RENORMALIZATION GROUP IN CASIMIR ENERGY CALCULATIONS[1]

I. O. Cherednikov[2]
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141 980 Dubna, Moscow region, Russian Federation
and
Institute for Theoretical Problems of Microphysics, Moscow State University, 119 899 Moscow, Russian Federation

Received 16 June 2001, in final form 26 June 2002, accepted 28 June 2002

The consequences of the renormalization group invariance in calculations of the ground state energy for models of confined quantum fields are discussed. The case of (1+1)D MIT quark bag model is considered in detail.

PACS: 12.39.Ba, 11.10.Gh, 11.10.Kk

1 Introduction

Recently, the application of the renormalization group methods in study of the Casimir energy effects for various situations attracted an attention of community [1]. The calculations of the Casimir energy for quantized fields under nontrivial boundary conditions encounter a number of difficulties (for the most recent review on the Casimir energy see ref. [2]). A majority of them are connected with ambiguities in results obtained by means of different regularization and renormalization methods. One of the physically interesting problems is the dependence of the (renormalized) energy from an additional mass parameter, which emerges inevitably in any regularization scheme. For example, in the widely used ζ-function regularization approach the one-loop vacuum energy for a fermion field may be defined as

$$\varepsilon_f = -K_D \left(\zeta_f'(0) - \zeta_f(0) \ln \frac{m}{\mu} \right) , \tag{1}$$

where K_D is the constant depending on the space-time dimension, m is the fermion field mass, and the arbitrary energy scale μ must be introduced in order to restore the correct dimension of the corresponding zeta-functions [3], which are defined as

$$\zeta_f(s) = \mu^{2s} \sum_n \omega_n^{-s} , \tag{2}$$

[1]Presented at 5th Int. Conf. Renormalization Group 2002, Tatranská Štrba (Slovakia), March 2002
[2]E-mail address: Igor.Cherednikov@jinr.ru

where ω_n are the energy eigenvalues. Whatever renormalization procedure one applies, the finite part of the energy would contain a μ-dependent contribution. Of course, there are several situations, for which $\zeta(0) = 0$ and this dependence is obviously cancelled due to some geometrical, or other, features of the given configuration. However, it would be very useful and interesting to investigate more general case.

In the present paper we consider the renormalization of the Casimir energy from the point of view of the convenient quantum field theory, and assume that variations of the mass scale μ must not produce any change of the physically observable quantities. This requirement naturally leads to a sort of the renormalization group equation, the solution of which allows to conclude that some of the parameters of the "classical" mass formula have to be considered as running constants. This may be important, *e. g.*, in some phenomenological applications, such as the quark bag models where the Casimir energy yields a nontrivial contribution to the mass of hadron, since it may provide a deeper understanding of relations between fundamental and effective theories of the hadronic structure.

2 (1+1)-Dimensional MIT Bag Model with Massive Fermions

For our purposes, it would be enough to use the following form of the ζ-regularized ground state energy of the confined fermion field:

$$E = -\sum \omega_n \to E_{reg}(\mu, \varepsilon) = -\mu^\varepsilon \sum \omega_n^{1-\varepsilon} \; . \tag{3}$$

The presence of a fermion mass may lead, in general, to some new effects compared to the massless case, and contains additional divergences that have to be subtracted. Consider in detail the (1+1)-dimensional MIT bag model with the massive fermions [4]. The Lagrangian of this system

$$\mathcal{L}_{MIT} = i\bar\psi\gamma\partial\psi - \theta(|x| < R)\, \left(m\bar\psi\psi + B\right) - \theta(|x| > R)\, M\bar\psi\psi \; , \tag{4}$$

where $\theta(x)$ is the usual θ-function, describes (in the limit for the external mass $M \to \infty$) the fermion field $\psi(x)$ confined to the segment $[-R, R]$ under the (1+1)-dimensional boundary condition:

$$(\pm i\gamma^1 + 1)\psi(\pm R) = 0 \; . \tag{5}$$

The exact spectrum of the elementary fermionic excitations reads:

$$\omega_n = \sqrt{\left(\frac{\pi}{2R} n + \frac{\pi}{4R}\right)^2 + m^2} \; . \tag{6}$$

In this simple case we don't need to use the heat-kernel expansion (see, *e.g.*, [2,3]) since the spectrum of eigenvalues is known explicitly. Here we will be interesting only in the small mass m limit, so we drop out all terms of the order m^4 and higher, what corresponds to the expansion in vicinity of the chiral limit [4]. Then the eigenvalues ω_n can be written as

$$\omega_n = \Omega_1 n + \Omega_0 + \frac{m^2}{2\left(\Omega_1 n + \Omega_0\right)} + O(m^4) \; , \tag{7}$$

where

$$\Omega_1 = \frac{\pi}{2R} \ , \ \Omega_0 = \frac{\pi}{4R} \ . \tag{8}$$

In order to analyze the singularities in the Casimir energy, we use the expansion for $n > 0$:

$$\omega_n = \Omega_1 n + \Omega_0 + \frac{\Omega_{-1}}{n} + O(n^{-2}) \ , \tag{9}$$

where $\Omega_{-1} = m^2R/\pi$, and assume the lowest quark state with $\omega_0 = \Omega_0 + 2\Omega_{-1}$ to be filled.

It can be shown, that the ζ-regularized sum (1) reads:

$$E_{reg} = -\Omega_{-1}\left(\frac{1}{\varepsilon} + \gamma_E\right) + \frac{\Omega_1}{12} + \frac{\Omega_0}{2} + \frac{\Omega_0^2}{2\Omega_1} + \Omega_{-1}\left(\ln\frac{\Omega_1}{\mu} + 1\right) , \tag{10}$$

where $\gamma_E = 0.5772...$ is the Euler constant. It's interesting to note, that the regularization by the exponential cutoff gives the equivalent result up to additional power-law divergences [6]. These divergences are a generic feature of the regularization schemes which use an ultra-violet cutoff, and, in contrast, never emerge in schemes without it, such as dimensional and ζ-function regularization.

The divergent part of (10) can be extracted in the form:

$$E_{div} = -\frac{m^2R}{\pi}\left(\frac{1}{\varepsilon} + \gamma_E - \ln\frac{\pi}{8} \ .\right) \tag{11}$$

We include in E_{div} the pole ε^{-1} as well as the transcendent numbers γ_E and $\ln\frac{\pi}{8}$ in analogy to the widely used modified minimal subtraction scheme in quantum field theory, but we should mention that this analogy is only formal one, since (11) has nothing to do with the singularities appearing in the conventional field theory since it depends on the geometrical parameter R. The removing of divergent part in (10) is performed by the absorption of E_{div} (11) into the definition of the "classical" bag constant B, which is introduced in the mass formula and characterizes the energy excess inside the bag volume as compared to the energy of nonperturbative vacuum outside [4].

The finite energy of our bag with one fermion on the lowest energy level is

$$E(R,\mu) = 2B_0R + \frac{11\pi}{48R} + \frac{3m^2R}{\pi} - \frac{m^2R}{\pi}\ln\mu R \ , \tag{12}$$

where B_0 is the renormalized bag constant. This quantity, being a physical observable, should not depend on a choice of arbitrary scale μ. Then the condition of the renormalization invariance has to be imposed on it, what yields

$$\mu\frac{d}{d\mu}E(B_0(\mu),\mu) = 2R\gamma_B - \frac{m^2R}{\pi} = 0 \ , \tag{13}$$

where

$$\gamma_B = \mu\frac{d}{d\mu}B_0(\mu) \ . \tag{14}$$

The solution of this equation can be written as

$$B_0(\mu) = B_0(\mu_0) + \frac{m^2}{2\pi}\ln\frac{\mu}{\mu_0}\ , \tag{15}$$

where the value $B_0(\mu_0)$ gives the boundary condition for the solution of the differential equation (13). It may seem that the running parameter $B_0(\mu)$ depend on both the initial value $B_0(\mu_0)$ and μ_0 itself, but indeed it must not depend on the starting point. Then it's convenient to express the running constant in terms of a single variable:

$$\Lambda_{MIT} = \mu_0 \exp\left[-\frac{2\pi}{m^2}B_0(\mu_0)\right]\ , \tag{16}$$

and write

$$B_0(\mu) = \frac{m^2}{2\pi}\ln\frac{\mu}{\Lambda_{MIT}}\ . \tag{17}$$

The scale Λ_{MIT} appears to be an analogue of the scale Λ_{QCD} in the Quantum Chromodynamics. One can easily check that the total bag energy $E(B_0(\mu), \mu)$ is independent on choice of the points μ and μ_0.

The actual size of the bag can be found from the equation which determines the energy minimum:

$$\frac{\partial E}{\partial R} = 0\ . \tag{18}$$

Then, the radius R_0 obeys the relation

$$\frac{11\pi}{48R_0^2} = 2\left(B_0 + \frac{m^2}{\pi}\right) - \frac{m^2}{\pi}\ln\mu R_0\ . \tag{19}$$

Taking into account the formula (17) for the running constant $B_0(\mu)$, one finds the following relation between the fundamental scale Λ_{MIT} and the bag's radius:

$$\Lambda_{MIT} = R_0^{-1}\exp\left[2 - \frac{11}{48}\left(\frac{\pi}{mR_0}\right)^2\right]\ . \tag{20}$$

Therefore, we find that in the renormalization group improved version of the (1+1)D MIT bag model, which takes into account the renormalized fermion ground state energy, the "bag constant" should be considered as a running parameter, and the value of the bag energy as well as it's radius are determined by the single (except of the current quark mass m) dimensional parameter Λ_{MIT} which plays a role of the fundamental energy scale, similar to Λ_{QCD}. Note, that the relation (16) can be considered as an improvement of the straightforward identification of the scale μ and Λ_{MIT} proposed in the first paper indicated in ref. [5]. It is clear, that in more complicated situations, for example, in (3+1)-dimensional MIT bag model with a larger amount of coupling constants, the results of the renormalization group analysis would be rather nontrivial.

3 Conclusion

We have considered the consequences of the renormalization invariance condition in the Casimir energy calculations on the simple example related to the quark bag models. It is shown that the value of the bag mass is controlled by the fundamental energy scale analogous to Λ_{QCD}, while the "bag constant" becomes the running parameter. The effects of the renormalization invariance in more realistic models—such as (3+1)-dimensional chiral hybrid quark bags, will be studied in future.

Acknowledgement: The author is grateful to the Organizers of the Renormalization Group '2002 for the hospitality and financial support. This work is supported in part by RFBR (projects nos. 02-02-16194, 01-02-16431 and 00-15-96577), and INTAS (project no. 00-00-366).

References

[1] I. Brevik, L. N. Granda, S. O. Odintsov: *Phys. Lett.* **B 367** (1996) 206 ,
I. L. Shapiro, J. Sola: *Phys. Lett.* **B 475** (2000) 236 ,
I. L. Shapiro, J. Sola: *J. High Energy Phys.* **2** (2002) 006 ,
A. Babic, B. Guberina, R. Horvat, H. Stefancic: *Phys. Rev.* **D 65** (2002) 085002

[2] M. Bordag, U. Mohideen, V. M. Mostepanenko: *Phys. Reports* **353** (2001) 1

[3] S. W. Hawking: *Commun. Math. Phys.* **55** (1977) 133

[4] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. Weisskopf: *Phys. Rev.* **D 9** (1974) 3471 ,
R. J. Perry, M. Rho: *Phys. Rev.* **D 34** (1986) 1169 ,
E. Elizalde, F. C. Santos, A. C. Tort: *E-print* **arxiv** (2002) hep-th/0206114

[5] S. K. Blau, M. Visser, A. Wipf: *Nucl. Phys.* **B 310** (1988) 163 ,
E. Elizalde, M. Bordag, K. Kirsten: *J. Phys.* **A 31** (1998) 1743

[6] I. Cherednikov: *Phys. Lett.* **B 498** (2001) 40