acta physica slovaca vol. 52 No. 4, 285 – 294 **August 2002**

CRITICAL DYNAMICS IN TWO LOOP ORDER[1]

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Received 15 June 2002, accepted 17 June 2002

We give an introduction to the critical dynamics and review recent progress made within the field theoretic renormalization group approach. Our main concern are liquids and superfluids. Two loop calculations are inevitable for different reasons in both cases. Calculations of the field theoretic functions take into account the decomposition of the dynamical vertex functions into the static vertex functions and genuine dynamical parts. This makes possible a complete two loop calculation of the critical dynamics near the superfluid transition of ^{3}He-^{4}He mixtures (model F'). As result we obtain the flow equations of the dynamical parameters and the amplitude functions of the various transport coefficients, which governs the nonasymptotic and non universal temperature dependence. From a reduction of our expressions we obtain the field theoretic functions of model F describing the critical dynamics of the supefluid transition in pure ^{4}He and of model C correcting long standing results.

PACS: 05.70.Jk, 64.60.Ht, 64.60.Ak

1 Introduction

Universality in dynamical critical phenomena has led to a series of models [1] labeled by the capital letters between A and J which describe the critical dynamics in different physical systems. Besides the properties defining the static universality classes in dynamics the conservation property of the order parameter (OP), the number of conserved densities coupling to the OP and the kind of coupling (mode coupling and/or only static coupling) to the OP play an essential role in defining different dynamical critical systems. In Tab. 1 we list the most important examples of these models. Among them are ferro- and antiferromagnets, pure and binary fluids and superfluids. Some of the models are extensions of simpler models such as pure liquids (model H) and mixtures (model H'). Model H' has one more (scalar) conserved density coupled to the order parameter. This makes both models different although the asymptotic critical dynmaics in both models are described by the same singularities and therefore one considers both models in the same universality class. However the measurable nonasymptotic behavior might be quite

[1] Invited talk given by R. F. at 5th Int. Conf. Renormalization Group 2002, Tatranská Štrba (Slovakia), March 2002
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Symbol	OP	SD	sc	mc	contains	System
A	n	0	0			relaxation
B	c	0	0			diffusion
C	n	1 scalar	1		A	structural phase transition
E	n	1 scalar	0	×	A	planar magnet $h_z = 0$
F	n	1 scalar	1	×	A,C,E	planar magnet $h_z \neq 0$ superfluid ^{4}He
F'	n	2 scalars	2	×	A,C,E,F	superfluid ^{3}He-^{4}He mixture
G	n	1 vector	0	×	A	Heisenberg antiferromagnet
H	c	1 vector	0	×	B	fluid (gas/liquid)
H'	c	1 scalar 1 vector	1	×	B	binary mixture (plait point, consolute point)
J	c	0	0	×	B	Heisenberg ferromagnet
SSS	n	1 tensor	0	×	G, A	

Tab. 1. Classes of different dynamic models. OP: order parameter (n=nonconserved, c=conserved), SD number of secondary densities coupling to the order parameter, sc: number of static couplings, mc: mode coupling (x: present). Symbols after [1]; for the SSS-model see [2]

different [3]. There are differences in the field theoretic functions however in the case of belonging to the same universality class these differences concern only terms which go to zero in the asymptotic limit. This allows to check the results for more complicated models by reduction to simpler models which are contained in the more complicated ones; e.g. model F' contains model F, E, C, A (see Tab. 1).

Comparison with experiment has reached a high level for the gas-liquid transition and most prominently for the superfluid transition. For both systems zero gravity experiments have been performed (see e.g. NASA project 'Fundamental Physics in Space' [4]). Zero gravity is of importance in order to reach the asymptotic region i.e. a small enough temperature distance from the critical temperature. On earth for several reasons the asymptotic region is not accsessible. At the gas-liquid phase transition the reason is the coupling of the OP to gravity. A rounding of the divergencies sets in before the asymptotic region has been reached. For the superfluid transition a slow transient [5, 6] reduces the asymptotic region such that it is inaccesible for realistic temperature distances from T_c. One has to go to such a small temperature region near the critical temperature that although the superfluid OP does not couple directly to gravity, shifts of the critical temparature due to density differences become important. Even then in zero gravity the asymptotics is not reached and the application of the nonasymptotic theory remains inevitable in the interpretation of the experimental results.

Recent measurements of transport coefficients like the shear viscosity in fluids [7] have demonstrated the first situation. The analysis of the experiment shows the deficiencies of a one loop calculation, which for the experimental analysis on earth seems to be quite sufficient [8–11]. But it turns out that neither the one loop value for the critical exponent nor the frequency dependence of a one loop calculation is sufficient to get agreement with all of the experimental data. In our comparison with the one loop order theoretical result the critcal exponent was adjusted

and a frequency scale parameter has to be introduced to get agreement with the data. Only recently a complete two loop calculation for model H has been performed [12] showing that earlier two loop calculations [13] are in error. However a two loop calculation of the frequency dependence seems to be cumbersome because of mathematical difficulties (several integrals cannot be calculated analytically as function of frequency).

An example for the second situation is the the thermal conductivity in ^{4}He near the superfluid transition [15]. A complete two loop field theoretic renormalization group calculation has been performed [16, 17] and accurate measurements are available covering a region in relative temperature distance from 10^{-2} to 10^{-7} [18–20]. Thus a highly quantitative comparison between theory and dynamical experiments is possible. One obtains by this comparison the non universal parameters - their background values - entering the theoretical expressions for the thermal conductivity. These together with the specific flow of the parameters are then used in the comparison of other theoretically calculated quantities with physical measurements, e.g. the critical sound attenuation, or light scattering.

The situation in ^{3}He-^{4}He mixtures was less favourable from the theoretical side although a lot of experimental results mostly by the group of H. Meyer are available (see references in [21]). So far the critical dynamics has only been treated [22] in a combination of a one loop model with static couplings to the secondary densities of entropy and concentration (model F') and the two loop terms without these couplings (model E' [23]). This approximation was used for a comparison with the temperature dependence of three transport coefficients - thermal conductivity, thermal diffusion ratio and mass diffusion. The interrelation between these coefficients allows a significant test of the theory.

In order to perform a complete two loop calculation for this complicated model one has to look for a new input into the straightforward but from the number of terms exploding loop expansion. We have reached this by (i) seperating stricltly statics from dynamics within the expression for the vertex functions and (ii) introducing 'summed up' static quantities like the correlation length as performed in [24]. This makes the calculation tractable and especially item (i) leads to further checks of the calculation and thus a high probability of the correctness of the results obtained.

Thus we achieved complete consistent two loop order results for model F' [25] and as a corollary the field theoretic functions of the simpler model F [16] and model C (anisotropic magnets and structural transitions) [26]. Both results are corrected by our calculations. For model C they agree with with unpublished results by Oerding [27]. The general method presented her has also been applied to calculate the field theoretic functions for other models listed in Tab. 1 like model G for general number of OP components n.

2 General Considerations

2.1 The structure of the models

Let us consider densities $a_i(x)$ which may be the order parameter ϕ and other conserved densities α_j necessary for describing the dynamics of the slow system considered. The dynamic equations of the Ma & Mazenko type [28], which are usually used in dynamic RNG, have the form

$$\frac{\partial a_i}{\partial t} = \sum_j \left[k_B T\{a_i, a_j^*\} - L_{ij}\right] \frac{\delta H(\{a\})}{\delta a_j^*} + \theta_i \tag{1}$$

The first sum at the right hand side of the above equation contains the reversible contributions, which are determined by a set of generalized Poisson brackets $\{a_i, a_j^*\}$ between the densities. The second term represents the dissipative part determined by a set of Onsager coefficients L_{ij}, where $L_{ij} = \Lambda_{ij}$ for non conserved densities and $L_{ij} = -\Lambda_{ij}\nabla^2$ for conserved densities. The θ_i are stochastic forces. $H(\{a\})$ is a convenient static functional including the static critical behavior of the considered system. For instance in models A, B and J (the OP is the only density, see Tab. 1) this is the well known ϕ^4 Landau Ginzburg functional with the fourth order coupling u. The other models include aditional conserved densities of Gaussian order (model E, G, H, SSS) or additional cubic terms γ between the square of the order parameter and the other densities (model C, F, F'). With methods of group theory it is possible to introduce generalized Poisson brackets in a systematic manner [29]. The Poisson brackets are defined in a way that the corresponding reversible hydrodynamic equations are reproduced. In the same way the Onsager coefficients are introduced in accordance to the considered hydrodynamics. We expect that the hydrodynamic basic structure is conserved in the Fourier transformed dynamic vertex functions when the statics is treated appropriately. More specifically (for notation and definitions see [30]) we expect that the seperate appearance of the static vertex function in the dynamic vertex function known from zero and one loop order is valid also in higher orders and one can write them most generally as functions of the correlation length ξ, the wave vector k and the frequency ω

$$\Gamma_{a_i\tilde{a}_j}(\xi, k, \omega) = -i\omega\Omega_{a_i\tilde{a}_j}(\xi, k, \omega) + \sum_m \Gamma^{(st)}_{a_i a_m}(\xi, k)\Gamma^{(d)}_{a_m\tilde{a}_j}(\xi, k, \omega)\,. \tag{2}$$

with the static vertex function $\Gamma^{(st)}_{a_i a_m}$. This structure justifies the method used so far to identify the hydrodynamic transport coefficients with the vertex functions. Indeed we verified the above structure of the vertex functions in two loop calculations for all models (A to J) mentioned. We have performed the dynamic perturbation expansion and used then exact algebraic rearrangements of the expressions to obtain the structure in (2). We want to emphasize that these rearrangements are quite non trivial and that they require some calculational expense since the structure is not valid for the topological different contributions of the loop expansion. E.g. let us consider the two loop contributions. Not every diagram (i) can be seperated into the structure given in (2) above because it contains additional contributions $\Delta^{(i)}$. However in the sum of all diagramatic contributions the additional terms $\Delta^{(i)}$ cancel, i.e. $\sum_i \Delta^{(i)} = 0$, and one recovers the structure of eq. (2). Observing this structure one obtains the two functions $\Omega_{a_i\tilde{a}_i}$ and $\Gamma^{(d)}_{a_m\tilde{a}_j}$ in a relatively lucid form.

In addition general relations for other vertex function, necessary for the calculation of the dynamic scattering functions, could be proofed in two loop order

$$\Gamma_{\tilde{\phi}\tilde{\phi}}(\xi, k, \omega) = -2\Re\left(\Omega_{\phi\tilde{\phi}}(\xi, k, \omega)\Gamma^{(d)}_{\phi\tilde{\phi}}(\xi, k, \omega)\right) \tag{3}$$

and a similar relation for the diagonal part of the secondary densities. We want to note here that so far no renormalization has been performed (all our quantities are unrenormalized ones), all the relations follow from the property of the perturbational expansion.

This genuine dynamic functions contain singularities not only at the critical dimension d_c, but they also contain poles at lower dimensions (eg. $d = 3$) due to a shift of T_c and the expansion of the correlation length ξ. These poles are purely static and can be eliminted by resumming the T_c-shift and the correlation length. This procedure is quite similar to the method introduced by Schloms and Dohm for static vertex functions [24] and guaranties regular expressions at $d = 3$ independent of the renormalization method used.

The big advantage of using the above structure (2) and the resummation is, that the former huge amount of integral expressions, which has to be calculated, is reduced to a minimal number. This makes it possible to calculate even models which have been considered as too extensive in the past years (like model F' for ^{3}He-^{4}He mixtures) in two loop order. Even in these most complex models only eight independent integrals remain for an explicite calculation after the rearrangement.

3 Critical dynamics near the superfluid transition

3.1 ^{3}He-^{4}He mixtures

Let us now consider the critical dynamics near the superfluid transition in ^{3}He-^{4}He mixtures. We have four dynamical equations: two for the complex OP ϕ (the macroscopic wave function), the entropy density α_1 and the concentration of ^{3}He in ^{4}He α_2. They contain the relaxation mode of the superfluid OP, and the hydrodynamic modes of thermal diffusion and mass diffusion,

$$\frac{\partial \phi}{\partial t} = -2\Gamma \frac{\delta H}{\delta \phi^*} + i\phi \sum_i g_i \frac{\delta H}{\delta \alpha_i} + \Theta_\phi \tag{4}$$

$$\frac{\partial \phi^*}{\partial t} = -2\Gamma^* \frac{\delta H}{\delta \phi^*} - i\phi^* \sum_i g_i \frac{\delta H}{\delta \alpha_i} + \Theta_\phi^* \tag{5}$$

$$\frac{\partial \alpha_i}{\partial t} = -\vec{\nabla} \vec{J}_i \tag{6}$$

$$\vec{J}_1 = -\lambda \vec{\nabla} \frac{\delta H}{\delta \alpha_1} - L \vec{\nabla} \frac{\delta H}{\delta \alpha_2} - 2 g_1 \Im(\phi^* \vec{\nabla} \phi) \tag{7}$$

$$\vec{J}_2 = -L \vec{\nabla} \frac{\delta H}{\delta \alpha_1} - \mu \vec{\nabla} \frac{\delta H}{\delta \alpha_2} - 2 g_2 \Im(\phi^* \vec{\nabla} \phi)\,. \tag{8}$$

The static functional H contains a fourth order term in the OP with coupling constant $\tilde{u}$ and a coupling term quadratic in the OP and linear in the densities a_i with coupling constants γ_i,

$$H = \int d^d x \left\{ \frac{1}{2}\tau |\phi|^2 + \frac{1}{2} |\nabla \phi|^2 + \frac{\tilde{u}}{4!} |\phi|^4 + \frac{1}{2} \sum_i \alpha_i^2 + \frac{1}{2} \sum_i \gamma_i \alpha_i |\phi|^2 \right\} \tag{9}$$

From the renormalization procedure (all our calculations are performed within the field theoretical formulation of renormalization group theory [30]) we get the Z-factors for the relaxation

coefficient Γ and the Onsager coefficients λ, L and μ. From the Z-factors we get the two loop result for the ζ-functions in model F' (all parameters are now renormalized ones, see [22]). In order to simplify this calculation is done in a rotated space where the Onsager matrix in equs. (7), (8) diagonalized [25] and $u = \tilde{u} - 3\sum_i \gamma_i^2$ denotes the fourth order coupling of the ϕ^4-model

$$\begin{aligned} \zeta_\Gamma &= \sum_j \mathcal{F}_j^2 - \frac{2}{3}\sum_j u\mathcal{F}_j a_j - \frac{1}{2}\sum_{j,m} \mathcal{F}_j\mathcal{F}_m b_{jm} \\ &+ \frac{u^2}{9}\left(L_0 + x_1 L_1 - \frac{1}{2}\right) \end{aligned} \tag{10}$$

$$\zeta_{\lambda_j} = \gamma_j^2 - \frac{F_j^2}{2w_j'}\Big(1 + \frac{1}{2}\Re[Q]\Big) \tag{11}$$

where we have introduced the complex coupling $\mathcal{F}_j = \mathcal{C}_j - i\mathcal{E}_j$ which is composed from couplings appearing in model C and E defined by

$$\mathcal{C}_j = \sqrt{\frac{w_j}{1+w_j}}\,\gamma_j\,, \qquad \mathcal{E}_j = \frac{F_j}{\sqrt{w_j(1+w_j)}} \tag{12}$$

In (11) and (12) we have defined the complex time scale ratios $w_j = \Gamma/\lambda_j = w_j' + iw_j''$ between the order parameter and the secondary densities Onsager coefficients λ_j and the mode couplings $F_j = g_j/\lambda_j$ quite analogous to model F in pure ^{4}He [16]. The sum is running over the number of all secondary densities. The quantites a_j, b_{jk} and Q in (10,11) are functions of F_j and w_j [25].

From these ζ-functions one calculates the flow equation for the dynamical parameters F_i and w_i introduced above

$$\ell\frac{dF_i}{d\ell} = \beta_{F_i} = -F_i(\frac{\epsilon}{2} + \zeta_{\lambda_i}) \tag{13}$$

$$\ell\frac{dw_i}{d\ell} = \beta_{w_i} = w_i(\zeta_\Gamma - \zeta_{\lambda_i}) \tag{14}$$

To these dynamical flow equations we have to add the static ones. The flow parameter ℓ can be related to the correlation length, thus we obtain from the solutions of the flow equations with the initial values (at a certain temperature distance from the λ-transition) as parameters the temprature dependence of all couplings and time ratios entering the expressions for the transport coefficients.

In comparing our results with measurements of the hydrodynamic transport coefficient - thermal conductivity, thermal diffusion ratio and mass diffusion we proceed along the lines of Ref. [21, 22]. The transport coefficients are determind by k^2 terms of $\Gamma^{(d)}_{\alpha_i\tilde{\alpha}_j}(\xi, k, \omega = 0)$ as functions of the static and dynamic parameters γ_i, u, F_i and w_i.

They also have been calculated in two loop order making use of the results for the integrals already found in model F [16]. The modification of the expressions for the transport coefficients by the two loop terms turn out to be numerically small contributions. The temperature dependence of the dynamical parameters is known from flow equations determined by the ζ-functions Eqs. (10) and (11). Taking all these results into account we fit the temperature dependence of two transport coefficients (thermal conductivity and thermal diffusion ratio), with the background values of the dynamic couplings and time ratios as adjustable parameters and predict the

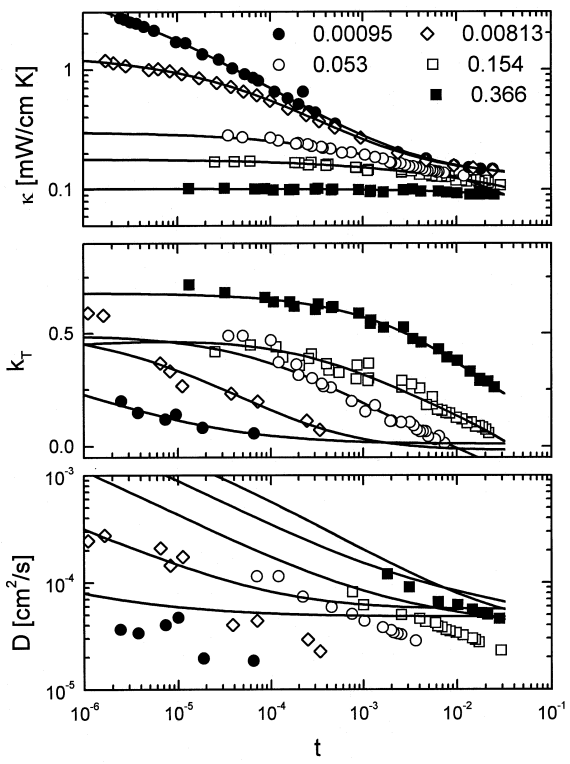

Fig. 1. Nonasymptotic theory compared with the data for ^{3}He-^{4}He-mixtures at different molar concentrations, from [25]. κ thermal conductivity, k_T thermal diffusion ratio and D mass diffusion as function of relative temeprature ditance t, data from the H. Meyer group [32, 33].

third one (mass diffusion) (see Fig. 1). There is (i) an improvement in the quality of the fit, (ii) some improvement in the prediction of the mass diffusion, but an obvious discrepancy remains and (iii) the background values for the imaginary parts of the time ratios change to more reliable values. This can be more explicitly demonstrated in pure ^{4}He.

3.2 Pure ^{4}He

Skipping the indices in equs. (10), (11) and the sums in (10) model F' reduces to model F. Regarding model F the corrections we found with respect to Ref. [16] concern terms, which go to zero at the fixed point, thus our results agree with model E [31]. The difference in the ζ-function for the OP between Dohm's result and ours (Eq. 10) produces drastic changes in the flow of the dynamical parameters and in consequence in the comparison with experiment.

We have recalculated the thermal conductivity in two loop order making use of the results for the integrals found in model F [16]. Therfrom we get the effective amplitude ratio R_λ^{eff} of the

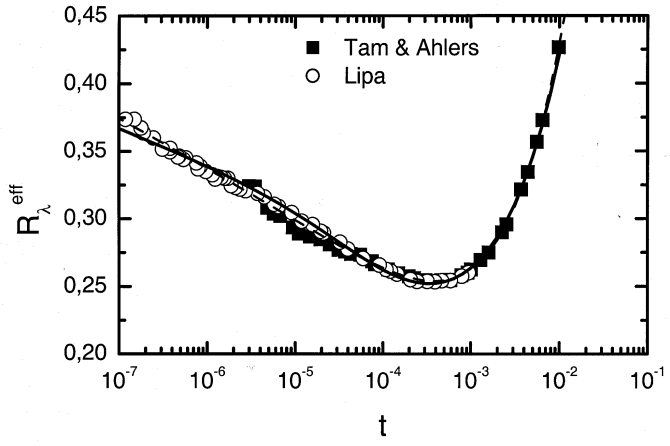

Fig. 2. Fit of the theoretically calculated effective amplitude of the thermal conductivity to its experimental counterpart with the correct (solid curve) flow equations. The initial conditions of the flow equations are the fit parameters. From ref. [25].

thermal conductivity $\lambda(t)$ and comparing with its experimental counterpart

$$R_\lambda^{exp}(t) = \frac{\lambda(t)}{g_0\sqrt{\xi(t)C_P(t)}} \tag{15}$$

where the temperature dependence ($t = (T - T_\lambda)/T_\lambda$) of the correlation length $\xi(t)$, of the specific heat at constant pressure $C_P(t)$ and of the thermal conductivity enters. g_0 is the unrenormalized mode coupling. All these quantities are known and a comparison e.g. at saturated vapour pressure has been made. Using these results a new fit of the the effective amplitude R_λ^{eff} to R_λ^{exp} has been performed (see Fig. 2). Apart from a slight bending over around $t \sim 10^{-6}$ the most important improvement is the change in the value of the background parameter for the renormalized imaginary part of the time ratio w. It is now of the expected size $w'' \approx 0.3$ instead of $w''_{Dohm} \approx 0.8$. In Ref. [34] the unrenormalized value was shown to be related to the background specific heat c_0, the background thermal conductivity λ_0 and the mass of ^{4}He m_4 by

$$w''_0 = \frac{\hbar c_0}{2m_4\lambda_0} \tag{16}$$

which is approximately $w''_0 \approx 0.21$ at saturated vapour pressure. One expects that the value of the renormalized counterpart is not so much different from this value.

4 Outlook

Further reducing our result for model F by setting equal to zero the mode coupling F and the imaginary part w'' one obtains model C with OP dimension $n = 2$. We also have calculated

model C for general OP dimension n the dynamical ζ-function for the relaxation coefficient. It reads [35]

$$\begin{aligned}\zeta_\Gamma &= \frac{w\gamma^2}{1+w} - \frac{1}{2}\frac{w\gamma^2}{1+w}\left[\frac{n+2}{3}u\Big(1-3\ln\frac{4}{3}\Big) + \frac{w\gamma^2}{1+w}\left(\frac{n}{2} - \frac{w}{1+w}\right.\right.\\ &- \left.\left.\frac{3(n+2)}{2}\ln\frac{4}{3} - \frac{1+2w}{1+w}\ln\frac{(1+w)^2}{1+2w}\right)\right] + \frac{n+2}{12}u^2\Big(\ln\frac{4}{3} - \frac{1}{6}\Big)\end{aligned} \tag{17}$$

This proves earlier results [26] and the limit of the model F of [16] to be incorrect; our result agrees with an unpublished result by Oerding [27]. We only remark that model A is obtained in our result in the limit $w \to 0$ and/or $\gamma \to 0$. This property is guaranted by the appearence of all genuine model C terms in ζ_Γ with a prefactor $w\gamma^2$. Based on this result a more complete discussion of the stabilty of the different fixed points as function of OP and spatial dimension and the flow equations will be given elsewhere [35].

The next step concerning the suprafluid transition in the mixture would be (i) to improve the accuracy of various static and dynamic quantities entering the comparison with theory, (ii) a comparison with sound velocity and sound absorption measurements both for pure ^{4}He and for the mixtures. In pure ^{4}He the analysis of the pressure dependence of the amplitude of the thermal conductivity will be reconsidered.

For all other models two loop calculations of dynamic amplitude ratios come now within the realm of possibility. As has been already mentioned "that there exists sufficient experimental and theoretical information to warrant a more complete analysis of correction terms, using non-linear recursion relations in three dimensions analogous to those for the superfluid." (cited from Ref. [36]).

Beside this line of research more work has to be done in calculating the shape of correlation functions and their crossovers (from background to the asymptotic region and/or from the hydrodynamic to the critical region). They may be compared with various scattering experiments.

Acknowledgement: The authors acknowledge support from the Fonds zur Förderung der wissenschaftlichen Forschung (project P15247).

References

[1] P. C. Hohenberg, B. I. Halperin: *Rev. Mod. Phys.* **49** (1977) 435
[2] L. Sasvary, F. Schwabl, P. Szepfalusy: *Physica* **A 81** (1975) 108
[3] R. Folk, G. Moser: *Phys. Rev.* **E 58** (1998) 6246
[4] see the web-page: http://funphysics.jpl.nasa.gov/
[5] C. De Dominicis, Peliti: *Phys. Rev. Lett.* **38** (1977) 505
[6] V. Dohm, R. A. Ferrell: *Phys. Lett.* **A 67** (1978) 387
[7] R. F. Berg, M. R. Moldover, G. A. Zimmerli: *Phys. Rev. Lett.* **82** (1999) 920
[8] R. Folk, G. Moser: *Phys. Rev.* **E 57** (1998) 683
[9] R. Folk, G. Moser: *Phys. Rev.* **E 57** (1998) 705
[10] G. Flossmann, R. Folk, G. Moser: *Phys. Rev.* **E 60** (1999) 779
[11] G. Flossmann, R. Folk: *Phys. Rev.* **E 62** (2000) 2460
[12] L. Ts. Adzhemyan et al.: *Theoret. Math. Phys.* **119** (1999) 454 and Folk, Moser: unpublished

[13] C. De Dominicis, L. Peliti: *Phys. Rev.* **B 18** (1978) 353
[14] R. F. Berg, M. R. Moldover: *J. Chem. Phys.* **93** (1990) 1926
[15] J. A. Lipa, D. R. Swanson, J. A. Nissen, T. C. P. Chui, U. E. Israelsson: *Phys. Rev. Lett.* **76** (1996) 944
[16] V. Dohm: *Phys. Rev.* **B 44** (1991) 2697 (model F ζ-functions)
[17] V. Dohm: *Z. Phys.* **B 61** (1985) 193 (amplitude function for the thermal conductivity)
[18] W. Y. Tam, G. Ahlers: *Phys. Rev.* **B 33** (1986) 133
[19] Q. Li, PhD Thesis, Stanford University (1991)
[20] J. A. Lipa, Q. Li: *Czech. J. Phys.* **46** (1996) 185
[21] G. Moser, R. Folk: *J. Low. Temp. Phys.* **86** (1992) 99
[22] G. Moser, R. Folk: *J. Low. Temp. Phys.* **86** (1992) 57
[23] V. Dohm, R. Folk: *Phys. Rev.* **B 28** (1983) 1332
[24] R. Schloms, V. Dohm: *Nucl. Phys.* **B 328** (1989) 639
[25] R. Folk, G. Moser: submitted to *Phys. Rev. Lett.*, Feb. 2002
[26] E. Brezin, C. De Dominicis: *Phys. Rev.* **B 12** (1975) 4954
[27] We thank V. Dohm for information about these unpublished calculations. We want to remark that in addition to reducing the model F' results the results for model F and model C have been checked by separate calculations.
[28] S-K. Ma, G. F. Mazenko: *Phys. Rev.* **B 11** (1975) 4077
[29] L. E Dzyaloshinskii, G. E. Volovick: *Ann. Phys. (N.Y.)* **125** (1980) 67
[30] R. Bausch, H. K. Janssen, H. Wagner: *Z. Physik* **B 24** (1976) 113
[31] V. Dohm: *Z. Physik* **B 31** (1978) 327
[32] D. Gestrich, R. Walsworth, H. Meyer: *J. Low Temp. Phys.* **54** (1984) 37
[33] F. Zhong, D. Gestrich, M. Dingus, H. Meyer: *J. Low Temp. Phys.* **68** (1987) 55
[34] V. Dohm: *Phys. Rev.* **B 29** (1984) 1497
[35] R. Folk, C. Moser, to be published.
[36] D. Privman, P. C. Hohenberg, A. Aharony, in: *Phase Transitions and Critical Phenomena*, ed. C. Domb, J. L. Lebowitz; Academic Press, London 1991