

**NONDECOUPLING EFFECTS OF HEAVY HIGGS PARTICLES  
IN TWO HIGGS DOUBLET MODEL<sup>1</sup>****M. Malinský<sup>2</sup>***Institute of Particle and Nuclear Physics, Charles University,  
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Non-decoupling properties of additional heavy degrees of freedom in the Higgs sector of the Two-Higgs-doublet extension of the Standard model are discussed in a particular case of production of a pair of longitudinally polarized  $W$ -bosons in the  $e^+e^-$  annihilation.

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**1 Introduction**

One of the least understood features of the Standard Model (SM) of electroweak interactions is the mechanism of electroweak symmetry breaking descending from the structure of the Higgs sector. Since the measurable quantities usually depend only weakly on its particular realization, many alternative models were proposed.

Perhaps the most popular (nonsupersymmetric) extension of the SM Higgs sector is the so called Two Higgs doublet model (THDM, 2HDM) [1], a theory with two Higgs doublets instead of one in the usual case. Not only it is capable to reproduce all the predictions of the standard theory but it also provides a nice framework for some possible new phenomena, among other things the CP-violation in the Higgs sector.

Having two doublets in the model the number of Higgs degrees of freedom gets doubled, i.e. we are left with 5 (8 total - 3 Goldstone modes) physical Higgs states. As in the case of SM the mass of the lightest state  $h^0$  is expected to be at the electroweak scale but the other states  $H^0$ ,  $A^0$ ,  $H^\pm$  can in principle be quite massive. Although they cannot be produced in low-energy experiments it is legitimate to ask whether they could contribute to low-energy amplitudes by means of virtual effects i.e. if they *decouple* in the heavy-mass limit or not.

There is a famous theorem by Appelquist and Carazzone [2] concerning general decoupling properties of heavy degrees of freedom in field theories. Unfortunately it does not work well in the case of the couplings among the light and heavy sectors growing too fast with the heavy-sector masses. Note that the Higgs couplings are typically proportional to the masses of interacting particles and this spoils the validity of this theorem in many situations involving heavy virtual

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Higgses coupled to light sector in the game. This is not in general the case of SUSY theories in which the form of the Higgs potential is rather strictly dictated by supersymmetry; from this point of view the heavy Higgs particles in these theories decouple in a usual Appelquist-Carazzone manner, see [3]

The problem of possibly large non-decoupling effects of heavy Higgs particles in THDM was discussed in several papers in 1990s, for instance [4] and [5].

As was shown in [5] we can expect relatively large non-decoupling effects of heavy Higgs bosons in this model for instance in cross-sections of processes involving longitudinal gauge bosons, in particular in  $e^+e^- \rightarrow W_L^+W_L^-$ . The magnitude of deviation of this quantity compared to the well-known SM value turns out to be of the order of several percent, which may (at least in principle) be measurable at future facilities.

In the calculation [5] several simplifications have been made:

1. The 'Equivalence theorem' [6] used therein works well only in the high-energy limit and therefore should not be used to estimate the non-decoupling features of the model (defined in the low-energy regime).
2. Only the ratio of total cross-sections is given which effectively washes out all the interesting (and probably larger) effects in differential quantities.

From this point of view we find it meaningful to *recalculate the ratio of the differential cross-sections of  $e^+e^- \rightarrow W_L^+W_L^-$  between THDM and SM* without use of the Equivalence theorem.

## 2 General analysis

Let us define the central quantity of our interest – the ratio of the differential cross-sections of  $e^+e^- \rightarrow W_L^+W_L^-$  in THDM and SM respectively:

$$\delta \equiv \frac{d\sigma^{\text{thdm}}(e^+e^- \rightarrow W^+W^-)}{d\sigma^{\text{sm}}(e^+e^- \rightarrow W^+W^-)} - 1. \quad (1)$$

Expanding now the THDM amplitude around the well-known SM value it is easy to obtain (at one-loop level)

$$\delta = 2\text{Re} \frac{\Delta\mathcal{M}_{\text{tree}} + \Delta\mathcal{M}_{1\text{-loop}}}{\mathcal{M}_{\text{tree}}^{\text{sm}}} + \frac{k_2}{k_1} \int dk_\gamma \frac{|\mathcal{B}^{\text{thdm}}|^2 - |\mathcal{B}^{\text{sm}}|^2}{|\mathcal{M}_{\text{tree}}^{\text{sm}}|^2} + \dots \quad (2)$$

Here the symbol “ $\Delta$ ” denotes differences of given quantities between THDM and SM, for example  $\Delta\mathcal{M}_{1\text{-loop}}$  is the difference of all one-loop contributions to amplitudes between the models;  $\mathcal{B}^{\text{model}}$  denote the corresponding bremsstrahlung amplitudes needed to regulate the IR divergences of  $\Delta\mathcal{M}_{1\text{-loop}}$  and  $k_i$ 's are some geometrical factors.

First we can get rid of the bremsstrahlung part of this expression: the IR divergent parts of  $\Delta\mathcal{M}_{1\text{-loop}}$  are combined with differences of bremsstrahlung terms to give a perfectly finite quantity which is suppressed by a factor of  $m_e/m_w$  (Yukawa couplings of Higgses to electrons in the initial state) in comparison with the rest of (2). Next, similar argumentation shows that the same proportionality factor occurs also in  $\Delta\mathcal{M}_{\text{tree}}$ . Neglecting such terms we are left with

$$\delta \doteq 2\text{Re} \frac{\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}}}{\mathcal{M}_{\text{tree}}^{\text{sm}}}. \quad (3)$$

All we need are therefore contributions of IR-finite one-loop graphs which are not common to both models.

### 3 Two Higgs doublet model (THDM)

Let us now specify the basic features of THDM in more detail. As we already know the presence of the second doublet gives rise to 5 Higgs states in the spectrum: neutral scalars  $h^0$  and  $H^0$ , charged scalars  $H^\pm$  and a neutral pseudoscalar  $A^0$ . The most general form of the Higgs potential (in terms of  $SU(2)$  doublets  $\Phi_1, \Phi_2$ , [1])

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \Phi_1^\dagger \Phi_2 - m_{12}^{2*} \Phi_2^\dagger \Phi_1 + \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \frac{\lambda_5^*}{2} (\Phi_2^\dagger \Phi_1)^2 + \\ & + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] (\Phi_1^\dagger \Phi_2) + [\lambda_6^* (\Phi_1^\dagger \Phi_1) + \lambda_7^* (\Phi_2^\dagger \Phi_2)] (\Phi_2^\dagger \Phi_1) \end{aligned}$$

is less restrictive than in SUSY theories; there is enough freedom for the coupling constants  $\lambda_i$  and the 'mass-parameters'  $m_{ij}$  to give rise to some new phenomena like the above mentioned CP-violation in the Higgs sector etc. Moreover, the bounds on the Higgs-mass pattern are not so stringent as for example in the SUSY theories, it is not a problem to have the masses of  $A^0$  or  $H^\pm$  at a scale of several TeV [1], [7].

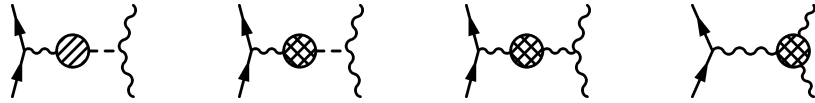
In general there are two basic types of THDM concerning the mass generation of up- and down-types of fermions. In type-I models both the up and down fermion masses are generated by one of the doublets only in analogy with the minimal Higgs model while in type-II theories one of the doublets generates the up-type and the second one the down-type masses in a similar way as in the MSSM. However, our analysis turns out to be model-independent because there are no relevant Yukawa couplings in the one-loop leading term computation.

### 4 Calculation of $\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}}$

Although the full set of Feynman diagrams contributing to  $\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}}$  is quite large, many of them can be safely neglected. It is mainly due to the presence of a suppressing Yukawa factor in all the diagrams involving Higgs couplings to electrons as indicated above.

Since we are using the on-shell renormalization scheme there is no need for renormalization of external legs. However, the vertex and propagator counterterms become nontrivial being not only the "1/ $\varepsilon$ " parts of the dimensionally-regularised expressions but fixed by on-shell renormalization conditions [8]. These additional structures need their own detailed discussion.

With these observations in mind we can classify all the one-loop topologies contributing to  $\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}}$ : it turns out that the only really important types of graphs are the following:


(4)

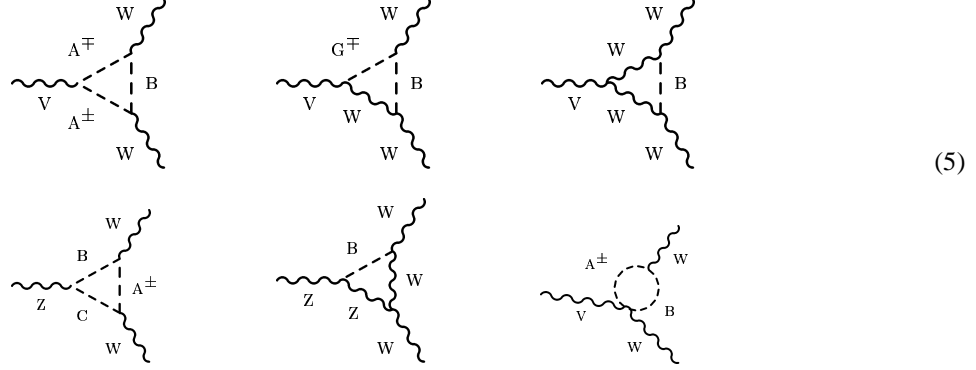
Here the dark shaded blob corresponds to loops involving at least one Higgs boson while the lighter stands for loops without any Higgs inside, all of them including finite parts of the relevant counterterms.

#### 4.1 Oblique-type corrections

Although there is no direct suppression by the Yukawa factors in first two graphs in (4) we can neglect them because of the one-loop mixed propagators whose contributions in general look like  $\Pi(p^2, m_i^2)(p_1 + p_2)_\alpha$ . Contracting this expression with the leptonic current and using the Dirac equation one reproduces again suppressing factor  $m_e/m_w$ . The case of the third topology is not so clear but it can be shown to exhibit the decoupling properties in the heavy (physical) Higgs mass limit [5]. (This can be easily seen in the particular case of the on-shell renormalization scheme [9].)

#### 4.2 Vertex corrections—one-loop TGV differences

Thus we are left with only the fourth topology in (4). To proceed, we need the differences of one-loop renormalized triple gauge vertex structures  $\gamma W^+ W^-$  and  $Z W^+ W^-$  [10]; let us denote them by  $\Delta\Gamma_{\sigma\mu\nu}^\gamma$  and  $\Delta\Gamma_{\sigma\mu\nu}^Z$  respectively. Relevant graphs can be divided into several clusters (charged bosons are denoted by a generic symbol  $A^\pm$  (for example  $G^\pm$  stands for the charged Goldstone bosons in  $R_\xi$  gauge), while the neutral ones by  $B, C$ ):



Their UV-divergences should be cancelled by counterterms descending from Ward-identity  $\delta Z_w = \delta Z_g$  connecting the  $W$ -boson wavefunction renormalization constant and the gauge coupling renormalization constant in the scheme fixed as in [11].

Having everything at hand we can write the leading contribution to  $\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}}$  in the form

$$\Delta\mathcal{M}_{1\text{-loop}}^{\text{ir-fin.}} \doteq \sum_{V=\gamma, Z} \bar{v}(p_1)\gamma_\lambda u(p_2) g_{e\nu} \frac{-ig^{\lambda\sigma}}{s - m_V^2} g_{\nu\omega\omega} \Delta\Gamma_{\sigma\mu\nu}^V \varepsilon^{*\mu}(q_1) \varepsilon^{*\nu}(q_2). \quad (6)$$

### 5 Results and conclusions

Since we are dealing with many complicated diagrams (there are 46 graphs in an  $R_{\xi=1}$  gauge in (5) and 9 others to calculate the finite parts of on-shell counterterms), we are forced to use

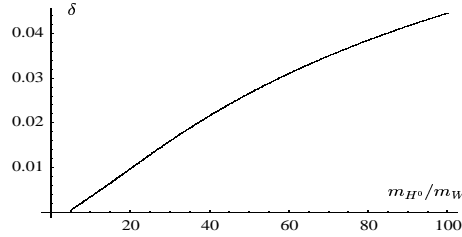


Fig. 1. Dependence of  $\delta[e_L^+e_R^- \rightarrow W_L^+W_L^-]$  as a function of  $m_{H^0}/m_W$ .  
Other parameters:  $\sqrt{s} \doteq 650$  GeV,  $m_{h^0} \sim m_\eta \doteq 130$  GeV,  $m_{A^0} \doteq 4$  TeV,  $m_{H^\pm} \doteq 2$  TeV

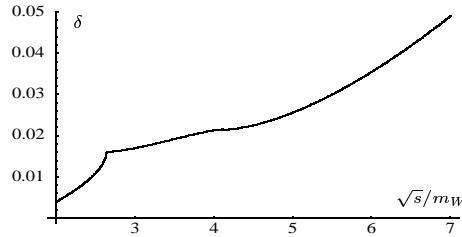


Fig. 2. Dependence of  $\delta[e_L^+e_R^- \rightarrow W_L^+W_L^-]$  as a function of  $\sqrt{s}/m_W$ .  
Other parameters:  $m_{h^0} \sim m_\eta \doteq 130$  GeV,  $m_{H^0} \doteq 240$  GeV,  $m_{A^0} \doteq 8$  TeV,  $m_{H^\pm} \doteq 800$  GeV

a computer. We have utilized Mathematica 4.0 with FeynCalc and LoopTools. The figures Fig. 1 and Fig. 2 correspond to ratios of differential cross-sections of particular initial state helicity configurations  $e_L^+e_R^- \rightarrow W_L^+W_L^-$  (in which  $\delta$  is expected to be largest). Moreover, in this particular case the leading contribution to  $\delta$  turns out to be  $\cos\theta^*$ -independent (CMS scattering angle). Although we use a slightly different Higgs mass pattern to exhibit mainly the basic features of  $\delta$ , its magnitude is in rough agreement with the expectation of [5]. (The discontinuities in derivatives of the second curve originate from the fact, that the loop integrals in (3) acquire non-zero imaginary parts above some values of  $\sqrt{s}$  which correspond to thresholds of productions of the loop particles in on-shell final states.)

As we see, at least in some regions of the parametric space we can expect relatively large non-decoupling (note the slope of the plot at Fig. 1) effects of heavy Higgs bosons in the considered quantity; they can reach the order of several percent.

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