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ELECTROMAGNETIC PROPERTIES OF ^{3}He

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In the frame of the hyperspherical formalism, the electromagnetic properties of ^{3}He are investigated for values of high momentum transfer up to 32 fm^{-2}. Calculations of electromagnetic form factors have been performed for momentum transfer up to 22 fm^{-2} and 32 fm^{-2} for the charge and magnetic form factors, respectively. The charge radii (rms) for ^{3}He are also calculated. Gaussian shaped spin-isospin potentials are used in generating complete mixed and mixed symmetric wave functions to be used in the form factors calculations. Contributions of partial waves are also taken into account. The present calculations are in good agreement with the experimental high momentum transfer measurements as well as the previous calculations using variational and Faddeev methods including the meson exchange current contributions.

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1 Introduction

The three body nuclear systems are shown very interesting to investigate due to the fact that it is the simplest non trivial nucleus that can be investigated by exact theoretical methods. The trinucleon system was studied applying different methods, using the nucleon-nucleon realistic forces as the basic input. The ability to predict the three body properties (such as binding energy, charge and magnetic form factors) gives strong indications on the ability of the considered nuclear force or the applied method of calculations. The produced current by the mesons exchanged between the nucleons strongly affect the electromagnetic properties of the trinucleon system. Therefore, investigating the electromagnetic form factors of ^{3}He nucleus provides us with interesting and useful information about the nucleonic and non nucleonic degrees of freedom of the complex nuclear systems. Experimental measurements of the ^{3}He magnetic form factors were performed [1] up to $q^2 = 10$ fm^{-2}, and then [2] to $q^2 = 30$ fm^{-2} to give a minimum at $q^2 = 15.0$ fm^{-2}. Therefore the experimental data on the ^{3}He magnetic form factors are available [2,3] for momentum transfer up to $q^2 = 30$ fm $^{-2}$. Experimental data [4] on the ^{3}He charge form factors are also available for momentum transfer up to 20 fm^{-2}. In the trinucleon system, the three body wave functions obtained with Reid or Hamada Johnston potentials and variational calculations in harmonic oscillator have yielded magnetic form factors minima in the neighborhood of $q^2 = 14.0$

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fm^{-2}. In the impulse approximation calculations [3], a minimum is obtained at $q^2 = 8.0$ fm^{-2} for the ^{3}He magnetic form factors which is too far from the experimental value minimum. Faddeev equations were solved using truncated Reid potentials to give a value of about 17.0 fm^{-2} for the ^{3}He magnetic form factor minimum. Different calculation were performed [5–7] in studying the charge form factors of ^{3}He. Variational calculations [5] were performed using Hamada Johnston and Reid potential to give a charge form factor minimum at 13 fm^{-2}. Their wave function includes the symmetric, anti-symmetric and mixed symmetric states. A very complete variational calculations [6] were performed using Reid potential obtaining a charge form factor minimum at 12.6 fm^{-2} and rms radius value of 2.06 fm. Faddeev equations were solved in configuration space [7] using Reid potentials and including the symmetric, anti-symmetric and mixed symmetric states obtaining a charge form factor minimum at 14.6 fm^{-2} and rms radius value of 1.9 fm. With these theoretical calculations, the electromagnetic properties of the trinucleon system were investigated using different methods and different two nucleon potentials [9–14]. In a previous work [8], we have calculated the trinucleon magnetic form factors using Faddeev method with realistic two body potentials. The obtained results were in a good agreement with the experimental values. It is the purpose of the present work to calculate the electromagnetic form factors and rms radii for ^{3}He nucleus in the frame of the hyperspherical formalism using realistic two body spin-isospin potentials of different shapes and different soft cores.

In the hyperspherical formalism [14,15], the trinucleon system is described in the Jacobian coordinates and then the system is expressed in terms of the hyperspherical coordinates. The kinetic energy operator of the three-nucleon system can be given in terms of the hyperspherical coordinates. The total trinucleon wave function in hyperspherical coordinates is expanded in a complete set of symmetrized angular eigen functions at the surface of a hypersphere. Then, the Schrodinger equation of the trinucleon system is transformed into an infinite set of coupled second order differential equations. The two-body central potentials are expanded in partial wave composition giving partial potentials. In case of symmetric or mixed symmetric states, the two-nucleon interaction $V(r_{ij})$ is half of the sum or the difference of the triplet and singlet even central potentials. Therefore, the coupling coefficients have been deduced [15] explicitly which are geometrical ones and independent on the interactions. The different properties of the ^{3}He nucleus were studied [16,17] in the frame of the hyperspherical formalism.

It is the purpose of our present work, to calculate the ^{3}He electromagnetic form factors following the hyperspherical formalism. The charge and magnetic form factors are evaluated using a hyperradial ^{3}He wave function obtained from spin-isospin two-nucleon potentials. Different spin-isospin two-nucleon interactions of the Gaussian form with soft cores are used [19,20]. The mixed symmetry contributions to the form factor are taken into account. We notice that in case of the potential in which the triplet and singlet even parts are equal, the mixed symmetry contribution vanishes. In Section 2, the different forms of the charge and magnetic form factors of the ^{3}He nucleus are given using the hyperspherical treatment. In Section 3, the numerical calculations and results are introduced. Discussion and conclusions are given in Section 4.

2 Electromagnetic form factors

The three nucleons are considered in the limit of the non relativistic case which are taken to contribute without mutual interference or distortions. Then, the magnetic moment density and electric charge density operators for the trinucleon system in momentum space are given as [8]

$$\rho_M(q) = \sum_{i=1}^{3}\left[\frac{1}{2}\sigma_z(i)\left(1+\tau_z(i)\right)\mu_p f^p_{mg}(q) + \frac{1}{2}\sigma_z(i)\left(1-\tau_z(i)\right)\mu_n f^n_{mg}(q)\right] \tag{1}$$

$$\rho_C(q) = \sum_{i=1}^{3}\left[\frac{1}{2}(1+\tau_z(i))f^p_{ch}(q) + \frac{1}{2}(1-\tau_z(i))f^n_{ch}(q)\right] , \tag{2}$$

where $\sigma(i)$ and $\tau(i)$ are the spin and isospin operators which operate on the spin and isospin nucleon states respectively while μ_p, $f^p_{mg(ch)}(q)$ and μ_n, $f^n_{mg(ch)}(q)$ are the static magnetic moments and magnetic (charge) form factors, for the proton and neutron, respectively. These form factors are the Fourier transforms of the normalized nucleon form factors $f^p_{mg(ch)}(\bar{x}-\bar{r}_i)$ and $f^n_{mg(ch)}(\bar{x}-\bar{r}_i)$, which are functions of $\bar{r}_i$ and $\bar{x}$ being the coordinates of the nucleon i and the center of mass, respectively. Therefore, the trinucleon magnetic and charge form factors can be given as

$$\mu(T_z)F^{T_z}_{mg}(q) = \langle\psi \sum_{i=1}^{3}\left[\frac{1}{2}\sigma_z(i)\left(1+\tau_z(i)\right)\mu_p f^p_{mg}(q) + \frac{1}{2}\sigma(i)\left(1-\tau_z(i)\right)\mu_n f^n_{mg}(q)\right] \times \\ \exp i\bar{q}.(\bar{r}_i-\bar{x})|\psi\rangle , \tag{3}$$

$$\left(\frac{3}{2}+T_z\right)F^{T_z}_{ch}(q) = \langle\psi \sum_{i=1}^{3}\left[\left(1+\tau_z(i)\right)f^p_{ch}(q) + \frac{1}{2}\left(1-\tau_z(i)\right)f^n_{ch}(q)\right] \times \\ \exp i\bar{q}.(\bar{r}_i-\bar{x})|\psi\rangle , \tag{4}$$

where $\mu(T_z)$ is the magnetic moment of the trinucleon system and T_z is the third component of the three nucleon system isospin which equals to $1/2$ for the ^{3}He nucleus and $-1/2$ for the ^{3}H nucleus. Introducing the scalar $G^S_{M(E)}(q)$ and vector $G^V_{M(E)}(q)$ nucleon magnetic (electric) form factors in equation (3) and (4), then the trinucleon magnetic form factors can be given as

$$\mu(T_z)F^{T_z}_{mg}(q) = \frac{3}{2}\langle\psi\left|\left[3G^S_M(q)+2G^V_M(q)\right]\mathrm{e}^{i(\bar{q}.\bar{x}/\sqrt{3})}\right|\psi\rangle \tag{5}$$

$$\left(\frac{3}{2}+T_z\right)F^{T_z}_{ch}(q) = 3\langle\psi\left|\left[G^S_E(q)+G^V_E(q)\right]\mathrm{e}^{i(\bar{q}.\bar{x}/\sqrt{3})}\right|\psi\rangle , \tag{6}$$

where the scalar and vector nucleon magnetic form factors are given as

$$G^S_M(q) = \frac{1}{2}\left(\mu_p f^p_{mg}(q) + \mu_n f^n_{mg}(q)\right) \tag{7}$$

$$G^V_M(q) = \frac{1}{2}\left(\mu_p f^p_{mg}(q) - \mu_n f^n_{mg}(q)\right) . \tag{8}$$

The scalar and vector nucleon charge form factor are given as

$$G^S_E(q) = \frac{1}{2}\left(f^p_{ch}(q) + f^n_{ch}(q)\right) \tag{9}$$

$$G^V_E(q) = \frac{1}{2}\left(f^p_{ch}(q) - f^n_{ch}(q)\right) . \tag{10}$$

The anti-symmetric normalized wave function ψ of the S- and D-state components [16], are used in equations (5) and (6) to give the magnetic and charge form factors as

$$\mu(T_z)F_{mg}^{T_z}(q) = \frac{3}{2}(8)\left[3G_M^S(q) + 2T_zG_M^V(q)\right]\sum_{\ell R} F_R^\ell(q) \tag{11}$$

$$\left(\frac{3}{2} + T_z\right)F_{ch}^{T_z}(q) = 8\left[3G_E^S(q) + 2T_zG_E^V(q)\right]\sum_{\ell R} F_R^\ell(q)\,, \tag{12}$$

where ℓ takes the value 0 for the central force term (S-state), and the value 2 for the tensor force term (D-state). R stands for the S, S' and D states, so that $F_R^\ell(q)$ represents the different form factors standing for the different contributions of the symmetric (S), mixed symmetric (S') and the D-states. In case of central force contribution where $\ell = 0$, the form factors $F_R^\ell(q)$ are expressed as [15]

$$F_R(q) = \sum_{KK'K''}(-)^{K''}\langle K|K'|K''\rangle\int_0^\infty \mathrm{d}\rho U_{2K}^R(\rho)U_{2K'}^R(\rho)\frac{J_{2K''+2}(q'\rho)}{(q'\rho)^2}\,, \tag{13}$$

where $U_{2K}^R(\rho)$ and $U_{2K'}^R(\rho)$ are the hyperradial wave functions. The geometrical coefficients $\langle K|K'|K''\rangle$ appearing in equation (13) couple the set of coupled equations with the main one for which $K = 0$ for each component of the central component of the two-body potentials. Explicit expressions for these coefficients are given in Ref. [16]. The summation indices K, K', and K'' in equation (13) satisfy the following condition:

$$|K - K'| \leq K'' \leq K + K'\,, \qquad \text{and} \qquad K + K' + K'' \equiv \text{even}\,.$$

According to these conditions, the symmetric S-state form factor contributions are given as [12]

$$F_S(q) = \sum_{K=0} F_{2K}(q) + \sum_{K\neq 0} F_{0,2K}(q)\,, \tag{14}$$

where

$$F_{2K}(q) = \frac{\langle 0|K|K\rangle^2}{(K+1)^2}\int \mathrm{d}\rho\left|U_{2K}^S(\rho)\right|^2\frac{J_2(q'\rho)}{(q'\rho)^2} \tag{15}$$

and

$$F_{0,2K}(q) = \frac{\langle 0|K|K\rangle^2}{(K+1)^2}\int \mathrm{d}\rho U_0^S(\rho)U_{2K}^S(\rho)\frac{J_{2K+2}(q'\rho)}{(q'\rho)^2}\,, \tag{16}$$

with $q' = q/\sqrt{3}$ and $J_{2K}(q'\rho)$ is the normal spherical Bessel function. It should be pointed out that we have neglected the cross-terms between the $U_{2K}(\rho)$ functions for which $K \neq 0$. Also, the mixed symmetric (S') form factor contributions are given to the first order approximation as

$$F_M(q) = \sqrt{2}F_{0,2}(q)\,. \tag{17}$$

Potential		a_i			b_i		
		a_1	a_2	a_3	b_1	b_2	b_3
Volkov [19]	t	144.86	-83.34	–	0.82	1.6	–
	s	144.86	-83.34	–	0.82	1.6	–
Peries [21]	t	509.00	-182.2	–	0.7	1.41	–
	s	509.00	-182.2	–	0.7	1.41	–
V^X [20]	t	130.00	-110.0	–	0.8	1.5	–
	s	130.00	-65.30	–	0.8	1.5	–

Tab. 1. Parameters of the Gaussian two-body potentials of the form $V^{t(s)}(r) = \sum_{i=1}^{3} a_i^{t(s)} \mathrm{e}^{-(r/b_i^{t(s)})^2}$

Potential	Volkov	Peries	V^X	Experiment
rms(^{3}He) [fm]	1.720	1.780	1.825	1.82

Tab. 2. Calculated values of the rms (charge radii) for the ^{3}He nucleus using different Gaussian shape potentials

3 Numerical calculations and results

In the present calculations, the hyperspherical formalism is used to get hyperradial wave functions to be used in generating the electromagnetic form factors of the ^{3}He nucleus. The symmetric (S) state hyperradial wave function which has the dominant contribution to charge and magnetic form factor is considered. The mixed symmetric (S') which is responsible for the difference between the ^{3}H and ^{3}He nuclei is also considered in this study. Accordingly, the mixed symmetric as well as the completely symmetric contributions to the ^{3}He form factors have been separately calculated. The form factors $F_S(q)$ and $F_M(q)$ are calculated using equations (14) and (17) with the understanding that the mixed symmetric magnetic form factor $F_M(q)$ vanishes and equal zero when the triplet and singlet even parts of the two-body potentials are equal. In these calculations, spin-isospin dependent two-nucleon potentials of Gaussian shape are used and expressed as [19–21]

$$V^{t(s)}(r) = \sum_{i=1}^{3} a_i^{t(s)} \exp\left[-(r/b_i^{t(s)})^2\right] \tag{18}$$

where $a_i^{t(s)}$ corresponds to the strength of the triplet and (singlet) parts of the potential, while $b_i^{t(s)}$ refers to the range of the triplet and (singlet) parts of the potential. The different parameters of the used potentials are given in Tab. 1. In calculating the magnetic form factor, we have used also the values $\mu(^3\mathrm{He}) = -2.1276$ nm, $\mu_p = 2.793$ nm, $\mu_n = -1.9135$ nm for the ^{3}He, proton, and neutron static magnetic moments, respectively. In calculating the nucleon electromagnetic form factors, a dipole fit [22] of the form $[1/(1+|q|^2/0.71(\mathrm{GeV/c})^2)^2]$ is used.

It was found that the contributions of the hyperradial wave functions for values of $K > 8$ corresponding to U_{18}, U_{20} are so small without any effect on the obtained values of the form

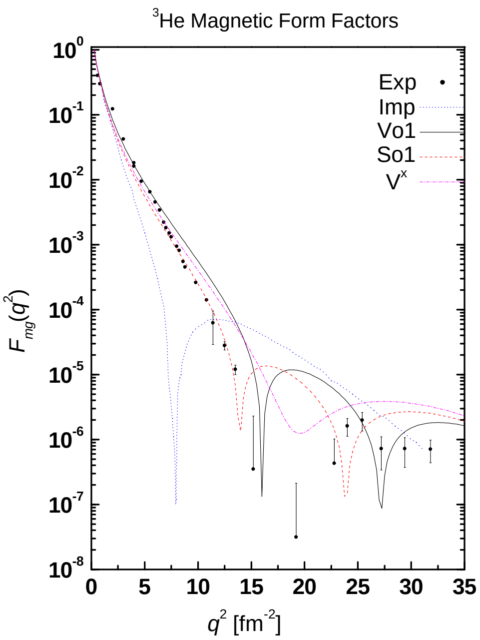

Fig. 1. ^{3}He magnetic form factors using Gaussian shape potentials. Experimental points are taken from Ref. [3].

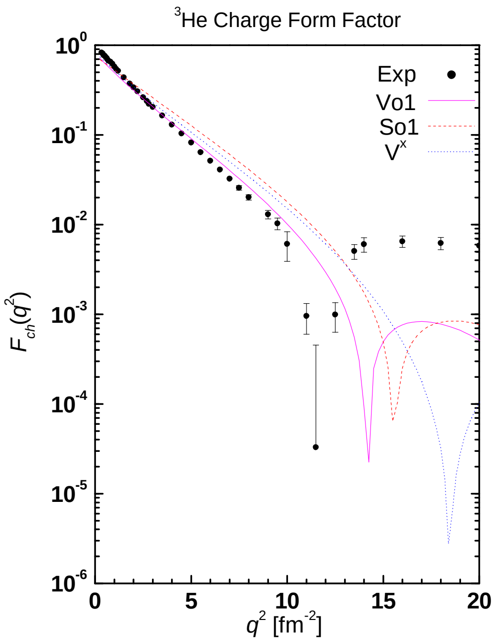

Fig. 2. ^{3}He charge form factors using Gaussian shape potentials. Experimental points are taken from Ref. [4].

factors. Therefore, it is justified to neglect terms for which $K > 8$ in the equations representing the symmetric and mixed symmetric form factors given by equations (14) and (17). Presently, numerical calculations are carried out for the U_{18} contributions using different potentials [19,21]. The calculated magnetic and charge form factors for the ^{3}He nucleus are shown in Figs. 1 and 2 respectively. The calculated values for the charge radii (rms) of ^{3}He are also given in Tab. 2.

4 Discussion and conclusions

The calculated ^{3}He magnetic form factors and the experimental data as well as the impulse [3] approximation results are shown in Fig. 1. The impulse approximation results are presented here for comparison, where the first minimum is obtained at $q^2 = 8$ fm^{-2}. We notice from Fig. 1 that the slopes of the calculated magnetic form factors for zero momentum transfer ($q^2 = 0$) are in good agreement with the experimental points. We also notice that the calculated form factors for Peries [21] and Volkov [19] potentials give first minimum around the experimental value ($q^2 = 15$ fm^{-2}), since Peries potential shows a minimum at $q^2 = 15.5$ fm^{-2}, while for Volkov potential a minimum is obtained at $q^2 = 14.0$ fm^{-2}. As for the modified Volkov Potential V^x [20] for which the triplet and singlet parts are not equal we have obtained a first minimum at $q^2 = 19.5$ fm^{-2} which does not fit the first minimum of the experimental points. In other words, this potential is too week to give the magnetic moment experimental values of the ^{3}He nucleus.

The calculated values as well as the experimental data for ^{3}He charge form factors are given in Fig. 2. Also the calculated (rms) values are given in Tab. 2. We notice from the figure that the calculated charge form factors are in good agreement with the experimental data at low momentum transfer ranging from $q^2 = 0$ fm^{-2} up to $q^2 = 5$ fm^{-2}. The calculated values for the charge radii (rms) of ^{3}He are in good agreement with the measured value. The calculated charge form factors for Volkov and Peries potential give a first minimum at $q^2 = 14$ fm^{-2} and $q^2 = 15$ fm^{-2} as can be seen from Fig. 2. For the modified Volkov potential V^x, a first minimum is obtained at $q^2 = 18.5$ fm^{-2} which is far from the measured value at $q^2 = 11.5$ fm^{-2}. So, we can say that the core is too week to produce a minimum that fit the experimental data. From the above result a general trend could be noticed for the used potential which is the good agreement between the calculated electromagnetic form factors and the experimental value for the Volkov and Peries potential while a poor agreement for the modified V^x potential. Also our calculated form factor are comparable with the results obtained applying the Faddeev and variational techniques [1–8] which show that the hyperspherical method is a reliable method for studying the different properties of the trinucleon system and is comparable with other methods such as the Faddeev and variational techniques. This shows that our considered potentials are reliable enough to be used to investigate the different properties of the ^{3}He nucleus and the trinucleon system in general. We expect that including the non-nucleonic degrees of freedom (i.e., meson exchange current contributions) and the three body effect might slightly change the position of the first minimum of the ^{3}He electromagnetic form factors.

From the above discussion, we conclude that in the frame of the hyperspherical formalism we have obtained, with realistic two body potentials, good results for the ^{3}He electromagnetic form factors which are in good agreement with the experimental measurements as well as the theoretical methods such as Faddeev [8] and variational [1] ones. We also conclude that the wave functions introduced by the hyperspherical formalism generate ^{3}He magnetic form factors, which are nearly independent of the potential used.

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