gamma^{tt} = (\rho_0 \partial_{\rho_0} + 1) \partial_{q^2} \Gamma^{tt}$, $\gamma^{ts} = (\rho_0 \partial_{\rho_0} + 2) \Gamma^{ts}$, $\partial^2_\omega \gamma^{ts} = (\rho_0 \partial_{\rho_0} + 2) \partial^2_\omega \Gamma^{ts}$, and

$$d_p = \mathbf{p}^2 (\rho_0 \partial_{\rho_0} + 1) \partial_{q^2} \Gamma^{tt} + \Gamma^{tt} + \frac{1}{4} \left[(\rho_0 \partial_{\rho_0} + 1) \Gamma^{ts} - \Gamma^{st} \right]^2 \frac{\mathbf{p}^2}{\mathbf{p}^2 \Gamma_L + \Gamma^{ss}}. \tag{24}$$

It is easy to see that the integration of these equations resums all correlation insertion of the particle-hole loop diagram in the given truncation of the gradient expansion, in particular the maximally crossed diagrams which are responsible for the cooperon pole needed for non-vanishing conductivity [14].

The terms $\mathcal{O}(\partial^2_{\rho_0})$ of the evolution equations correspond to a formal diffusion process on the plane (ρ_0, g) considered as space-time. One can verify that the remaining terms which contain $\rho_0 \partial^3_{\rho_0}$ generate similar spread, as well. The diffusion constant in the ρ_0-space, the integral on the right hand sides, is small at the initial condition for strong disorder and becomes large for weak disorder. According to the numerical solution of the evolution equations the spread of the Γ-functions is such that the conductivity decreases as the strength of the disorder is increased for weak disorder, as expected from weak localization.

The localization transition should occur when some or all $\mathcal{O}(\partial^2)$ terms of the effective action are vanishing. In fact, according to the Kubo formulae the mobility is proportional to certain $\mathcal{O}(\partial^2)$ terms in the effective action. Another reasoning is to recall the reduction formulae for a single particle which gives the scattering amplitudes in terms of the residuum of the connected propagator on the mass shell. It remains to be seen by detailed numerical studies of the evolution equation in 3 dimensions whether the ρ_0-dependence of the integrals on the right hand sides provides such a self-acceleration of the diffusion process in the ρ_0-space which cancels some Γ-functions at finite value of g.

We note finally that there is a formal analogy between localization and the phenomenon of the quark confinement. According to the haaron-model of the QCD vacuum the quark propagator is $\mathcal{O}(p^{-4})$ and the quark confinement appears as a localization in the space-time [15].

5 Summary

Two applications of the internal space renormalization group were given demonstrating the possibility of a new, systematical non-perturbative method to tackle the bound state problem at least when the bound state formation is triggered by an external potential.

This method is in its infancy and the outline of its formal structure is sketched only. But we believe that it can provide an accuracy and flexibility superior to other procedures when formal computer algebra and numerical integration are combined in deriving and solving the evolution equation in a sufficiently rich functional space.

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