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QCD COUPLING CONSTANT VALUE AND DEEP INELASTIC MEASUREMENTS[1]

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We reanalyze deep inelastic scattering data of Bologna-CERN-Dubna-Munich-Saclay (BCDMS) Collaboration by including proper cuts of ranges with large systematic errors. We perform also fits of high statistic deep inelastic scattering data of BCDMS, SLAC, new muon (NM) and Berkeley-FNAL-Princeton (BFP) Collaborations taking the data separately and in combined way and find good agreement between these analyses. We extract the values of the QCD coupling constant $\alpha_s(M_Z^2)$ up to next-to-leading order level.

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1 Introduction

The deep inelastic scattering (DIS) leptons on hadrons is the basical process to study the values of the parton distribution functions (PDF) which are universal (after choosing of factorization and renormalization schemes) and can be used in other processes. The accuracy of the present data for deep inelastic structure functions (SF) reached the level at which the Q^2-dependence of logarithmic QCD-motivated terms and power-like ones may be studied separately (for a review, see the recent papers [1] and references therein).

In the present letter we sketch the results of our analysis [2] at the next-to-leading order (NLO) of perturbative QCD for the most known DIS SF $F_2(x,Q^2)$ [4] taking into account experimental data [3]- [6] of SLAC, NM, BCDMS and BFP Collaborations. We stress the power-like effects, so-called twist-4 (i.e. $\sim 1/Q^2$) contributions. To our purposes we represent the SF $F_2(x,Q^2)$ as the contribution of the leading twist part $F_2^{pQCD}(x,Q^2)$ described by perturbative QCD, when the target mass corrections are taken into account (and coincides with $F_2^{tw2}(x,Q^2)$ when the target mass corrections are withdrawn), and the nonperturbative part ("dynamical" twist-four terms):

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[4] Here $Q^2 = -q^2$ and $x = Q^2/(2pq)$ are standard DIS variables, where q and p are photon and hadron momentums, respectively.

$$F_2(x,Q^2) \equiv F_2^{full}(x,Q^2) = F_2^{pQCD}(x,Q^2)\left(1+\frac{\tilde{h}_4(x)}{Q^2}\right), \tag{1}$$

where $\tilde{h}_4(x)$ is magnitude of twist-four terms.

Contrary to standard fits (see, for example, [7]- [9]) when the direct numerical calculations based on Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [10] are used to evaluate structure functions, we use the exact solution of DGLAP equation for the Mellin moments $M_n^{tw2}(Q^2)$ of SF $F_2^{tw2}(x,Q^2)$:

$$M_n^k(Q^2) = \int_0^1 x^{n-2}\, F_2^k(x,Q^2)\, dx \qquad (k = full, pQCD, tw2, ...) \tag{2}$$

and the subsequent reproduction of $F_2^k(x,Q^2)$ at every needed Q^2-value with help of the Jacobi Polynomial expansion method [11, 12] (see similar analyses at the NLO level [12, 13] and at the next-next-to-leading order (NNLO) level and above [14].

In this letter we do not present exact formulae of Q^2-dependence of SF F_2 which are given in [2]. We note only that the moments $M_n^{tw2}(Q^2)$ at some Q_0^2 is theoretical input of our analysis and the twist-four term $\tilde{h}_4(x)$ is considered as a set of free parameters (one constant $\tilde{h}_4(x_i)$ per x_i-bin): $\tilde{h}_4^{free}(x) = \sum_{i=1}^{I} \tilde{h}_4(x_i)$, where I is the number of bins.

2 Fits of F_2: procedure

Having the QCD expressions for the Mellin moments M_n^k we can reconstruct the SF $F_2^k(x)$ as

$$F_2^{k,N_{max}}(x,Q^2) = x^a(1-x)^b \sum_{n=0}^{N_{max}} \Theta_n^{a,b}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}^k\left(Q^2\right), \tag{3}$$

where $\Theta_n^{a,b}$ are the Jacobi polynomials [5] and a, b are fitted parameters.

First of all, we choose the cut $Q^2 \geq 1$ GeV^2 in all our studies. For $Q^2 < 1$ GeV^2, the applicability of twist expansion is very questionable. Secondly, we choose quite large values of the normalization point Q_0^2: our perturbative formulae should be applicable at the value of Q_0^2. Moreover, the higher order corrections $\sim \alpha_s^k(Q_0^2)$ and $\sim (\alpha_s(Q^2) - \alpha_s(Q_0^2))^k$ ($k \geq 2$) should be less important at higher Q_0^2 values.

We use MINUIT program [18] for minimization of $\chi^2(F_2) = |(F_2^{exp} - F_2^{teor})/\Delta F_2^{exp}|^2$. We consider free normalizations of data for different experiments. For the reference, we use the most stable deuterium BCDMS data at the value of energy $E_0 = 200$ GeV (E_0 is the initial energy lepton beam). Using other types of data as reference gives negligible changes in our results. The usage of fixed normalization for all data leads to fits with a bit worser χ^2.

3 Results of fits

Hereafter we choose Q_0^2 = 90 GeV^2 (Q_0^2 = 20 GeV^2) for the nonsinglet (combine nonsinglet and singlet) evolution, that is in good agreement with above conditions. We use also $N_{max} = 8$.

[5] We note here that there is similar method [15], based on Bernstein polynomials. The method has been used in the analyses at the NLO level in [16] and at the NNLO level in [17].

N	0	1	2	3	4	5	6
Y_{cut3}	0	0.14	0.16	0.16	0.18	0.22	0.23
Y_{cut4}	0	0.16	0.18	0.20	0.20	0.23	0.24
Y_{cut5}	0	0.20	0.20	0.22	0.22	0.24	0.25

Tab. 1. The values of Y_{cut3}, Y_{cut4} and Y_{cut5}.

3.1 BCDMS $^{12}\mathbf{C} + \mathbf{H_2} + \mathbf{D_2}$ data

We start our analysis with the most precise experimental data [5] obtained by BCDMS muon scattering experiment at the high Q^2 values. The full set of data is 762 (607) points (for the bounded x range: $x \geq 0.25$).

It is well known that the original analyses given by BCDMS Collaboration itself (see also Ref. [8]) lead to quite small values $\alpha_s(M_Z^2) = 0.113$. Although in some recent papers (see, for example, [7, 19]) more higher values of the coupling constant $\alpha_s(M_Z^2)$ have been observed, we think that an additional reanalysis of BCDMS data should be very useful.

Based on study [20] we proposed in [2] that the reason for small values of $\alpha_s(M_Z^2)$ coming from BCDMS data was the existence of the subset of the data having large systematic errors. We studied this subject by introducing several so-called Y-cuts [6] (see [2]). Excluding this set of data with large systematic errors leads to essentially larger values of $\alpha_s(M_Z^2)$ and very slow dependence of the values on the concrete choice of the Y-cut (see below).

We use the following x-dependent Y-cuts:

$$\begin{aligned} &y \geq 0.14 \quad \text{when} \quad 0.3 < x \leq 0.4, \qquad y \geq 0.16 \quad \text{when} \quad 0.4 < x \leq 0.5 \\ &y \geq Y_{cut3} \quad \text{when} \quad 0.5 < x \leq 0.6, \qquad y \geq Y_{cut4} \quad \text{when} \quad 0.6 < x \leq 0.7 \\ &y \geq Y_{cut5} \quad \text{when} \quad 0.7 < x \leq 0.8 \end{aligned} \tag{4}$$

and several N sets for the cuts at $0.5 < x \leq 0.8$:

The systematic errors for BCDMS data were given [5] as multiplicative factors to be applied to $F_2(x, Q^2)$: f_r, f_b, f_s, f_d and f_h are the uncertainties due to spectrometer resolution, beam momentum, calibration, spectrometer magnetic field calibration, detector inefficiencies and energy normalization, respectively. For this study each experimental point of the undistorted set was multiplied by a factor characterizing a given type of uncertainties and a new (distorted) data set was fitted again in agreement with our procedure considered in the previous section. The factors (f_r, f_b, f_s, f_d, f_h) were taken from papers [5] (see CERN preprint versions in [5]). The α_s values for the distorted and undistorted sets of data are given in the Figs. 1 and 2 (for the cases of non-singlet and complete evolutions, respectively) together with the total systematic error estimated in quadratures.

From the Figs. 1 and 2 we can see that the α_s values are obtained for $N = 1 \div 6$ of Y_{cut3}, Y_{cut4} and Y_{cut5} are very stable and statistically consistent. The case $N = 6$ of the Tab. 1 reduces the systematic error in α_s by factor 1.8 and increases the value of α_s, while increasing the statistical error on the 30%.

[6] Hereafter we use the kinematical variable $Y = (E_0 - E)/E_0$, where E is scattering energies of lepton.

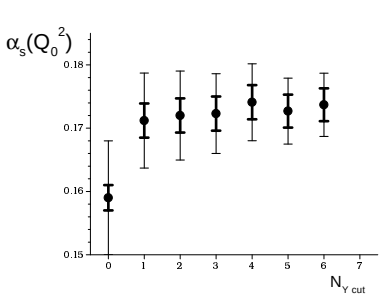

Fig. 1. The study of systematics at different Y_{cut} values in the fits based on nonsinglet evolution. The QCD analysis of BCDMS $^{12}\mathrm{C}, \mathrm{H}_2, \mathrm{D}_2$ data (nonsinglet case) is given at $x_{cut} = 0.25$ and $Q_0^2 = 90$ GeV^2. The inner (outer) error-bars show statistical (systematic) errors.

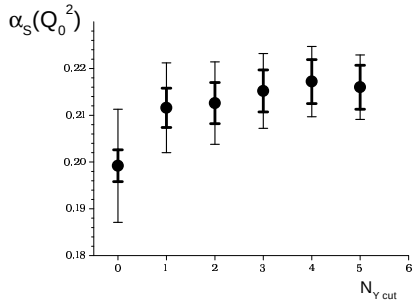

Fig. 2. The study of systematics at different Y_{cut} values in the fits based on combine singlet and nonsinglet evolution. All other notes are as in Fig. 1 with two exceptions: no a x_{cut} and $Q_0^2 = 20$ GeV^2. Moreover, the points $N_{Y\,cut} = 1, 2, 3, 4, 5$ correspond the values $N = 1, 2, 4, 5, 6$ in the Tab. 1.

After the cuts have been implemented (we use the set $N = 6$ of the Tab. 1), we have 590 (452) points (for the bounded x range: $x \geq 0.25$). Fitting them in agreement with the same procedure considered in the previous Section, we obtain the following results:

from fits, based on nonsinglet evolution (i.e. when $x \geq 0.25$):

$$\alpha_s(M_Z^2) \quad = \quad 0.1153 \pm 0.0013\,\text{(stat)} \pm 0.0022\,\text{(syst)} \pm 0.0012\,\text{(norm)},$$

from fits, based on combined singlet and nonsinglet evolution:

$$\alpha_s(M_Z^2) \quad = \quad 0.1175 \pm 0.0014\,\text{(stat)} \pm 0.0020\,\text{(syst)} \pm 0.0011\,\text{(norm)}, \tag{5}$$

where hereafter the symbol "norm" marks the error of normalization of experimental data.

The results are agree each other within considered errors. In Ref. [2] we have also analyzed the combine SLAC, NM and BFP data and found good agreement with (5). So, we have a possibility to fit together all the data. It is the subject of the following subsection.

3.2 SLAC, BCDMS, NM and BFP data

After these Y-cuts have been incorporated (with $N = 6$) for BCDMS data, the full set of combine data is 1309 (797) points (for the bounded x range: $x \geq 0.25$).

To verify the range of applicability of perturbative QCD, we analyze firstly the data without a contribution of twist-four terms, i.e. when $F_2 = F_2^{pQCD}$. We do several fits using the cut $Q^2 \geq Q_{cut}^2$ and increase the value Q_{cut}^2 step by step. We observe good agreement of the fits with the data when $Q_{cut}^2 \geq 10 \div 15$ GeV^2 (see the Figs. 3 and 4). Later we add the twist-four corrections and fit the data with the standard cut $Q^2 \geq 1$ GeV^2. We have find very good agreement with the data. Moreover the predictions for $\alpha_s(M_Z^2)$ in both above procedures are very similar (see the Figs. 3 and 4). The results of the fits are compiled in Summary.

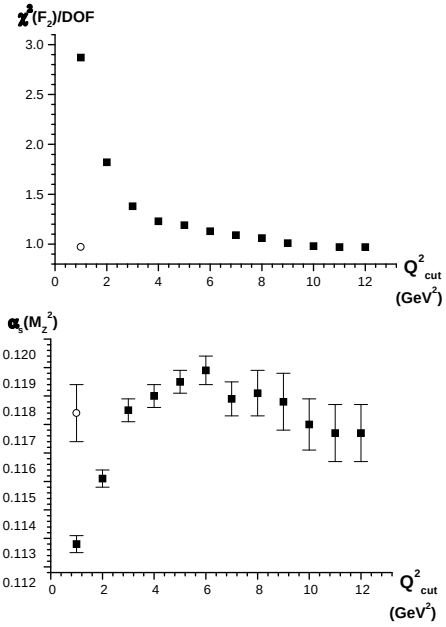

Fig. 3. The values of $\alpha_s(M_Z^2)$ and χ^2 at different Q^2-values of data cuts in the fits based on nonsinglet evolution. The black (white) points show the analyses of data without (with) twist-four contributions. Only statistical errors are shown.

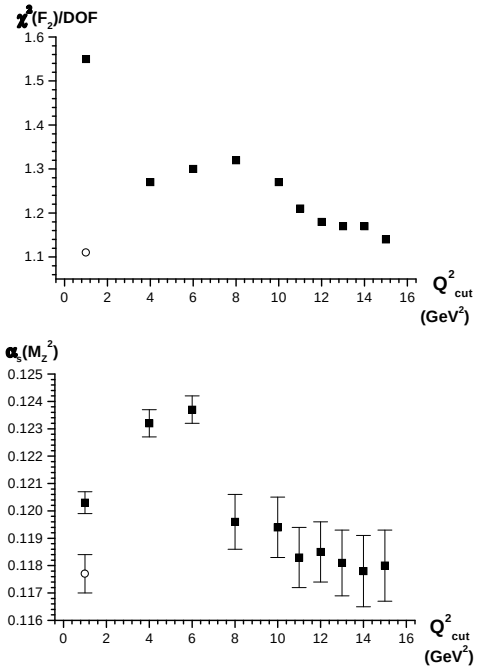

Fig. 4. The values of $\alpha_s(M_Z^2)$ and χ^2 at different Q^2-values of data cutes in the fits based on combine singlet and nonsinglet evolution. All other notes are as in Fig. 3.

4 Summary

We have demonstrated several steps of our study [2] of the Q^2-evolution of DIS SF F_2 fitting all modern fixed target experimental data.

From the fits we have obtained the value of the QCD coupling constant $\alpha_s(M_Z^2)$.

First of all, we have reanalyzed the BCDMS data cutting the range with large systematic errors: the value of $\alpha_s(M_Z^2)$ rises strongly and does not dependent on the concrete type of the Y-cut within modern statistical errors (see the Figs. 1 and 2).

We have found that at $Q^2 \geq 10 \div 15$ GeV^2 the formulae of pure perturbative QCD (i.e. twist-two approximation together with target mass corrections) are in good agreement with all data. The results for the coupling constant $\alpha_s(M_Z^2)$ based on combined evolution are:

$$\alpha_s(M_Z^2) \quad = \quad 0.1180 \pm 0.0013\,(\text{stat}) \pm 0.0021\,(\text{syst}) \pm 0.0009\,(\text{norm}) \tag{6}$$

When we have added twist-four corrections, we have very good agreement between QCD (i.e. first two coefficients of Wilson expansion) and all data with $Q^2 \geq 1$ GeV^2. The results for

the coupling constant $\alpha_s(M_Z^2)$ based on combined evolution have the following form:

$$\alpha_s(M_Z^2) \quad = \quad 0.1177 \pm 0.0007\,(\text{stat}) \pm 0.0021\,(\text{syst}) \pm 0.0009\,(\text{norm}) \tag{7}$$

Thus, there is very good agreement (see Eqs. (6) and (7)) between results based on pure perturbative QCD at quite large Q^2 values (i.e. at $Q^2 \geq 10 \div 15$ GeV^2) and the results based on first two twist terms of Wilson expansion.

We would like to note that we have good agreement also with the analysis [19] of combined H1 and BCDMS data, which has been given by H1 Collaboration very recently. Our results for $\alpha_s(M_Z^2)$ are in good agreement also with the average value for coupling constant, presented in the recent studies (see [7, 17, 21] and references therein) and in famous reviews [22].

The last result (7) based on all data with $Q^2 \geq 1$ GeV^2 can be considered as "best value" for the coupling constant $\alpha_s(M_Z^2)$ coming in our analysis.

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