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RENORMALIZATION FLOW FROM UV TO IR DEGREES OF FREEDOM[1]

H. Gies[2]*, Ch. Wetterich[3]†
*TH Division, CERN, CH-1211 Geneva 23, Switzerland
†Institut für Theoretische Physik, Universität Heidelberg,
Philosophenweg 16, D-69120 Heidelberg, Germany

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Within the framework of exact renormalization group flow equations, a scale-dependent transformation of the field variables provides for a continuous translation of UV to IR degrees of freedom. Using the gauged Nambu-Jona-Lasinio model as an example, this translation results in a construction of partial bosonization at all scales. A fixed-point structure arises which makes it possible to distinguish between fundamental-particle and bound-state behavior of the scalar fields.

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1 Introduction

For an investigation of a system of interacting quantum fields, it is mandatory to identify the "true" degrees of freedom of the system. As we know from many physical systems such as QCD, or the plethora of condensed-matter systems, the nature of these degrees of freedom of one and the same system can be very different at different momentum (or length) scales. Of course, the first physical task is the identification of these relevant degrees of freedom at the various scales. Simplicity can be an appropriate criterion for this, in particular, simplicity of the effective action governing these degrees of freedom.

Whereas quantum field theory is usually defined in terms of a functional integral over quantum fluctuations of those field variables that correspond to the degrees of freedom in the ultraviolet (UV), we are often interested in the properties of the system in the infrared (IR). In some but rare instances, we know not only the true degrees of freedom at these different scales, but also the formal translation prescription of one set of variables into the other in terms of a discrete integral transformation. An example is given by the Nambu-Jona-Lasinio (NJL) model [1] in which self-interacting fermions (UV variables; "quarks") can be translated into an equivalent system of (pseudo-)scalar bosons (IR variables; "mesons") with Yukawa couplings to the fermions. This

[1] Presented by H. G. at 5th Int. Conf. Renormalization Group 2002, Tatranská Štrba (Slovakia), March 2002
[2] E-mail address: holger.gies@cern.ch
[3] E-mail address: C.Wetterich@thphys.uni-heidelberg.de

is done by means of a Hubbard-Stratonovich transformation, also called partial bosonization. A purely bosonic theory can then be obtained by integrating out the fermions.

Integrating out the fermions at once leads, however, to highly nonlocal effective bosonic interactions. This problem can be avoided by integrating out the short distance fluctuations stepwise by means of the renormalization group. In this context, a continuous translation from multifermion to bosonic interactions would be physically more appealing, since it would reflect the continuous transition from the ultraviolet to the infrared more naturally. Furthermore, phases in which different degrees of freedom coexist could be described more accurately.

In the following, we will report on a new approach which is capable of describing such a continuous translation. The approach is based on an exact renormalization group flow equation for the effective average action [2] allowing for a scale-dependent transformation of the field variables [3].[4] In order to keep this short presentation as transparent as possible, we will discuss our approach by way of example, focussing on the gauged version of the NJL model which shares many similarities with, e.g., building blocks of the standard model.

We shall consider the gauged NJL model for one fermion flavor in its simplest version characterized by two couplings in the UV: the gauge coupling e of the fermions to an abelian gauge field $\sim \bar{\psi} A\!\!\!/ \psi$, and the chirally invariant four-fermion self-interaction in the (pseudo-)scalar channel $\sim \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}$ with coupling λ_{NJL}. Depending on the values of these couplings, the gauged NJL model interpolates between the pure NJL model entailing chiral-symmetry breaking (χSB) for strong λ_{NJL} coupling and (massless) QED for weak λ_{NJL}; for simplicity, the gauge coupling is always assumed to be weak in the present work. The physical properties and corresponding degrees of freedom in the infrared depend crucially on λ_{NJL}: we expect fermion condensates and bosonic excitations on top of the condensate in the case of strong coupling, but bound states such as positronium at weak coupling. We shall demonstrate that our flow equation describes these features in a unified manner. The question as to whether the fields behave like fundamental particles or bound states thereby receives a scale-dependent answer; in particular, this behavior can be related to a new infrared fixed-point structure with interesting physical implications.

2 Fundamental particles versus bound states

Let us study the scale-dependent effective action Γ_k for the abelian gauged NJL model ($N_{\mathrm{f}} = 1$) including the scalars arising from bosonization in the following simple truncation,

$$\begin{aligned}\Gamma_k \quad = \quad & \int d^4x \Big\{ \bar{\psi}\mathrm{i}\partial\!\!\!/ \psi + 2\bar{\lambda}_{\sigma,k}\, \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} - e\bar{\psi} A\!\!\!/ \psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \\ & \qquad + Z_{\phi,k}\partial_\mu\phi^*\partial_\mu\phi + \bar{m}_k^2\,\phi^*\phi + \bar{h}_k(\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}\phi - \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}\phi^*) \Big\},\end{aligned}$$

where we take over the conventions from [3]. Beyond the kinetic terms, we focus on the fermion self-interaction $\sim \bar{\lambda}_{\sigma,k}$, the scalar mass $\sim \bar{m}_k^2$, and the Yukawa coupling between the fermions and the scalars $\sim \bar{h}_k$. In the framework of exact renormalization group (RG) equations, the infrared scale k divides the quantum fluctuations into modes with momenta $k < p < \Lambda$ that have been integrated out, so that Γ_k governs the dynamics of those modes with momenta $p < k$ which

[4]For an earlier approach, see [4]. A general account of field transformations within flow equations has been given in [5].

still have to be integrated out in order to arrive at the full quantum effective action $\Gamma_{k\to 0}$. The RG flow of Γ_k to the quantum effective action is described by a functional differential equation [2] which we solve within the truncation given by Eq. (1). The flow is initiated at the UV cutoff Λ, which in our case also serves as the bosonization scale, and we fix the couplings according to

$$\lambda_{\mathrm{NJL}} = \frac{1}{2}\,\frac{\bar{h}_\Lambda^2}{\bar{m}_\Lambda^2}, \quad \bar{\lambda}_{\sigma,\Lambda} = 0, \quad Z_{\phi,\Lambda} = 0. \tag{1}$$

In other words, all fermion self-interactions are put into the Yukawa interaction $\bar{h}_k$ and the scalar mass $\bar{m}_k^2$ at the bosonization scale Λ, and the standard form of the gauged NJL model in a purely fermionic language could be recovered by performing the Gaussian integration over the scalar field.

Concentrating on the flow of the couplings $\bar{m}_k^2, \bar{h}_k, \bar{\lambda}_{\sigma,k}$, we find[5] ($\partial_t \equiv k(d/dk)$):

$$\begin{aligned}
\partial_t \bar{m}_k^2 &= \frac{k^2}{8\pi^2}\,\bar{h}_k^2, \\
\partial_t \bar{h}_k &= -\frac{1}{2\pi^2}\,e^2\,\bar{h}_k + \mathcal{O}(\bar{\lambda}_{\sigma,k}), \\
\partial_t \bar{\lambda}_{\sigma,k} &= -\frac{9}{8\pi^2 k^2}\,e^4 + \frac{1}{32\pi^2 Z_{\phi,k}^2 k^2}\,\frac{3+\frac{\bar{m}_k^2}{Z_{\phi,k}k^2}}{(1+\frac{\bar{m}_k^2}{Z_{\phi,k}k^2})^3}\,\bar{h}_k^4 + \mathcal{O}(\bar{\lambda}_{\sigma,k}).
\end{aligned} \tag{2}$$

We observe that, although the four-fermion interaction has been bosonized to zero at Λ, $\bar{\lambda}_{\sigma,\Lambda} = 0$, integrating out quantum fluctuations reintroduces four-fermion interactions again owing to the RHS of the last equation; for instance, the first term $\sim e^4$ arises from gauge boson exchange. Bosonization in the standard approach is obviously complete only at Λ. However, guided by the demand for simplicity of the effective action at any scale k, we would like to get rid of the fermion self-interaction at all scales, i.e., re-bosonize under the flow. Here the idea is to employ a flow equation for a scale-dependent effective action $\Gamma_k[\phi_k]$, where the scalar field variable ϕ_k is allowed to vary during the flow; this flow equation is derived in [3], and can be written in a simple form as

$$\partial_t \Gamma_k[\phi_k] = \partial_t \Gamma_k[\phi_k]\big|_{\phi_k} + \int_q \left(\frac{\delta \Gamma_k}{\delta \phi_k(q)}\,\partial_t \phi_k(q) + \frac{\delta \Gamma_k}{\delta \phi_k^*(q)}\,\partial_t \phi_k^*(q) \right), \tag{3}$$

where the notation omits the remaining fermion and gauge fields for simplicity. The first term on the RHS is nothing but the flow equation for fixed variables evaluated at fixed ϕ_k instead of $\phi = \phi_\Lambda$. The second term reflects the flow of the variables. In the present case, we may choose

$$\partial_t \phi_k(q) = -(\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}})(q)\,\partial_t \alpha_k, \quad \partial_t \phi_k^*(q) = (\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}})(-q)\,\partial_t \alpha_k, \tag{4}$$

[5] The numerical coefficients on the RHS's of Eqs. (2) depend on the implementation of the IR cutoff procedure at the scale k and on the choice of the Fierz decomposition of the four-fermion interactions. For the former point, we use a linear cutoff function [6] (see also D.F. Litim's contribution to this volume). For the latter, we choose a $(S-P)$, (V) decomposition, but display only the (pseudo-)scalar channels here; the vectors are discussed in [3]. Furthermore, we work in the Feynman gauge.

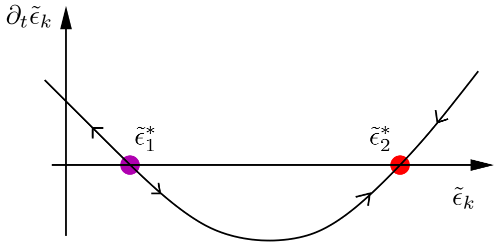

Fig. 1. Schematic plot of the fixed-point structure of the $\tilde{\epsilon}_k$ flow equation after fermion-boson translation. Arrows indicate the flow towards the infrared, $k \to 0$.

where the transformation parameter $\alpha_k(q)$ is an a priori arbitrary function. Projecting Eq. (3) onto our truncation (1), we arrive at modified flows for the couplings (the equation for $\bar{m}_k^2$ remains unmodified):

$$\begin{aligned} \partial_t \bar{h}_k &= \partial_t \bar{h}_k\big|_{\phi_k} + \bar{m}_k^2\, \partial_t \alpha_k, \\ \partial_t \bar{\lambda}_{\sigma,k} &= \partial_t \bar{\lambda}_{\sigma,k}\big|_{\phi_k} - \bar{h}_k\, \partial_t \alpha_k. \end{aligned} \tag{5}$$

We can now obtain bosonization at all scales, $\bar{\lambda}_{\sigma,k} = 0$, if we adjust α_k in such a way that the RHS of the $\partial_t \bar{\lambda}_{\sigma,k}$ equation equals zero for all k. This, of course, affects the flow of the Yukawa coupling $\bar{h}_k$. The physical effect can best be elucidated with the aid of the convenient coupling $\tilde{\epsilon}_k := \frac{\bar{m}_k^2}{k^2 \bar{h}_k^2}$ and its RG flow:

$$\partial_t \tilde{\epsilon}_k = -2\tilde{\epsilon}_k + \frac{1}{8\pi^2} + \frac{e^2}{\pi^2}\tilde{\epsilon}_k + \frac{9e^4}{4\pi^2}\tilde{\epsilon}_k^2 - \frac{1}{16\pi^2}\frac{\epsilon_k^2(3+\epsilon_k)}{(1+\epsilon_k)^3}, \tag{6}$$

where we also abbreviated $\epsilon_k := \frac{\bar{m}_k^2}{Z_{\phi,k} k^2}$. A schematic plot of $\partial_t \tilde{\epsilon}_k$ is displayed in Fig. 1 where the occurence of two fixed points is visible (note that all qualitative features discussed here are insensitive to the last term of Eq. (6)). The first fixed point $\tilde{\epsilon}_1^*$ is infrared unstable and corresponds to the inverse critical λ_{NJL} coupling. Starting with an initial value of $0 < \tilde{\epsilon}_\Lambda < \tilde{\epsilon}_1^*$ (strong coupling), the flow of the scalar mass-to-Yukawa-coupling ratio will be dominated by the first two terms in the flow equation (6) $\sim -2\tilde{\epsilon}_k + 1/(8\pi^2)$. This is a typical flow of a theory involving a "fundamental" scalar with Yukawa coupling to a fermion sector. Moreover, we will end in a phase with (dynamical) chiral symmetry breaking, since $\tilde{\epsilon} \sim \bar{m}_k^2$ is driven to negative values.

On the other hand, if we start with $\tilde{\epsilon}_\Lambda > \tilde{\epsilon}_1^*$, the flow will necessarily be attracted towards the second infrared-stable fixed point $\tilde{\epsilon}_2^*$. There will be no dynamical symmetry breaking, since the mass remains positive. The effective four-fermion interaction corresponding to the second fixed point reads: $\lambda_\sigma^* = \frac{1}{2k^2\tilde{\epsilon}_2^*} \approx \frac{9}{16\pi^2}\frac{e^4}{k^2}$, which coincides with the perturbative value of massless QED. We conclude that the second fixed point characterizes massless QED. The scalar field shows a typical bound-state behavior with mass and couplings expressed by e and k. A more detailed analysis reveals that the scalar field corresponds to positronium at this fixed point [3].

Our interpretation is that the "range of relevance" of these two fixed points tells us whether the scalar appears as a "fundamental" or a "composite" particle, corresponding to the state of the system being governed by $\tilde{\epsilon}_1^*$ or $\tilde{\epsilon}_2^*$, respectively.

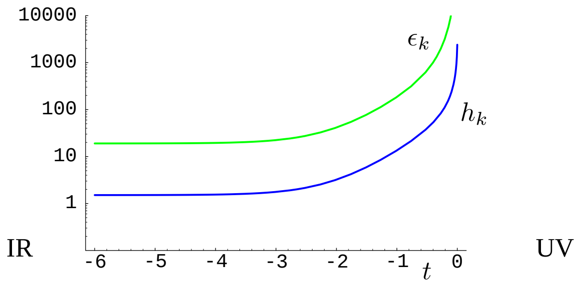

Fig. 2. Flows of ϵ_k, h_k in the symmetric phase according to Eqs. (8) for the initial values $\epsilon_\Lambda = 10^6$, $\tilde{\epsilon}_\Lambda = 0.17 > \tilde{\epsilon}_1^*$, $e = 1$, $Q_\sigma = -0.1$ ($t = \ln k/\Lambda$).

3 Physics at the bound-state fixed point

The bound-state fixed point $\tilde{\epsilon}_2^*$ shows further interesting physical properties. In order to unveil them, we have to include momentum dependences of the couplings; in particular, we study the momentum dependence of $\bar{\lambda}_{\sigma,k}$ in the s channel. Then we can generalize the fermion-to-boson translation (4),

$$\partial_t \phi_k(q) = -(\bar{\psi}_{\mathrm{L}} \psi_{\mathrm{R}})(q)\, \partial_t \alpha_k(q) + \phi_k(q)\, \partial_t \beta_k(q), \tag{7}$$

(and similarly for ϕ^*) with another a priori arbitrary function $\beta_k(q)$. Now we can fix $\alpha_k(q)$ and $\beta_k(q)$ in such a way that $\bar{\lambda}_{\sigma,k}(s)$ vanishes simultaneously for all s and k and that $\bar{h}_k$ becomes momentum-independent. Defining the dimensionless renormalized couplings $\epsilon_k = \bar{m}_k^2/(Z_{\phi,k} k^2)$, $h_k = \bar{h}_k\, Z_{\phi,k}^{-1/2}$, this procedure leads us to the final flow equations [3],

$$\begin{aligned} \partial_t \epsilon_k &= -2\epsilon_k + \frac{h_k^2}{8\pi^2} - \frac{\epsilon_k(\epsilon_k+1)}{h_k^2}\left(\frac{9e^4}{4\pi^2} - \frac{h_k^4}{16\pi^2}\frac{3+\epsilon_k}{(1+\epsilon_k)^3}\right)\big(1+(1+\epsilon_k)Q_\sigma\big), \\ \partial_t h_k &= -\frac{e^2}{2\pi^2}\, h_k - \frac{2\epsilon_k + 1 + (1+\epsilon_k)^2 Q_\sigma}{h_k}\left(\frac{9e^4}{8\pi^2} - \frac{h_k^4}{32\pi^2}\frac{3+\epsilon_k}{(1+\epsilon_k)^3}\right). \end{aligned} \tag{8}$$

Using $\tilde{\epsilon}_k = \epsilon_k/h_k^2$, Eq. (6) can be rediscovered from Eqs. (8). Defining $\Delta\bar{\lambda}_{\sigma,k} := \bar{\lambda}_{\sigma,k}(k^2) - \bar{\lambda}_{\sigma,k}(0)$, the quantity $Q_\sigma \equiv \partial_t \Delta\bar{\lambda}_{\sigma,k}/\partial_t \bar{\lambda}_{\sigma,k}(0)$ measures the suppression of $\bar{\lambda}_{\sigma,k}(s)$ for large external momenta. Without an explicit computation, we may conclude that this suppression implies $Q_\sigma < 0$ in agreement with unitarity (e.g., $Q_\sigma \simeq -0.1$). In Fig. 2, a numerical solution of Eqs. (8) is presented in which we release the system at Λ at $\tilde{\epsilon}_\Lambda > \tilde{\epsilon}_1^*$, so that it approaches $\tilde{\epsilon}_2^*$ in the IR.

We observe that both h_k and ϵ_k approach fixed points in the IR. (For analytical results for the fixed points, see [3].) In particular, this implies that the scalar mass term $m_k^2 = \epsilon_k k^2$ decreases with k^2 in the symmetric phase. This is clearly a nonstandard running of a scalar particle mass. As a consequence, a large scale separation $\Lambda \gg k$ gives rise to a large mass scale separation $m_\Lambda \gg m_k$ without any fine-tuning of the initial parameters.

4 Conclusions and Outlook

Within the framework of exact renormalization group flow equations for the effective average action, a scale-dependent transformation of the field variables provides for a continuous translation of UV to IR degrees of freedom. This concept is able to realize the physical criterion of desired simplicity of the effective action. Using the gauged NJL model as an example, this translation can be regarded as partial bosonization at all scales. Here we identified an infrared fixed-point structure which can be associated with a bound-state behavior. One main result is that the RG flow of the scalar mass at the bound-state fixed point is "natural" in 't Hooft's sense so that no fine-tuning problem arises if we want to have small masses at scales far below the UV cutoff $k \ll \Lambda$.

It should be interesting to see if this possibility of a naturally small scalar mass is applicable for the gauge hierarchy problem of the standard model. For this purpose, a mechanism has to be identified that causes the system to flow into the phase with spontaneous symmetry breaking after it has spent some "RG time" at the bound-state fixed point. Phrased differently, the bound-state fixed point has to disappear in the deep IR. Taking a first glance at Eq. (6), or its immediate nonabelian generalization for SU($N_{\rm c}$) gauge groups (here we use the Landau gauge),

$$\partial_t \tilde{\epsilon}_k = -\left(2 - \frac{3\,C_2}{4\pi^2}\,{g_k}^2\right)\tilde{\epsilon} + \frac{N_{\rm c}}{8\pi^2} + \frac{9}{8\pi^2}\frac{C_2}{N_{\rm c}}\left(C_2 - \frac{1}{2N_{\rm c}}\right){g_k}^4\,\tilde{\epsilon}^2 + \mathcal{O}(\epsilon^2), \tag{9}$$

where g_k is the running gauge coupling and $C_2 = (N_{\rm c}^2 - 1)/(2N_{\rm c})$, we find that the parabola depicted in Fig. 1 is lifted and the fixed points vanish for large gauge coupling. In this case, the system would finally run into the χSB phase once the gauge coupling has grown large enough. The question as to whether this mechanism can successfully be applied to a sector of the standard model is currently under investigation.

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