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RENORMALIZATION GROUP FLOW IN FIELD THEORIES WITH BROKEN SYMMETRY[1]

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We consider renormalization group (RG) equations in field theories with broken symmetry. The key idea is to treat them like unbroken theories in an external field and absorb the symmetry breaking terms into the redefinition of parameters. Then, RG equations for the broken theory follow directly from those of unbroken one. A particular example of a supersymmetric gauge theory is considered in more detail.

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1 Introduction

It is very common to consider theories with broken symmetry. Moreover, a typical situation is when symmetry is broken spontaneously due to non-zero expectation value of some field. This suggests the treatment of a broken theory as unbroken one in an external field. When this field has a vanishing expectation value the symmetry is unbroken, and when it has a non-vanishing value the symmetry is broken. So, if one is able to consider a theory in an arbitrary field, one can simultaneously treat both the broken and unbroken cases.

$$\begin{array}{ccc} \text{Symmetric theory} & \rightarrow & \text{Spontaneously Broken Symmetry} \\ \Downarrow & & \Downarrow \\ < H >= 0 & & < H >\neq 0 \end{array}$$

Assume now that treatment of a theory in external field can be considered as modification of original parameters in a sense that they become external field dependent quantities: $\{g\} \rightarrow \{g(H)\}$. This is not always the case, but if possible, leads to considerable progress.

Consider now the renormalization group (RG) equations for the couplings of the original unbroken theory. If, as has been mentioned above, one can treat the theory in an arbitrary field, one can make the replacement $g \rightarrow g(H)$ in the RG equation and get

$$\dot{g} = \beta(g) \ \rightarrow \ \dot{g}(H) = \beta(g(H)). \tag{1}$$

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Notice that the β function remains the same, just the argument is changed. This is the crucial point which is not obvious and has to be proved. Whence it is so, one can perform the Taylor expansion of the coupling over the external field

$$g(< H >) = g(0) + g'(0) < H > + \frac{< H >^2}{2} g''(0) + ...$$

in eq.(1) and get the RG equations for the symmetry breaking terms (g', g'', etc) directly from the one of unbroken theory without any further calculation

$$\left\{ \begin{array}{lcl} \dot{g} & = & \beta(g), \\ \dot{g}' & = & \frac{d\beta(g)}{dg} g', \\ \dot{g}'' & = & \frac{d^2\beta(g)}{dg^2}(g')^2 + \frac{d\beta(g)}{dg} g''. \end{array} \right. \tag{2}$$

We give explicit examples below.

The same procedure works also for solutions to the RG equations. Consider solution to the RG equation for effective coupling (one coupling case for simplicity) and make the same substitution

$$t = \int_{g_0}^{g_t} \frac{dx}{\beta(x)} \quad \rightarrow \quad t = \int_{g_0(H)}^{g_t(H)} \frac{dx}{\beta(x)}.$$

Performing Taylor expansion in the limits of integration one gets the solutions for the symmetry breaking terms

$$g'(t) = c_1 \beta(g), \quad g''(t) = c_1^2 \beta \frac{d\beta}{dg} + c_2 \beta(g), \tag{3}$$

where $g = g(t)$ is the solution in the unbroken case, and c_1 and c_2 are some constants.

Below we demonstrate how the advocated procedure works in the case of broken supersymmetric field theory.

2 Supersymmetric Field Theory

We give here a brief description of the model. Supersymmetry is a fermion-boson symmetry which converts bosons into fermions and vice versa, and changes the statistics. Supersymmetry algebra is an extension of Poincaré algebra by anticommuting generators [1]. Supersymmetric theory assumes that physical states (particles) come in pairs, called superpartners, each pair containing states with spin different by 1/2. If it is relevant to Nature, it has to be broken, since we do not observe a degenerate spectrum of fermions and bosons.

The simplest way to write the supersymmetric Lagrangian is to take the usual field theory in Minkowski space and go from space to superspace, which is an extension of Minkowski space by anticommuting Grassmannian variables. Then one has to replace ordinary fields depending on a space-time point by the superfields depending on superspace coordinates. These superfields contain both the fermionic and bosonic states transforming one into another under supersymmetry operators. Omitting the details for the lack of space, we write down a typical SUSY Lagrangian for a general gauge field theory [2]

$$\mathcal{L}_{rigid} = \int d^2\theta\ \frac{1}{4g^2}\mathrm{Tr}W^\alpha W_\alpha + \int d^2\theta d^2\bar\theta\ \ \bar\Phi^i(e^V)^j_i\Phi_j + \int d^2\theta\ \ \mathcal{W} + h.c, \tag{4}$$

where

$$W_\alpha = -\frac{1}{4}\bar D^2 e^{-V} D_\alpha e^V,\ \ \mathcal{W} = \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M^{ij}\Phi_i\Phi_j.$$

Here $V(x,\theta,\bar\theta)$ and $\Phi(x,\theta)$ are the gauge and matter superfields, respectively, W_α is the gauge field strength tensor and $\mathcal{W}$ is the so-called superpotential; θ and $\bar\theta$ are the Grassmannian coordinates. The Lagrangian (4) contains two dimensionless couplings g and y^{ijk} and one mass parameter M^{ij}.

This is a rigid or unbroken theory. The possible SUSY breaking terms are [3]:

$$-\mathcal{L}_{soft-breaking} \ \ = \ \ \left[\frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + h.c.\right] + (m^2)^i_j\phi^*_i\phi^j, \tag{5}$$

They break the mass degeneracy in the supermultiplets and can be rewritten in terms of superfields as

$$\begin{aligned}\mathcal{L}_{soft} \ &= \ \int d^2\theta\ \frac{1}{4g^2}(1-2M\eta)\mathrm{Tr}W^\alpha W_\alpha + \int d^2\theta d^2\bar\theta\ \ \bar\Phi^i(\delta^k_i - (m^2)^k_i\eta\bar\eta)(e^V)^j_k\Phi_j \\ &+ \ \int d^2\theta \left[\frac{1}{6}(y^{ijk} - A^{ijk}\eta)\Phi_i\Phi_j\Phi_k + \frac{1}{2}(M^{ij} - B^{ij}\eta)\Phi_i\Phi_j\right] + h.c.,\end{aligned} \tag{6}$$

where $\eta = \theta^2$ is an external spurion field. It is responsible for supersymmetry breaking. Obviously, eq.(6) can be treated as replacement of the couplings of unbroken Lagrangian (4) by the external field dependent quantities

$$\frac{1}{g^2} \to \frac{1}{\tilde g^2} = \frac{1 - M\eta - \bar M\bar\eta}{g^2},\ \ y^{ijk} \to \tilde y^{ijk} = y^{ijk} - A^{ijk}\eta,\ \ M^{ij} \to \tilde M^{ij} = M^{ij} - B^{ij}\eta,$$

The mass term m^2 can be absorbed into the redefinition of the Yukawa coupling y as well (see below).

Thus, a softly broken SUSY theory can be treated as a rigid SUSY theory in an external spurion superfield η with the couplings g, y and M being the external superfilelds $S(\eta,\bar\eta)$. Both the theories have the same singular part of the effective action.

The statement: In external spurion field η the singular part of the effective action depends on the couplings, but does not depend on their derivatives:

$$S^{eff}_{Sing}(g) \Rightarrow S^{eff}_{Sing}(S, \cancel{D^2}S, \cancel{\bar D^2 S}, D^2\cancel{\bar D^2}S),$$

and as a result has the same form in unbroken and broken cases with replacement of the couplings by external fields. As a consequence, the following theorem is valid [4]:

The theorem *Let a rigid theory be renormalized via introduction of the renormalization constants Z_i, defined within some minimal subtraction massless scheme. Then, a softly broken theo-*

ry is renormalized via introduction of the renormalization superfields $\tilde{Z}_i$ which are related to Z_i by the coupling constant redefinition

$$\tilde{Z}_i(g^2, y, \bar{y}) = Z_i(\tilde{g}^2, \tilde{y}, \tilde{\bar{y}}),$$

where the redefined couplings are

$$\begin{aligned}
\tilde{g}_i^2 &= g_i^2(1 + M_i\eta + \bar{M}_i\bar{\eta} + (2M_i\bar{M}_i + \Delta_i)\eta\bar{\eta}), \\
\tilde{y}^{ijk} &= y^{ijk} - A^{ijk}\eta + \frac{1}{2}(y^{njk}(m^2)^i_n + y^{ink}(m^2)^j_n + y^{ijn}(m^2)^k_n)\eta\bar{\eta}, \\
\tilde{\bar{y}}_{ijk} &= \bar{y}_{ijk} - \bar{A}_{ijk}\bar{\eta} + \frac{1}{2}(y_{njk}(m^2)^n_i + y_{ink}(m^2)^n_j + y_{ijn}(m^2)^n_k)\eta\bar{\eta},
\end{aligned} \tag{7}$$

Here Δ_i are unphysical ghost field masses to be eliminated from physical RG equations [5].

This leads to RG functions for the soft terms of a broken theory in terms of unbroken one. The results for the soft term β functions are summarized below.

Summary of the Soft Term Renormalizations

The Rigid Terms	The Soft Terms
$\beta_{\alpha_i} = \alpha_i\gamma_{\alpha_i}$	$\beta_{M_{Ai}} = D_1\gamma_{\alpha i}$
$\beta_M^{ij} = \frac{1}{2}(M^{il}\gamma^j_l + M^{lj}\gamma^i_l)$	$\beta_B^{ij} = \frac{1}{2}(B^{il}\gamma^j_l + B^{lj}\gamma^i_l) - (M^{il}D_1\gamma^j_l + M^{lj}D_1\gamma^i_l)$
$\beta_y^{ijk} = \frac{1}{2}(y^{ijl}\gamma^k_l+$ perm's)	$\beta_A^{ijk} = \frac{1}{2}(A^{ijl}\gamma^k_l+$ perm's) - $(y^{ijl}D_1\gamma^k_l+$ perm's)
$\Uparrow$	$(\beta_{m^2})^i_j = D_2\gamma^i_j$
chiral anomalous dim.	$\beta_{\Sigma_{\alpha_i}} = D_2\gamma_{\alpha_i}$

$$D_1 = M_{A_i}\alpha_i\frac{\partial}{\partial\alpha_i} - A^{ijk}\frac{\partial}{\partial y^{ijk}}, \quad \bar{D}_1 = M_{A_i}\alpha_i\frac{\partial}{\partial\alpha_i} - A_{ijk}\frac{\partial}{\partial y_{ijk}}$$

$$\begin{aligned}
D_2 = \bar{D}_1 D_1 + \Sigma_{\alpha_i}\alpha_i\frac{\partial}{\partial\alpha_i} + \tfrac{1}{2}(m^2)^a_n\Big(& y^{nbc}\frac{\partial}{\partial y^{abc}} + y^{bnc}\frac{\partial}{\partial y^{bac}} + y^{bcn}\frac{\partial}{\partial y^{bca}} \\
&+ y_{abc}\frac{\partial}{\partial y_{nbc}} + y_{bac}\frac{\partial}{\partial y_{bnc}} + y_{bca}\frac{\partial}{\partial y_{bcn}}\Big),
\end{aligned}$$

$$\Sigma_{\alpha_i} = M_{A_i}\bar{M}_{A_i} + \Delta_i$$

3 Illustration

Consider, as an illustration of the above formulas, the simplest case of a pure gauge theory [6]. In a rigid theory the coupling is renormalized as

$$\alpha^{Bare} = Z_\alpha\alpha, \quad \alpha \equiv g^2/16\pi^2.$$

Making the substitution $\alpha \to \tilde{\alpha}$ one gets $\tilde{\alpha}^{Bare} = \tilde{Z}_\alpha\tilde{\alpha}$ or (up to linear terms in η)

$$\alpha^{Bare}(1 + M_A^{Bare}\eta) = \alpha(1 + M_A\eta)Z_\alpha(\alpha(1 + M_A\eta)).$$

After expansion over η this leads to equations

$$\begin{aligned} \alpha^{Bare} &= \alpha Z_\alpha(\alpha), \\ M_A^{Bare}\alpha^{Bare} &= M_A\alpha Z_\alpha(\alpha) + \alpha D_1 Z_\alpha, \end{aligned}$$

where $D_1 = M_A\alpha\frac{\partial}{\partial\alpha}$ is the differential operator extracting linear terms over η. As a result, we get the bare mass

$$M_A^{Bare} = M_A + D_1 \ln Z_\alpha. \tag{8}$$

Differentiating eq.(8) with respect to a scale, one gets

$$\beta_\alpha = \alpha\gamma_\alpha, \quad \beta_{M_A} = D_1\gamma_\alpha,$$

where γ_α is the gauge field anomalous dimension $\gamma_\alpha = d\log Z_\alpha/d\log\mu^2$.

Consider now a solution to the RG equation in a rigid theory written in quadratures

$$\int_{\alpha_0}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} = \ln\frac{Q^2}{\mu^2}. \tag{9}$$

Performing the replacement of the couplings one gets

$$\int_{\alpha_0(1+M_{A0}\eta)}^{\alpha(1+M_A\eta)}\frac{d\alpha'}{\beta(\alpha')} = \ln\frac{Q^2}{\mu^2}$$

which after expansion over η leads to the solution for the soft mass term

$$\frac{\alpha M_A}{\beta(\alpha)} = \frac{\alpha M_{A0}}{\beta(\alpha_0)} \;\Rightarrow\; M_A = const\,\frac{\beta(\alpha)}{\alpha},$$

where α is taken from eq.(9). Thus, solution for the soft mass term directly follows from the one for the rigid coupling.

4 Examples

Consider now some particular examples in gauge field theories.

1. General gauge theory

In the one-loop order the rigid β functions are

$$\begin{aligned} \beta_\alpha &= \alpha\gamma_\alpha, \;\; \gamma_\alpha^{(1)} = \alpha Q, \quad Q = T(R) - 3C(G), \\ \beta_y^{ijk} &= \frac{1}{2}(y^{ijl}\gamma_l^k + perm's), \;\; \gamma_j^{i\,(1)} = \frac{1}{2}y^{ikl}y_{jkl} - 2\alpha C(R)_j^i, \end{aligned}$$

where $T(R), C(G)$ and $C(R)$ are the Casimir operators of the gauge group. This leads to the following soft β functions:

$$\begin{aligned} \beta_{M_A}^{(1)} &= \alpha M_A Q, \\ \beta_B^{ij\,(1)} &= \frac{1}{2}B^{il}(\frac{1}{2}y^{jkm}y_{lkm} - 2\alpha C(R)_l^j) + M^{il}(\frac{1}{2}A^{jkm}y_{lkm} + 2\alpha M_A C(R)_l^j) \\ &\quad + (i\leftrightarrow j), \end{aligned}$$

$$
\begin{aligned}
\beta_A^{ijk\ (1)} &= \frac{1}{2}A^{ijl}(\frac{1}{2}y^{kmn}y_{lmn} - 2\alpha C(R)_l^k) + y^{ijl}(\frac{1}{2}A^{kmn}y_{lmn} + 2\alpha M_A C(R)_l^k) \\
&\quad +(i \leftrightarrow j,k), \\
[\beta_{m^2}]_j^{i\ (1)} &= \frac{1}{2}A^{ikl}A_{jkl} - 4\alpha M_A^2 C(R)_j^i + \frac{1}{4}y^{nkl}(m^2)_n^i y_{jkl} + \frac{1}{4}y^{ikl}(m^2)_j^n y_{nkl} \\
&\quad +\frac{4}{4}y^{isl}(m^2)_s^k y_{jkl}.
\end{aligned}
$$

We used here the fact that in the given order the solution for Σ_α is $\Sigma_\alpha = M_A \bar{M}_A$.

At two loops the gauge anomalous dimension γ_α is

$$
\gamma_\alpha = \alpha Q + 2\alpha^2 QC(G) - \frac{2}{r}\alpha\gamma_j^{i\,(1)}C(R)_i^j \tag{10}
$$

and the β function for the gaugino mass takes the form

$$
\begin{aligned}
\beta_{M_A} &= \alpha M_A Q + 4\alpha^2 M_A QC(G) - \frac{2}{r}\alpha M_A \gamma_j^{i\,(1)} C(R)_i^j \\
&+ \frac{1}{r}\alpha(A^{ikl}y_{jkl} + y^{ikl}A_{jkl})C(R)_i^j - \frac{4}{r}\alpha M_A C(R)_j^i C(R)_i^j.
\end{aligned}
$$

In this case, again the solution for the ghost mass Δ_α can be found analytically [5]

$$
\Sigma_\alpha{}^{(2)} = \Delta_\alpha^{(2)} = -2\alpha[\frac{1}{r}(m^2)_j^i C(R)_i^j - M_A^2 C(G)].
$$

In case when one knows the β function in all loops as in the NSVZ scheme [7]

$$
\gamma_\alpha^{NSVZ} = \alpha\frac{Q - 2r^{-1}\mathrm{Tr}[\gamma C(R)]}{1 - 2C(G)\alpha},
$$

one has the all loop β function for a gaugino mass as well as the all loop solution for Δ_α:

$$
\Delta_\alpha^{NSVZ} = -2\alpha\frac{r^{-1}\mathrm{Tr}[m^2 C(R)] - M_A^2 C(G)}{1 - 2C(G)\alpha}.
$$

2. MSSM in the low tan β regime

The Minimal Supersymmetric Standard Model (MSSM) is the generalization of the Standard Model of fundamental interactions [8]. The Lagrangian of the MSSM repeats that of the SM with replacements of all the fields by appropriate superfields. It contains three gauge and a set of Yukawa couplings the same as in the SM. However, the RG equations are slightly modified due to the presence of extra particles. In the so-called low $\tan\beta$ regime one is left with three gauge and one Yukawa coupling. The one loop rigid RG equations are

$$
\begin{cases}
\dot{\alpha}_i = -b_i\alpha_i^2, \quad i = 1,2,3 \\
\dot{Y}_t = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t).
\end{cases}
$$

Their analytical solution has the form [9]

$$
\begin{cases}
\alpha_i(t) = \dfrac{\alpha_0}{1 + b_i\alpha_0 t}, \quad t = \ln\frac{M_X^2}{Q^2} \\
Y_t(t) = \dfrac{Y_0 E(t)}{1 + 6Y_0 F(t)}, \quad E(t) = \prod_i (1 + b_i\alpha_0 t)^{c_i/b_i}, \quad F(t) = \int_0^t E(t')dt'.
\end{cases} \tag{11}
$$

To get solutions for the soft terms one has to make the substitution: $\alpha \to \tilde{\alpha}, Y \to \tilde{Y}$ and expand over $\eta, \bar{\eta}$. For the gauge coupling one has [6]

$$\alpha_i M_{A_i} = \frac{\alpha_0 M_0}{1+b_i\alpha_0 t} - \frac{\alpha_0 b_i \alpha_0 M_0 t}{(1+b_i\alpha_0 t)^2} = \frac{\alpha_0}{1+b_i\alpha_0 t}\cdot\frac{M_0}{1+b_i\alpha_0 t} \;\Rightarrow\; M_{A_i} = \frac{M_0}{1+b_i\alpha_0 t}.$$

Analogously

$$A_t(t) = \frac{A_0}{1+6Y_0F} - M_0[\frac{t}{E}\frac{dE}{dt} - \frac{6Y_0}{1+6Y_0F}(tE-F)].$$

Expanding up to $\eta\bar{\eta}$ one can get the solution for the mass term ($\Sigma_t = \tilde{m}_U^2 + \tilde{m}_Q^2 + \tilde{m}_H^2$):

$$\Sigma_t(t) = \frac{\Sigma_0 - A_0^2}{1+6Y_0F} + \frac{[A_0 + 6M_0Y_0(tE-F)]^2}{(1+6Y_0F)^2} + M_0^2[\frac{d}{dt}(\frac{t^2}{E}\frac{dE}{dt}) - \frac{6Y_0}{1+6Y_0F}t^2\frac{dE}{dt}].$$

These solutions exhibit the so-called fixed point behaviour [10]. Namely, if one neglects ”1” in the denominator of expression for the Yukawa coupling in eq.(11), the initial value Y_0 cancels and one gets the fixed point value at the infrared region (which corresponds to $t \to \infty$)

$$t \to \infty \;\;\Rightarrow Y_t(t) \to Y_t^{FP} = \frac{E}{6F}. \tag{12}$$

The same fixed point behaviour is true for the soft parameters. To find it, one has to replace the couplings in (12) by the proper superfields and expand the FP over $\eta, \bar{\eta}$. This gives directly the FP for the soft terms.

$$\begin{aligned} A_t^{FP} &= -M_0\left(\frac{t}{E}\frac{dE}{dt} - \frac{tE-F}{F}\right), \\ \Sigma_t^{FP} &= M_0^2\left[(\frac{tE-F}{F})^2 + \frac{d}{dt}(\frac{t^2}{E}\frac{dE}{dt}) - \frac{t^2}{F}\frac{dE}{dt}\right]. \end{aligned}$$

Thus, the FP's for the soft terms follow those for Yukawa couplings and have the same stability properties.

3. MSSM in the high tan β regime

Here we have three Yukawa couplings and the corresponding RG equations look like

$$\begin{cases} \dot{\alpha}_i = -b_i\alpha_i^2, \quad i = 1,2,3 \\ \dot{Y}_t = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 - 6Y_t - Y_b), \\ \dot{Y}_b = Y_t(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{7}{15}\alpha_1 - Y_t - 6Y_b - Y_\tau), \\ \dot{Y}_\tau = Y_\tau(3\alpha_2 + \frac{9}{5}\alpha_1 - 3Y_b - 4Y_\tau). \end{cases}$$

Despite a simple form of these equations, there is no explicit analytic solution similar to (11). One has either approximate solution [11] or the iterative one [12]. In both the cases the Grassmannian expansion over η leads to the corresponding solutions for the soft terms.

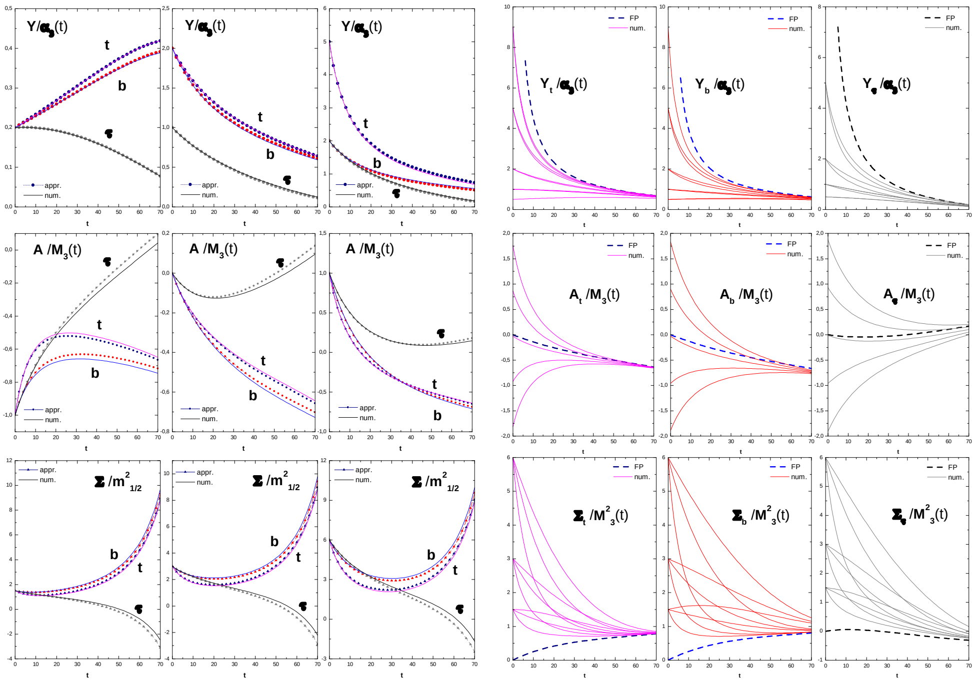

Fig. 1. Comparison of numerical and approximate solutions. Dotted lines correspond to the analytical approximate solutions, solid lines to the numerical solution.

We consider as an illustration the approximate solution. It can be taken in the form [11]

$$\begin{aligned}
Y_t^{app}(t) &= \frac{Y_{t0}E_t(t)}{[1+\frac{7}{2}(Y_{t0}F_t(t)+Y_{b0}F_b(t))]^{2/7}[1+7Y_{t0}F_t(t)]^{5/7}} \\
Y_b^{app}(t) &= \frac{Y_{b0}E_b(t)}{[1+\frac{7}{2}(Y_{t0}F_t(t)+Y_{b0}F_b(t))]^{2/7}[1+7Y_{t0}F_t(t)]^{2/7}}, \\
&\qquad \times \frac{1}{[1+\frac{7}{3}(3Y_{b0}F_b(t)+Y_{\tau 0}F_\tau)]^{3/7}}, \\
Y_\tau^{app}(t) &= \frac{Y_{\tau 0}E_\tau(t)}{[1+\frac{21}{4}Y_{\tau 0}F_\tau]^{4/7}[1+\frac{7}{3}(3Y_{b0}F_b(t)+Y_{\tau 0}F_\tau)]^{3/7}}.
\end{aligned} \tag{13}$$

To demonstrate the accuracy of the approximate solution (13) and the efficiency of the Grassmannian expansion, we present in Fig.1 the comparison of numerical and approximate solutions for the Yukawa couplings of a rigid theory as well as the soft terms. One can notice perfect agreement of numerical and analytical curves. Shown also are the fixed point behaviour, again for the Yukawa couplings and for the soft terms obtained via the expansion procedure for the approximate solutions (13). The numerical curves approach the analytically calculated FP's in the infrared region.

5 Totally all loop finite N=1 SUSY gauge theories

Another example of application of the same procedure is the so-called finite field theories in the framework of SUSY GUTs. These are the theories where all the UV divergences cancel and hence all the β functions vanish. This can be achieved in a rigid theory if the following two conditions are satisfied [13, 14]:

• The group representations are chosen in a way to obey the sum rule

$$\sum_R T(R) = 3C_2(G)$$

• The Yukawa couplings are the functions of the gauge one

$$Y_i = Y_i(\alpha),\ Y_i(\alpha) = c_1^i \alpha + c_2^i \alpha^2 + ... \tag{14}$$

where the coefficients c_n^i are calculated algebraically in the n-th order of PT.

To achieve the complete finiteness of the model including the soft terms, one has to modify the finiteness condition (14) as

$$\tilde{Y}_i = Y_i(\tilde{\alpha})$$

and perform the expansion over $\eta, \bar{\eta}$. This gives [15]

$$\left\{ \begin{array}{l} A_i = -M_A \dfrac{d \ln Y_i}{d \ln \alpha}, \\ \\ \Sigma_i = M_A^2 \dfrac{d}{d\alpha} \alpha \dfrac{d \ln Y_i}{d \ln \alpha}, \end{array} \right.$$

where $Y(\alpha)$ is assumed to be known from a rigid theory. These expressions lead to a totally finite softly broken SUSY field theory!

Alternatively one can formulate the same conditions in terms of the bare couplings. They are finite in this case. In dimensional regularization one has instead of eq.(14)

$$Y_i^{Bare} = \alpha_{Bare} \cdot f_i(\varepsilon), \quad f_i(\varepsilon) = c_i^{(1)} + c_i^{(2)}\varepsilon + c_i^{(3)}\varepsilon^2 + ... \tag{15}$$

where the coefficients $c_i^{(n)}$ are in one-to-one correspondence to those in eq.(14). Replacing the couplings in eq.(15) in a usual way one finds that the function $f(\varepsilon)$ cancels and one has simple relations for the soft terms valid in all orders of PT [6]

$$\tilde{Y}_i^{Bare} = \tilde{\alpha}_{Bare} \cdot f_i(\varepsilon) \ \Rightarrow \left\{ \begin{array}{l} A_i^{Bare} = -M_A^{Bare}, \\ \Sigma_i^{Bare} = (M_A^{Bare})^2. \end{array} \right.$$

These relations for the bare quantities provide the vanishing of the β functions for the soft terms.

6 Conclusion

It is very useful to consider a spontaneously broken theory in terms of a rigid one in an external field. In case when one is able to absorb the external field into the redefinition of parameters of the original theory and perform the renormalizations for an arbitrary field, one can reproduce renormalization properties of a spontaneously broken theory from a rigid one. The following statements are valid:

- All the renormalizations are defined in a rigid theory. There are no independent renormalizations in a softly broken theory.
- RG flow in a softly broken theory follows that in a rigid theory.
- This statement is true for RG equations, solutions to these equations, particular (fixed point) solutions, approximate solutions, etc.

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