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A MODEL FOR THE DOUBLETS OF THE $K^\pi = 0^+$ STATES IN DEFORMED NUCLEI

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The aim of this paper is to present a model based on the neutron-proton pairing vibrations for the doublets of the excited 0^+ states that were observed in some deformed nuclei in the rare earth region. To prove our hypothesis on these states we have calculated their excitation energies using the Quasiparticle Random Phase Approximation (QRPA) method.

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1 Introduction

Understanding of the nature of the collective $K^\pi = 0^+$ states in even-even deformed nuclei continues to be a challenging theoretical problem, where the K is the projection quantum number of the total angular momentum on the z-axis and the π is the quantum number related with parity. During the last decade, the situation has changed gradually by the development of new experimental methods which increased sensitivity of the measurements. As a result, the proposal of the collective model which describes the lowest 0^+ state as β-vibration band head has been discussed intensively by many researchers [1–13]. Recently, many studies dealing with the different aspects of the problem were published [2,14–20]. Meanwhile, according to the calculations in [21], the 0^+ states below 2 MeV in deformed nuclei should be pairing vibrational type. In addition recent evaluation of the experimental data and present models by Garrett also shows that new models should be based on largely pairing interactions [2]. All of the discussions show that new microscopic models and approximations are necessary because the $K^\pi = 0^+$ states exhibits different properties in different nuclei i.e. they have not a certain systematic.

In the nuclei with $N \approx Z$, since the neutron and proton Fermi levels are close each other and hence the neutron-proton (np) pairing correlations are expected to play a significant role, the earlier studies on np pairing interactions concentrated mostly on light nuclei [22–24]. The possible effects of np pairing correlations on heavier nuclei was, unfortunately, neglected because of the neutron excess. However, the developments in the structure of nuclei lying close to the proton drip line, super dense matter and of double beta decay made in the last two decades indicate that it is difficult to make a serious advance without considering the np interactions in

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medium and heavy mass nuclei [25–28]. In this context, Cheoun *et al.* showed for medium and heavy mass nuclei that in many cases the solution of the nuclear Hamiltonian with np pairing gives rise to the minimum which is deeper than that of without np pairing correlation [25]. This and similar results reveal the importance of np correlations for medium and heavy mass nuclei clearly. Moreover, the existing theoretical approximations for np pairing correlations are not satisfactory or too complicated to apply to any real problem [29–32]. Although there has been many efforts, the problem could not, however, be solved satisfactorily yet.

The special type of neutron-proton (snp) pairing interaction was first used in [33,34], then in [39,40] and by the present author and collaborators [41,42] independently in the studies of nuclear structure. Grigorescu presented it as gauge restoring interaction which breaks the number symmetry and gave its physical meaning. In addition, it was shown the cranking method could be used to restore the broken symmetry, and proposed that such an interaction could produce scissors vibrations in the BCS gauge space which may be considered as a different collective mode for the atomic nucleus [33–38]. On the other hand, Delion *et al.* used the snp interaction for a microscopic description of the Alpha decay to the intruder 0^+ states of Pb, Po, Hg and Pt isotopes. Using the snp interaction, they showed that the calculated hindrance factors with respect to Alpha transitions into the ground state were in good agreement with the experimental data [39,40]. Lately, the present author and collaborators used the snp interaction [41,42] to obtain an agreement between theoretical and experimental values of moments of inertia of deformed nuclei and also to determine a range of the strength of this interaction by fitting of the theoretical values to the experimental data. They showed in the framework of the independent quasiparticle model that this interaction affects so that neutron gap renormalized by proton gap and vice versa. They found a good agreement with the experiment, in some given values of snp pairing interaction strength constant [41,42]. Recently, the possibility of obtaining a new 0^+ state using the snp interaction below the gap in even-even deformed nuclei was shown. This important result was reported in [43,44]. Thus it was proved that the snp interaction could be effectual both in the ground and excitation state phenomena in deformed heavy nuclei.

As is mentioned above, experimentally the weak electromagnetic E2 and E0 transitions and the strong two-nucleon transfer strengths [2,19–21] imply the importance of the pairing interaction for the explanation of the structure of the 0^+ states. If the present models including pairing interactions are unsuccessful in order to explain the experimental data, then there may be two probabilities: a) The states may have pairing vibrational character, however, the present models at hand cannot truly describe their nature. The models should be developed for true description. b) There can be some other unknown mechanisms that are effective in the formation of these states. These mechanisms should be searched and taken into account. In this paper, we are interested in the second probability and we suggest an hypothesis based on the np pairing vibrations for the doublets of 0^+ states which lie closely each other in the low-lying spectra of some even-even deformed nuclei. Here, firstly the structure of these special 0^+ states which are observed in a part of the rare earth region nuclei will be discussed and then it will be shown that the source of the doublets of 0^+ states might be the np pairing vibrations. It is beneficial to state that there is not a direct experimental method in nuclear spectroscopy to distinguish the effect of interactions between like-nucleons from the effect of the interactions between unlike-nucleons. Experimental data denotes only these states may have pairing vibrational nature and cannot reveal anything about if their sources are related with like-nucleons pairing or np pairing. Therefore, only nuclear models predict their effects.

According to the best of our knowledge that there is no study which paid attention to the relationship between these close lying 0^+ states and have explained their origin in the same model in the literature. At this point it is important to emphasize that the usual np pairing interaction is not suitable to express the mechanism that we suggest.

2 Model

In the low lying spectra of some even-even nuclei in the rare earth region, for instance in ^{158}Dy, ^{164}Er, ^{166}Er, ^{172}Hf, ^{178}Hf, there are the pairs of 0^+ states which are lying closely each other. We mean the states which have energy gap of approximately 10–50 keV. We consider that the np pairing vibrational mode resulting from the snp interaction might cause these states. Our hypothesis may be explained as follows:

At the beginning, the pairing vibrations which may be viewed as a vibrational mode of a nucleus in number space have been studied by Bes and Broglia using the QRPA method [45]. They considered only pairing correlations between like-nucleons, either protons or neutrons. From the well known solution, up to 2 MeV, the pairing vibrations results in generally two states with excitation energy of approximately 2Δ, where Δ is the gap parameter. In this model, one of the solutions belongs to the pairing vibrations in proton system and other one is related with the pairing vibrations in neutron system. However it should be recognized that this theory could not give theoretical energy values close enough one to another for the pairs of 0^+ states [45].

As is stated above, using the Hamiltonian in [45], one could search on only the pairing vibrational mode between like-nucleons. However we think that one need an effective interaction between neutron and proton pairs and the special type of neutron-proton pairing interaction can produce the pairs of 0^+ states in the above nuclei that we have considered.

The essential part of our hypothesis has been established on the formation of 0^+ states during the two-nucleon transfer reactions. The excited 0^+ states may be populated strongly in two-nucleon transfer reactions. For the transfer from a spin-0 ground state to a spin-0 final state, the transferred angular momentum must be zero. This condition causes to very characteristic diffraction patterns in angular distributions observed in two-nucleon transfer reactions. The transfer of a pair of nucleons coupled to $J = 0$, $L = 0$ includes pairs located close to Fermi surface, where J and L denote the total and the orbital angular momentum quantum numbers, respectively. These pairs can be regarded as nucleons in time-reversed orbits such as Nilsson states. Thus, two-nucleon transfer probes the microscopic pairing components in the total wavefunction of the nucleus. The ground states of well deformed nuclei, which are typically in a superconducting state and form a pairing rotational band are populated strongly in (p,t) and (t,p) reactions. Here, we will consider a two-neutron transfer reaction; a two-neutron stripping reaction. A mechanism during a two-neutron transfer reaction (N→N+2) could be given as follows below:

The bands of excited 0^+ states that are produced by neutron-neutron pairing vibrational mode and by a special neutron-proton pairing vibrational mode in the target nucleus can be seen in Fig. 1. First, we assume that there is no correlation between neutron and proton systems. During a two-neutron stripping reaction, the neutron system is excited by joining of two neutrons to the ground state of even-even nucleus and proton system could not be excited. If we note from the Fig. 1 that after the excitation of a neutron pair from the upper single-particle level in a subshell to a 0^+ state, the second one should be excited from the lower single-particle level in that subshell.

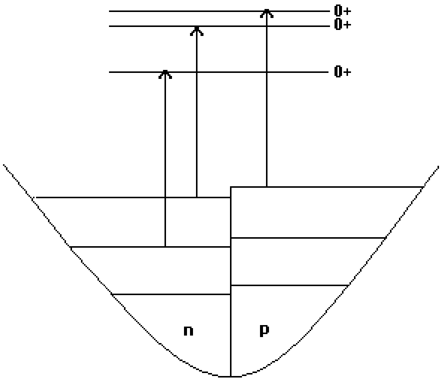

Fig. 1. Pairing vibrational 0^+ excitations in a nucleus

However, in order to excite the second pair to the same 0^+ state just as the first one, more energy is needed. In the end, the second neutron pair could be excited to a lower 0^+ state. Thus, the gap between the states is not so little. This situation can be seen in the left hand side of Fig. 1. Thus, a pairing rotational band involves the first and the second 0^+ states in Fig. 1 occurs. In fact, this situation corresponds to the study that has been made by Bes and Broglia in [45]. Moreover, it should be noted to the fact that the excitation arrows in Fig. 1 have the same length means we assume all pairs are under the effect of the equal amount of external nuclear force.

Second if we assume the existence of a correlation between the neutron and proton systems. In this case, by joining of a pair of neutron to the ground state of the nucleus can excite not only the neutron pairs in the neutron system but also the proton pairs in the proton system. In this case, the Fig. 1 should be taken into account completely, not only its one side. Therefore, the two neutrons that have been taken up by the target nucleus, can excite a neutron pair from a single-particle level of a subshell in the neutron system and at the same time a proton pair from the corresponding equivalent single-particle level of that subshell in the proton system to the 0^+ state. In this process, it is clear that the little energy difference between the equivalent single-particle levels are effective. The reason of the energy difference in question is that the average potential for a proton is less deep than that for neutron due to Coulomb repulsion in medium and heavy nuclei [46]. This situation can be seen in the Fig. 1. Thus, the states that are close lying each other in the excitation energy spectrum, i.e., a pairing rotational band involves the first, the second and the third states in Fig. 1 occur. In conclusion, if we allow an interaction between neutron and proton systems we should obtain a denser band for the 0^+ states. Because, in this case not only neutron-neutron pairing interactions could give rise to 0^+ states but also neutron-proton and proton-proton interactions could give rise also to 0^+ states. For the realisation of these ideas, we propose here a model Hamiltonian of the following form in the second quantization representation:

$$H = \hat{H}_{SP} - G_n \hat{S}_n^+ \hat{S}_n - G_p \hat{S}_p^+ \hat{S}_p - \frac{1}{2} q G \Big\{ \hat{S}_n^+ \hat{S}_p + \hat{S}_p^+ \hat{S}_n \Big\} - \Omega \hat{T}_0 \,, \tag{1}$$

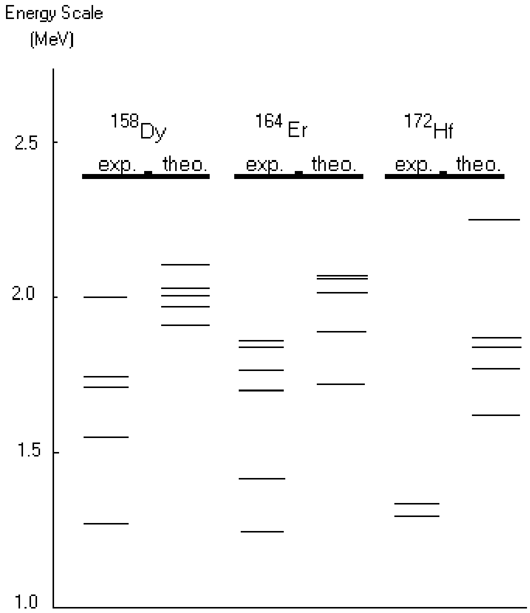

Fig. 2. Experimental and calculated 0^{+} excitations of ^{158}Dy, ^{164}Er and ^{172}Hf isotopes

where the single-particle Hamiltonian is

$$\hat{H}_{SP} = \sum_{\tau} (\varepsilon_{\tau} - \lambda_{\tau}) a_{\tau}^{+} a_{\tau}. \tag{2}$$

In the Hamiltonian (2), the ε is the single-particle energy, the λ is the chemical potential parameter and the $a^{+}(a)$represents the single-particle creation (destruction) operator correspond to the τ-th single-particle state, respectively. The isospin index in the sum is $\tau = n, p$. In the Hamiltonian (1), the snp pairing strength constant was taken as an approximation,

$$G = \frac{G_n + G_p}{2}\,, \tag{3}$$

where $G_n(G_p)$ denotes neutron (proton) pairing strength constant. Besides, in order to change the strength of the neutron-proton interaction we use a free parameter q in the Hamiltonian (1). The pairing operators are

$$\hat{S}_n^{+} = \sum_{s} a_{s+}^{+} a_{s-}^{+} \qquad \hat{S}_p^{+} = \sum_{r} a_{r+}^{+} a_{r-}^{+} \tag{4}$$

$$\hat{S}_n = \sum_{s} a_{s-} a_{s+} \qquad \hat{S}_p = \sum_{r} a_{r-} a_{r+}\ . \tag{5}$$

Also, the cranking term $-\Omega \hat{T}_0$ is introduced into the equation (1) for an approximate projection of proton and neutron numbers Z, N since the snp interaction does not commute with the third component of isospin, $\hat{T}_0$. The Ω which is a Lagrange parameter can be fixed by the constraint

$$\left\langle \hat{T}_0 \right\rangle = \frac{1}{2}(N - Z)\,. \tag{6}$$

To find the energies of the excited 0^+ states, using the well known procedure of the QRPA method one has to solve the following equation of motion

$$\left[\hat{H}, \hat{Q}_n^+\right] = \omega_n \hat{Q}_n^+\,, \tag{7}$$

where $\hat{Q}_n^+$ is the phonon creation operator corresponds to the n-th 0^+ state given by

$$\hat{Q}_n^+ = \sum_s \psi_s^n A_s^+ - \sum_s \varphi_s^n A_s + \sum_r \psi_r^n A_r^+ - \sum_r \varphi_r^n A_r\,, \tag{8}$$

in which the $\hat{A}^+$ and $\hat{A}$ are the boson operators of the quasiboson approximation and ψ and φ are the QRPA amplitudes. Moreover, in equation (7) ω_n is the excitation energy belongs to n-th 0^+ state. The vacuum state is given by

$$\hat{Q}_n|0\rangle = 0\,. \tag{9}$$

Following the QRPA procedure in [47,48], the solution is found as the below secular equation for the excitation energies,

$$\begin{aligned} &abgq^2\omega_n^2(i - j\omega_n^2) - ad\omega_n^4(g^2 - hj) + aeq^2(g^2\omega_n^2 - hi) - b^2fq^2\omega_n^2(iq^2 - j\omega_n^2) \\ &\quad + b^2g^2\omega_n^4(q^2 - 1)^2 + b^2h\omega_n^2(iq^2 - j\omega_n^2) - bgcq^2\omega_n^2(i - j\omega_n^2) \\ &\qquad - bgq^2\omega_n^2(h - f)(d\omega_n^2 - e) - cdq^2\omega_n^4(fj - g^2) + ceq^4(fi - g^2\omega_n^2) = 0\,, \end{aligned} \tag{10}$$

where

$$\begin{aligned} a &= \sum_s \frac{\omega_n^2 - 4\Delta_n^2}{E_s(4E_s^2 - \omega_n^2)} & h &= \sum_r \frac{\omega_n^2 - 4\Delta_p^2}{E_r(4E_r^2 - \omega_n^2)} \\ b &= \sum_s \frac{2(\varepsilon_s - \lambda_n - \Omega)}{E_s(4E_s^2 - \omega_n^2)} & g &= \sum_r \frac{2(\varepsilon_r - \lambda_p + \Omega)}{E_r(4E_r^2 - \omega_n^2)} \\ c &= \sum_s \frac{(\varepsilon_s - \lambda_n - \Omega)^2}{E_s(4E_s^2 - \omega_n^2)} & f &= \sum_r \frac{(\varepsilon_r - \lambda_p + \Omega)^2}{E_r(4E_r^2 - \omega_n^2)} \\ d &= \sum_s \frac{1}{E_s(4E_s^2 - \omega_n^2)} & j &= \sum_r \frac{1}{E_r(4E_r^2 - \omega_n^2)} \\ e &= \sum_s \frac{4E_s}{4E_s^2 - \omega_n^2} & i &= \sum_r \frac{4E_r}{4E_r^2 - \omega_n^2} \end{aligned} \tag{11}$$

in which the E_s is the quasiparticle energy belongs to the s-th single-particle state, $\lambda_n(\lambda_p)$ and $\Delta_n(\Delta_p)$ are the chemical potential and the gap parameters respectively for neutrons (protons).

^{158}Dy	Exp. [keV]	Theo. [keV] $q = 0.3$	Theo. [keV] $q = 0.5$	Theo. [keV] $q = 1.0$
0_1^+	991	1243	1314	1004
0_2^+	1269	1978	1980	1910
0_3^+	1549	2017	2017	1972
0_4^+	1710	2044	2059	2006
0_5^+	1743	2341	2405	2028
0_6^+	2000	2440	2714	2105

Tab. 1. Experimental and theoretical excitation energies of the 0^+ states for ^{158}Dy

^{164}Er	Exp. [keV]	Theo. [keV] $q = 0.3$	Theo. [keV] $q = 0.5$	Theo. [keV] $q = 1.0$
0_1^+	1246	1148	1192	1719
0_2^+	1417	1897	2525	1890
0_3^+	1702	2054	2710	2011
0_4^+	1766	2712	2752	2057
0_5^+	1841	2738	3176	2067
0_6^+	1861	3178	3660	2424

Tab. 2. Experimental and theoretical excitation energies of the 0^+ states for ^{164}Er

Moreover the index $_s$ denotes neutron and the index $_r$ denotes proton single-particle states, respectively. The quasiparticle energies for neutrons and protons are,

$$E_s = \sqrt{(\varepsilon_s - \lambda_n - \Omega)^2 + \Delta_n^2}\,, \quad E_r = \sqrt{(\varepsilon_r - \lambda_r + \Omega)^2 + \Delta_p^2}\,. \tag{12}$$

3 Numerical results and discussion

In order to obtain the numerical results for the ^{158}Dy, ^{164}Er and ^{172}Hf that are well deformed nuclei, the single particle Hamiltonian given by Lamm [49] and also developed by Boisson and Piepenbring [50] has been used in the above solution. To simplify the expression of matrix elements, the asymptotic basis of eigenvector $|NN_z\Lambda\Sigma\rangle$ was preferred, where the quantum numbers N, N_z, Λ and Σ represent the total number of oscillator quanta, the projection of the total number of oscillator quanta on the z-axis, the projection of the orbital angular momentum on the z-axis and the projection of the spin angular momentum on the z-axis, respectively. Meanwhile it is beneficial to mention the equality here $K = \Lambda + \Sigma$. All of the calculation have been performed using the deformation values in [51]. Besides, all states of the $N = 4, 5, 6$ shells for neutrons and protons (64 levels for each) were taken into account. The experimental energies have been taken from [52,53]. The calculated energies of the first six states for the specific values of the parameter q and experimental ones are given in keV in Tables 1,2,3. In Fig. 2, the experimental 0^+ states and calculated 0^+ states located above 1 MeV have been presented for three nuclei.

^{172}Hf	Exp. [keV]	Theo. [keV] $q = 0.3$	Theo. [keV] $q = 0.5$	Theo. [keV] $q = 1.0$
0_1^+	871	1729	1729	947
0_2^+	1296	1769	1769	1619
0_3^+	1336	1826	1841	1770
0_4^+		1842	1869	1841
0_5^+		2314	2112	1869
0_6^+		2375	2247	2249

Tab. 3. Experimental and theoretical excitation energies of the 0^+ states for ^{172}Hf

From the calculated energies in the Tables 1,2,3 and Fig. 2 it can be seen that the snp interaction produces the pairs of 0^+ states, as we expected theoretically. It can also be seen that the location of some levels remains almost the same as the parameter q changes. We think that such levels are created by like-particle pairing correlations, i.e., neutron-neutron and proton-proton pairing correlations and the interactions of the pairs that occupy different shells in neutron and proton systems. Some other states in the Tables 1,2,3 however, depend sensitively on the q-strength parameter. Acording to these results, the number of the 0^+ states with low energy becomes greater as the value of the q-parameter increases when the possibility of interaction of these pairs increase.

Especially for the pairs with low energy as in the case of ^{172}Hf, obtaining a quantitative agreement seems more difficult. To investigate the dependence of these states created by the snp interaction on the neutron and proton numbers the first six energies obtained for $q = 1.0$ and they were listed in Tab. 4 for $^{160-170}$Er isotopes ($Z = 68$) and in Tab. 5 for isotopes from ^{160}Gd to ^{170}W ($N = 96$). In the end, we have observed that the doublets are not produced for all nuclei. For instance, the low-lying 0^+ doublets below 2 MeV exist only for ^{170}Er in Tab. 4 and ^{168}Hf in Tab. 5 and the high-lying ones above 2 MeV exist only for ^{160}Er , ^{164}Er in the Tab. 4 and ^{164}Er, ^{166}Yb, ^{170}W in Tab. 5. Hence we conclude that the 0^+ doublets do not exhibit a systematic behaviour with the neutron or proton number. Theoretic production of the 0^+ states by the snp interaction depend directly on the microscopic structure of the nucleus that is considered .

Finally, it should be mentioned here a special case of ^{166}Er that has a 0^+ doublets at energies of 1934 keV and 1943 keV. The former state has been commented as β-vibrational state and the latter has been commented as γ-vibrational state with two phonon already [4,5].Therefore this doublet cannot be produced by our model.

4 Conclusion

In conclusion our hypothesis on the 0^+ states is corroborated by the use of the snp interaction. Since there is not any alternative theory for the pairs of 0^+ states, which connects the origins of these states each other, this model might be a good candidate to search for their physics. Moreover, the theory in this paper can give a qualitative expression for the origin of the states in question and it is not possible to obtain a doublet below 1.75 MeV using this model. Because the other known residual interactions were not considered in this study to investigate the pure

	^{160}Er [keV]	^{162}Er [keV]	^{164}Er [keV]	^{166}Er [keV]	^{168}Er [keV]	^{170}Er [keV]
0_1^+	1345	1101	1719	667	1718	1749
0_2^+	1742	1745	1890	1739	1876	1856
0_3^+	1866	1955	2011	1958	1950	1896
0_4^+	2059	2064	2057	2040	2220	2260
0_5^+	2109	2140	2066	2144	2285	2530
0_6^+	2181	2406	2424	2274	2343	2900

Tab. 4. Dependence of the energies of the 0^+ states on the neutron number

	^{160}Gd [keV]	^{162}Dy [keV]	^{164}Er [keV]	^{166}Yb [keV]	^{168}Hf [keV]	^{170}W [keV]
0_1^+	1753	1951	1719	1656	866	1925
0_2^+	1898	2050	1890	1749	1642	1959
0_3^+	2234	2196	2011	1880	1750	1983
0_4^+	2435	2373	2057	1977	1760	2004
0_5^+	2593	2591	2066	2023	1859	2028
0_6^+	2651	2771	2424	2066	1913	2120

Tab. 5. Dependence of the energies of the 0^+ states on the proton number

effect of the snp interaction on this states and to verify our proposal. Therefore it should not be expected an exact agreement between the theoretical and experimental energy values. Important result is theoretical production of these states by the snp interaction in the atomic nuclei.

The 0^+ doublets are not produced for all nuclei by the snp. Interestingly they are produced only in nuclei which have the experimental 0^+ doublets, as in the case ^{162}Dy, ^{164}Er and ^{172}Hf. Moreover, from the Tabs. 4 and 5, it is observed that the production of 0^+ doublets does not depend on the neutron or proton number, i.e., it does not exhibit a general behaviour. Instead, the production of the 0^+ doublets is related with the microscopic structure of the nucleus that is considered.

In summary this study showed the importance of np interactions for the physics of the heavy nuclei in which the np interactions has been neglected so far because of the neutron excess.

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