

**COHERENT CONTROL OF QUANTUM JUMPS IN AN OPTICAL LATTICE  
BY A WEAK AXIAL MAGNETIC FIELD**

A. Bhattachjee<sup>1</sup>

*Department of Physics, Atma Ram Sanatan Dharma College, University of Delhi*

*(South Campus), Daula Kuan, New Delhi-110 021, India*

*and*

*The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy*

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As an extension to our earlier work we study how a weak axial magnetic field applied along the axis of an optical lattice can coherently control the photon statistics of a trapped cold metastable Helium atoms.

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## 1 Introduction

Recent spectacular advances in laser cooling and trapping provides an opportunity to explore the fluorescent light emitted by a single confined atomic ion. This light is expected to contain information about the quantum nature of the single-atom source. When the source is a three-level atom, with the fluorescence originating from two separate transitions, a quantum jump between two levels can have an immediate detectable effect on the probability of photon emission from the other transition [1-6]. Theoretical analysis have demonstrated that the resonance fluorescence of three level system in the presence of two laser fields under certain conditions, may exhibit light and dark periods [7-11]. During the dark period the atom is predominantly in the metastable state (electron shelving). A primary condition for such a situation to occur is that one of the lasers should have sufficiently small Rabi frequency as compared to the other. The switching off and on of the fluorescence would be a direct manifestation of the instantaneous quantum jump of the atom between the weakly coupled state and the driven state. Hegerfeldt and Plenio [12] showed the presence of a narrow peak in the spectrum of resonance fluorescence, the origin of which could be traced back to electron shelving. Recently, we investigated the effect of atomic motion and an axial magnetic field on the spectrum of resonance fluorescence of a  ${}^4\text{He}$  atom in the  $\Lambda$  configuration in an optical lattice [13]. We found that polarization gradient induces electron shelving at selected points on the optical lattice. In particular localization of the atom inhibits electron shelving but can be recovered in the presence of an axial magnetic field. We also found

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<sup>1</sup>E-mail address: aranyab@bol.net.in

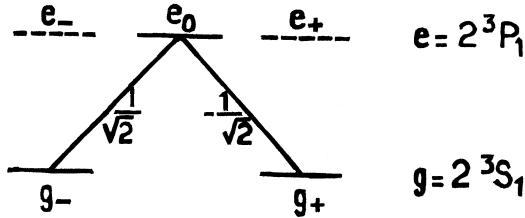


Fig. 1. The Zeeman sublevels of  ${}^4\text{He}$  and the relevant Clebsch-Gordan coefficients. Since the  $e_o \leftrightarrow g_o$  transition is forbidden, all atoms are pumped into  $g_+$  and  $g_-$  after few fluorescence cycles. These two levels are coupled only to  $e_o$ , and a closed three-level  $\Lambda$  configuration is realized.

that the magnetic field significantly broadens the narrow peak spectrum. A similar broadening effect in resonance fluorescence spectrum due to magnetic field was observed experimentally in Rubidium atoms in an optical Molasses [14]. In this paper as an extension to our earlier work, we study the intensity correlation function ( $f(t)$ ), waiting time distribution ( $w(t)$ ) and the rate constants which determine the statistical distribution for the dark and bright periods in the fluorescent emission as a function of atomic motion and the strength of the applied axial magnetic field. We apply a general formalism developed by Nienhuis [10] for the separation of rapid and slow time scales in linear evolution equations to three state atoms with a strong and a weak radiative transition. The starting point for the above study would be the optical Bloch equations(OBE).

## 2 The optical Bloch equations

Fig. 1 shows a  $\Lambda$  atomic system driven by two counterpropagating orthogonal plane-polarized laser fields with the same frequency  $\omega_L$ . For the experiments with  ${}^4\text{He}$ ,  $g_{\pm}$  are the two Zeeman sublevels  $m = \pm 1$  of the  $2^3\text{S}_1$  state and  $e_o$  is the  $m = 0$  Zeeman sublevel of  $2^3\text{P}_1$ . The light shifts of the Zeeman sublevels are position dependent, since the couplings (C.G. couplings) vary with polarization. The superposition of the two counter propagating orthogonally-polarized plane waves gives rise to a standing wave for which the polarization is space dependent. The polarization is alternatively circular  $\sigma^-$  in  $Z = 0$ , linear in  $Z = \lambda/8$ , circular  $\sigma^+$  in  $Z = \lambda/4$  etc., with a spatial periodicity  $\lambda/2$ . This polarization gradient leads to a periodic spatial modulation of the degenerate ground state Zeeman sublevels, which acts as a potential for the motion of the atomic centre of mass. An axial magnetic field provides the necessary coupling between the two degenerate ground states even for atoms with a nonzero momentum. The Sisyphus cooling mechanism of Fig. 2 traps the atom in the laser induced optical potential well only for blue detuning. For red detuning the atom gets heated and is expelled out of the optical potential. The time evolution of the atomic density matrix  $\rho$ , in the Schrodinger picture, obeys

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\hat{H}, \rho] + (\frac{d\rho}{dt})_{sp}. \quad (1)$$

Here the total Hamiltonian  $H$  consists of an atomic Hamiltonian  $H_A$ , a laser – atom interaction Hamiltonian  $H_{LA} = -\vec{D} \cdot \vec{E}$ , where  $\vec{D}$  is the electric dipole moment operator, and the positive

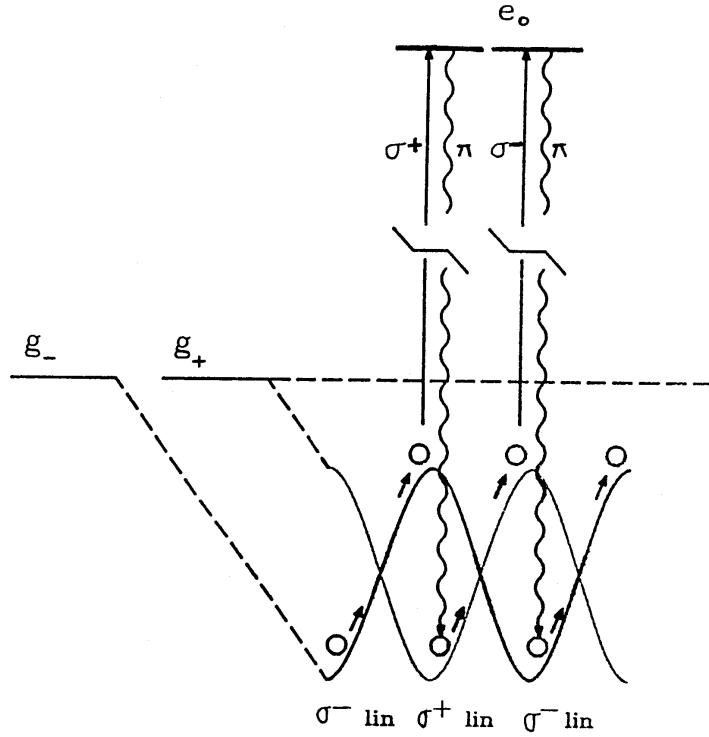


Fig. 2. One-dimensional optical potential induced by interference between oppositely travelling blue detuned orthogonally polarized plane waves.

frequency part of the electric field  $\vec{E}$  propagating along the z-axis is

$$\vec{E}^+(z, t) = \frac{E}{\sqrt{2}} [\vec{\varepsilon}_+ \cos(kz) + \vec{\varepsilon}_- \sin(kz)] \quad (2)$$

with  $\vec{\varepsilon}_\mp = \mp(\vec{e}_x \pm i\vec{e}_y)/\sqrt{2}$ , where  $\vec{\varepsilon}_+$  and  $\vec{\varepsilon}_-$  are the polarization unit vectors, a term  $H_B$  corresponding to the shift in the energy due to an applied axial magnetic field  $B$ , and  $(d\rho/dt)_{sp}$  is the damping term by spontaneous emission processes.

$$H_B = g\mu_B B(|g_+\rangle\langle g_+| - |g_-\rangle\langle g_-|), \quad (3)$$

$g$  is the gyromagnetic ratio for the  $2^3S_1$  state,  $\mu_B$  is the Bohr magneton =  $-9.2741 \times 10^{-24}$  J/T.  $(d\rho/dt)_{sp}$  is the damping due to spontaneous emission. The time evolution of the atomic density matrix  $\rho_{ij}(t) = \langle \sigma_{ij}(t) \rangle$  (where  $\sigma_{ij}(0) = |i\rangle\langle j|$ ) is governed by the quantum Bloch equations arising from Eq. (1). The fast oscillations with the optical frequency  $\omega_L$  can be eliminated by taking

$$\begin{aligned} \tilde{\rho}_{e_o e_o}(t) &= \rho_{e_o e_o}(t), \quad \tilde{\rho}_{g_\pm g_\pm}(t) = \rho_{g_\pm g_\pm}(t), \quad \tilde{\rho}_{g_+ g_-}(t) = \rho_{g_+ g_-}(t), \\ \tilde{\rho}_{e_o g_\pm}(t) &= \rho_{e_o g_\pm}(t) \exp(-i\omega_L t), \end{aligned}$$

and

$$\tilde{\rho}_{g-g-} + \tilde{\rho}_{g+g+} + \tilde{\rho}_{e_o e_o} = 1.$$

The optical Bloch equations are:

$$\begin{aligned}\dot{\tilde{\rho}}_{e_o e_o}(t) &= i\frac{\Omega}{2}[\cos(kz)(\tilde{\rho}_{e_o g_+} - \tilde{\rho}_{g+e_o}) - \sin(kz)(\tilde{\rho}_{g-e_o} - \tilde{\rho}_{e_o g_-})] \\ &\quad - 2(\Gamma_{g-g-} + \Gamma_{g+g+})\tilde{\rho}_{e_o e_o}, \\ \dot{\tilde{\rho}}_{g+g+}(t) &= i\frac{\Omega}{2}\cos(kz)(\tilde{\rho}_{g+e_o} - \tilde{\rho}_{e_o g_+}) + 2\Gamma_{g+g+}\tilde{\rho}_{e_o e_o}, \\ \dot{\tilde{\rho}}_{g-g-}(t) &= i\frac{\Omega}{2}\sin(kz)(\tilde{\rho}_{g-e_o} - \tilde{\rho}_{e_o g_-}) + 2\Gamma_{g-g-}\tilde{\rho}_{e_o e_o}, \\ \dot{\tilde{\rho}}_{g+g-}(t) &= i\frac{\Omega}{2}[\sin(kz)\tilde{\rho}_{g+e_o} - \cos(kz)\tilde{\rho}_{e_o g_-}] - 2i\beta\tilde{\rho}_{g+g-}, \\ \dot{\tilde{\rho}}_{e_o g+}(t) &= i(\delta + \omega_R + \beta)\tilde{\rho}_{e_o g+} + i\frac{\Omega}{2}[\cos(kz)(\tilde{\rho}_{e_o e_o} - \tilde{\rho}_{g+g+}) - \sin(kz)\tilde{\rho}_{g-g+}] \\ &\quad - \Gamma_{g+g+}\tilde{\rho}_{e_o g+}, \\ \dot{\tilde{\rho}}_{g-e_o}(t) &= -i(\delta + \omega_R - \beta)\tilde{\rho}_{g-e_o} - i\frac{\Omega}{2}[\sin(kz)(\tilde{\rho}_{e_o e_o} - \tilde{\rho}_{g-g-}) - \cos(kz)\tilde{\rho}_{g-g+}] \\ &\quad - \Gamma_{g-g-}\tilde{\rho}_{g-e_o},\end{aligned}\tag{4}$$

where  $\Omega$  is the Rabi frequency,  $\delta$  is the detuning,  $\omega_R = \hbar k^2/2m$  is the recoil frequency shift and  $\beta = (kp/m + gB\mu_B/\hbar)$  is the Doppler shift associated with the velocity  $p/m$  in absence of the axial magnetic field. When  $\beta \neq 0$  the energies of  $|g-\rangle$  and  $|g+\rangle$  differ by  $2\beta$ .

### 3 Separation of time scales and photon statistics

The set of Bloch equations may be viewed as the single evolution equation  $\frac{d\rho}{dt} = L\rho(t)$  with an evolution operatore  $L$  that is defined by the right-hand side of equations (4). This operatore  $L$  is time independent. We need to find an approximate evolution equation by using the fact that the transition  $|e_o\rangle \leftrightarrow |g_+\rangle$  is very weak, which means that  $\cos^2(kz) \ll \sin^2(kz)$  and  $\Gamma_{g+g+} \ll \Gamma_{g-g-}$ . This condition makes the  $\sigma_-$  polarized field, which drives the  $|e_o\rangle \leftrightarrow |g_+\rangle$  transition weak as compared to the  $\sigma_+$  polarized field, which drives the  $|e_o\rangle \leftrightarrow |g-\rangle$  transition. Hence  $|e_o\rangle \leftrightarrow |g_+\rangle$  becomes the metastable transition and during this time the atom is predominantly in the metastable state(electron shelving). Dark periods (absence of resonance fluorescence) is a signature of electron shelving. This occurs at points near  $z = n\lambda/4$ , i.e., where the polarization is either  $\sigma^+$  or  $\sigma^-$  since here only one of the transition is more favoured. From experimental view  $z = n\lambda/4$  are the points corresponding to bottom of the potential well where the atoms tend to accumulate once their kinetic energy is sufficiently small. Hence it is natural to treat  $\sin^2(kz)$  as being linear in some small expansion parameter, and to assume  $\Gamma_{g+g+}$  is of second order in this parameter  $L$ . Therefore, we formally write

$$L = L_0 + L_1 + L_2,\tag{5}$$

where  $L_2$  is defined by the terms on the right-hand side of Eqs. (4) with  $\Gamma_{g+g+}$ , whereas  $L_1$  is defined by the terms with  $\cos^2(kz)$ . The remaining terms ,i.e.,  $\Gamma_{g-g-}$ ,  $\sin^2(kz)$ ,  $(\delta' \pm \beta)$  define

the zeroth order operator  $L_0$ . In using this separation of the evolution operator in rapid and slow parts to derive simplified expressions for ‘ $f(t)$ ’ and ‘ $w(t)$ ’, we follow Nienhuis [10]. The initial state ( $t = 0$ ) of the atom is assumed to be  $|g_-\rangle$ . Since the time scales defined by  $L_0$  are very short compared with the time scales of  $L_1$  and  $L_2$ , the rapid evolution defined by  $L_0$  drives the atom to a steady state w.r.t  $L_0$ , before  $L_1$  and  $L_2$  have a chance to contribute appreciably to the evolution. Consequently, the steady state density matrix  $\bar{\rho}$  for  $t \leq \Gamma_{g-g-}^{-1}$  is simply a matrix within the two-dimension subspace spanned by  $|g_-\rangle$  and  $|e_o\rangle$ . This means that the excited state population is the steady state density matrix of a driven two-level atom, with matrix element determined by.

$$\bar{\rho}_{e_o e_o}^{(2)} = \frac{s}{(1 + 2s)}, \quad (6)$$

with

$$s = \frac{\Omega^2 \sin^2(kz)}{4[(\frac{\Gamma_{g-g-}}{2})^2 + (\delta' - \beta)^2]}.$$

On a time scale long compared to  $\Gamma_{g-g-}^{-1}$ , the evolution operators  $L_1$ ,  $L_2$  and the parameter  $\beta$  start to contribute to the evolution of the atom by causing a slow transfer of probability from brightness to darkness and vice versa. This can be represented by the following two coupled linear evolution equation for the parameters  $n_b(t)$  and  $n_d(t)$

$$\frac{dn_b(t)}{dt} = R_- n_d(t) - R_+ n_b(t) = -\frac{dn_d(t)}{dt}, \quad (7)$$

where  $n_b$  is the probability for the atom to be in the bright period, and  $n_d$  to be in the dark period then  $n_b + n_d = 1$ , where  $n_b = \rho_{e_o e_o} + \rho_{g-g-}$  and  $n_d = \rho_{g+g+}$ . With the initial conditions  $n_b(0) = 1$  and  $n_d(0) = 0$ , the two parameters  $n_b(t)$  and  $n_d(t)$  determine the density matrix for  $t > \Gamma_{g-g-}^{-1}$ . For a finite  $\beta$  and for  $t \geq \Gamma_{g-g-}^{-1}$ , the elements of the density matrix are the populations appropriate to a three level  $\Lambda$  system with the conditions  $\cos^2(kz) \ll \sin^2(kz)$  and  $\Gamma_{g+g+} \ll \Gamma_{g-g-}$ . The expression for the steady excited state of the three level atom is:

$$\bar{\rho}_{e_o e_o}^{(3)} = \frac{16\beta^2\Omega^2 \sin^2(kz)}{\Omega^4 - 16\beta(\delta' - \beta)\Omega^2 \cos^2(kz) + 32\beta^2(\Omega^2 \sin^2(kz) + 2\Gamma_{g-g-}^2 + 2(\delta' - \beta)^2)}. \quad (8)$$

The above rate equation is a two valued Markov process, or the random telegraph signal. The coefficient  $R_-$  is the switching rate from a state of the atom with no fluorescence, to the state with the fluorescence switched on, and  $R_+$  the switching rate for the reverse process. The quantum jump approach allows one to calculate the mean length of the dark and bright period. These two quantities completely determine the statistical properties of the light emitted by a single atom. The average duration of the dark period is  $T_d = 1/R_-$  and according to the rate Eq. (7), the average duration of the bright period is  $T_b = 1/R_+$ . We find for a cold  ${}^4\text{He}$  atom in an optical lattice the expressions for  $R_+$  and  $R_-$  as

$$R_- = \frac{32\beta^2\Gamma_{g-g-}\Omega^2 \cos^2(kz)}{64\beta^2\Gamma_{g-g-}^2 + (\Omega^2 \sin^2(kz) + 8\beta(\delta' + \beta))^2}, \quad (9)$$

$$R_+ = \frac{\Omega^4 \sin^4(kz)}{32\beta^2(\Omega^2 \sin^2(kz) + 2(\delta' - \beta)^2 + 2\Gamma_{g-g-}^2)} R_- . \quad (10)$$

The basic quantity which monitors the correlation between two photons emitted by an atom with a time separation “ $t$ ” is the intensity correlation function  $f(t)$ . From the behavior of  $f(t)$  both for short and long time values one can conclude the existence of quantum jumps of the atoms, which are observed as intermittent dark and bright periods of fluorescence. An expression for the intensity correlation function  $f(t)$  easily follows from standard consideration from the theory of photon correlations in fluorescence. The condition of an earlier emission at time zero ensures that the atom was in state  $|g-\rangle$  this moment, so that the initial density matrix  $\rho(0)$  was the projector  $P_0 = \rho_0 = |g-\rangle\langle g-|$  on this state. The probability density for another photon emission at time  $t$  is then equal to  $\Gamma_{g-g-}\rho_{e_0 e_0}(t)$ , which is the rate of spontaneous decay multiplied by the probability that the atom is in the upper state  $|e_0\rangle$ .

$$f(t) = \Gamma_{g-g-} \text{Tr} P_1 \exp(Lt) P_0 , \quad (11)$$

with  $P_1$  the projector on state  $|e_0\rangle$ . Eq. (11) illustrates that  $f(t)$  monitors the probability that the atom is in the state  $|e_0\rangle$  at time  $t$ , when it was known to be in state  $|g-\rangle$  at time  $t = 0$ . For times  $t \leq \Gamma_{g-g-}^{-1}$ ,  $L_1$  and  $L_2$  can be ignored in the evolution of  $\rho$  and the density matrix is just the two state density matrix that approaches the steady state eqn.(6) after a few lifetimes  $\Gamma_{g-g-}^{-1}$ . For short times, the coupling to the state  $|g_+\rangle$  has not yet been effective. Consequently for this range of small time scales the intensity correlation is simply that for two state atom. One can show that the explicit expression for the intensity correlation function valid for arbitrary values of the time  $t$  and for  $\beta \neq 0$  is given as:

$$f(t) = f_0(t) - \frac{\Gamma_{g-g-}\bar{\rho}_{e_0 e_0}^{(3)} R_+ \{1 - \exp[-(R_+ + R_-)t]\}}{(R_+ + R_-)} , \quad (12)$$

where  $f_0(t) = \Gamma_{g-g-}\bar{\rho}_{e_0 e_0}^{(2)}(1 - \exp(-\Gamma_{g-g-}t))$  is the intensity correlation function for a two state atom.  $f_0(t) = 0$  for  $t = 0$ , reflects the antibunching property and that it approaches the steady state value  $\Gamma_{g-g-}\bar{\rho}_{e_0 e_0}^{(2)}$  in a few life times  $\Gamma_{g-g-}^{-1}$ . Here  $(R_+ + R_-)^{-1}$  is the correlation time for the random telegraph process  $f(t)$  which stimulates the light and dark periods. It is this structure that is observed in the spectrum of resonance fluorescence as an additional narrow peak [12]. In the absence of the ground state coherence ( $\beta$ ),  $\bar{\rho}_{e_0 e_0}^{(3)}$  in the second term of Eq. (12) is to be replaced by  $\bar{\rho}_{e_0 e_0}^{(2)}$ . This is because in the absence of ground state coupling the population of state  $|g_+\rangle$  is not effected by  $L_o$  and hence the population of the state  $|e_0\rangle$  for  $t > \Gamma_{g-g-}^{-1}$  is thus equal to  $\bar{\rho}_{e_0 e_0}^{(2)} n_b(t)$ .

#### 4 Results and discussion

The behaviour of  $R_-$  and  $R_+$  as a function of  $\beta$  is shown in Fig. 3a. We find an initial steady increase in  $R_-$  and then it approaches zero slowly for high values of the coupling parameter.  $R_+$  on the other hand decays rapidly to zero for large  $\beta$ . The average rate  $1/(T_D + T_L)$  at which we observe quantum jumps (i.e., the onset of dark periods) is also shown in Fig. 3a as a function of  $\beta$ . We observe a initial increase in the quantum jump rate (QJR) and it reaches a maxima

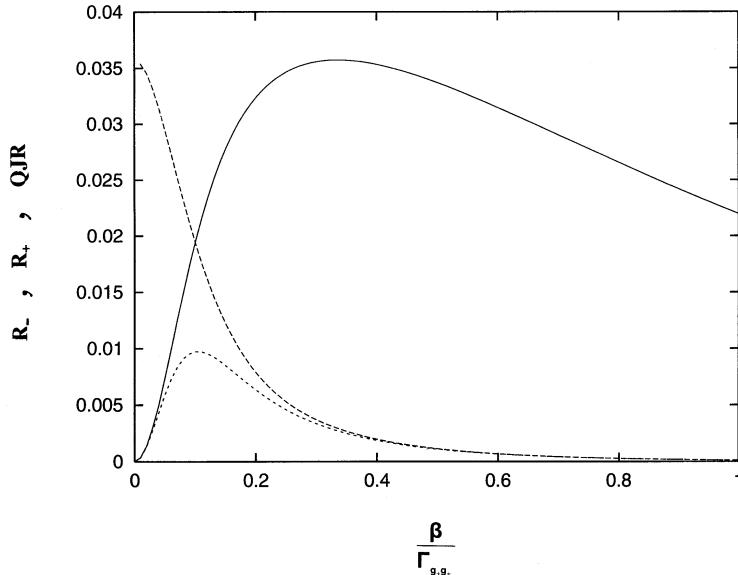


Fig. 3a. The switching rate  $R_-$  (solid line),  $R_+$  (long dashed line) and Quantum jump rate (short dashed line) as a function of ground state coherence parameter  $(\beta/\Gamma_{g+g_+})$  for  $\delta'/\Gamma_{g+g_+} = 0$ ,  $\Omega/\Gamma_{g+g_+} = 10$ ,  $\Gamma_{g-g_-}/\Gamma_{g+g_+} = 9$ ,  $\cos^2(kz) = 0.1$ , and  $\sin^2(kz) = 0.9$ .

for that value of  $\beta$  for which the two rates  $R_-$  and  $R_+$  are equal. For higher values of  $\beta$  QJR steadily decreases. Fig. 3b shows asymmetric behaviour of  $R_+$ ,  $R_-$  and QJR with respect to the detuning. A very high value of these parameters is noticed for red detuned laser as compared to that for blue detuning. A transparent description of this problem can be reached in the coupled  $|\Psi_c\rangle$  and noncoupled  $|\Psi_{NC}\rangle$  state basis. This basis is composed of the following two orthogonal linear combinations of  $|g_-\rangle$  and  $|g_+\rangle$

$$|\Psi_C\rangle = \frac{1}{\sqrt{2}}\{\cos(kz)|g_+\rangle + \sin(kz)|g_-\rangle\}, \quad (13)$$

$$|\Psi_{NC}\rangle = \frac{1}{\sqrt{2}}\{\sin(kz)|g_+\rangle - \cos(kz)|g_-\rangle\}. \quad (14)$$

The state  $|\Psi_{NC}\rangle$  is always decoupled from the laser field for any value of  $\beta$ :

$$\langle e_o | V | \Psi_{NC} \rangle = 0. \quad (15)$$

On the other hand, the atomic motion and/or the applied weak axial magnetic field induces a coupling between  $|\Psi_c\rangle$  and  $|\Psi_{NC}\rangle$ ,

$$\langle \Psi_C | H_B | \Psi_{NC} \rangle = -g\mu_B B. \quad (16)$$

Finally, the laser field couples  $|\Psi_c\rangle$  to the excited state,

$$\langle e_o | V | \Psi_C \rangle = \frac{\hbar\Omega}{\sqrt{2}} \exp(-i\omega_L t). \quad (17)$$

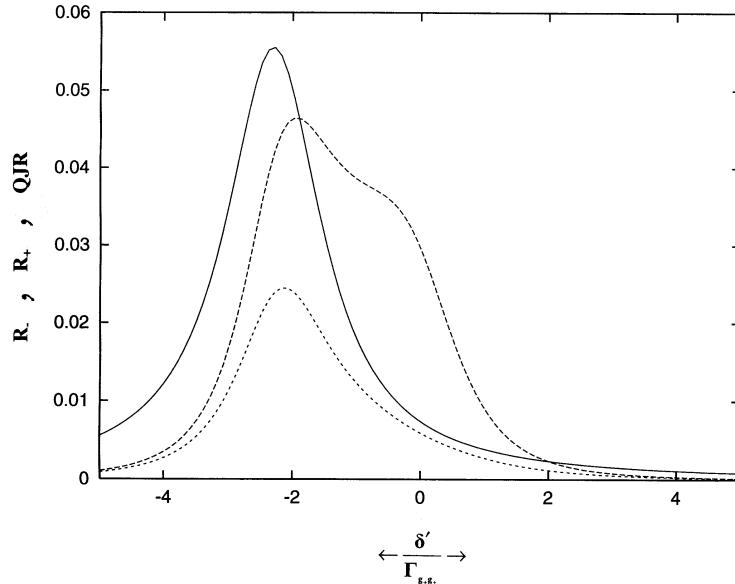


Fig. 3b. The switching rate  $R_-$  (solid line),  $R_+$  (long dashed line) and Quantum jump rate (short dashed line) as a function of  $\delta'/\Gamma_{g+g+}$  for  $\beta/\Gamma_{g+g+} = 0.05$ ,  $\Omega/\Gamma_{g+g+} = 10$ ,  $\Gamma_{g-g-}/\Gamma_{g+g+} = 9$ ,  $\cos^2(kz) = 0.1$ , and  $\sin^2(kz) = 0.9$ .

So in the absence of coupling (i.e.  $\beta = 0$ ) between the  $|\Psi_c\rangle$  and the  $|\Psi_{NC}\rangle$  states,  $R_-$  is zero (i.e. dark periods of infinite duration) and the probability of the atom to switch from the on state to the off state is high. If  $\beta$  approaches zero, the upper level occupation probability tends to zero so that the intensity of the resonance fluorescence vanishes in this limit. This is the well known Velocity selective coherent population trapping(VSCPT) [ 16 ] and finds its explanation in the fact that the uncoupled state is stable against laser excitation (once the atom is trapped in the uncoupled state, it is unavailable for excitation to the excited state) but can only be populated via spontaneous decay from the upper level. For quantum jumps to occur (i.e for the atom to switch between on and off states), the atom must make a transition to the excited state. This can happen only for a finite value of the coupling parameter. Once the atom is in the excited state, it has a finite rate to jump either to the weak ground state  $|g_+\rangle$  or the strong state  $|g_-\rangle$ . For larger values of  $\beta$  larger than the Rabi frequencies and the spontaneous emission rate of the excited state), the atom switching between the lower levels take place at a very small but finite occupation probability of the upper level. This explains the practically zero quantum jump rate for high values of  $\beta$ .

The time development of intensity correlation function  $f(t)$  for different three different values of  $\beta$  is shown in Fig. 4a. For  $\beta = 0$ , the time development of  $f(t)$  is that for a two level atom decreasing to zero for large times. The decrease in  $f(t)$  for long time values is an indication of the switching on of the dark period. However for a finite  $\beta$  the atom is transferred from the  $|\Psi_{NC}\rangle$  to the  $|\Psi_c\rangle$  state and then to the excited state by the coupling of the laser field with the coupled state. This induces the correlation function to settle down to a finite non zero value

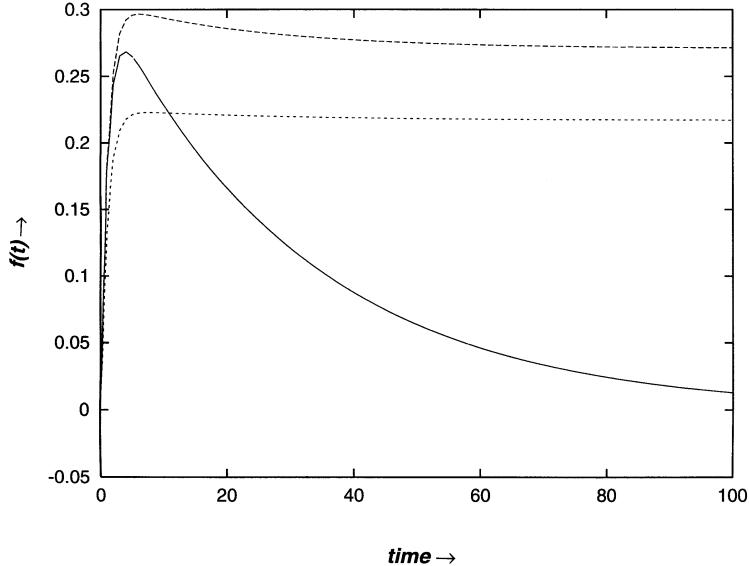


Fig. 4a. Time development of the correlation function for three different values of  $\beta/\Gamma_{g_+g_+} = [0$  (solid line), 0.1 (long dashed line), and 0.5 (short dashed line)]. Other parameters are same as that for Fig. 3a.

for large times. With an applied magnetic field greater than a certain value (for which QJR is maximum), the incoherent Raman contribution to the scattering process increases and causes the correlations to decrease with increasing  $\beta$ . This is illustrated in Fig. 4b where we have plotted the steady state value of the intensity correlation function as a function of  $\beta$ . This decrease in the degree of second order coherence with increasing magnetic field has been observed by Bali et al. [14]. In order to decide on the existence of dark periods another relevant quantity is waiting time distribution. The distribution of the time lapses one has to wait for until the emission of the first next photon after an earlier emission is called the waiting time distribution function  $w(t)$ . It obeys the normalization condition  $\int_0^\infty w(t)dt = 1$  and  $w(t)$  approaches zero for  $t \rightarrow \infty$ .  $W(t)$  is a more appropriate quantity than  $f(t)$  for discussing quantum jumps in three-level systems [17, 18]. We now write down the expression for  $w(t)$  in our model, without going into the detailed derivation:

$$w(t) = w_0(t) + \frac{R_+ R_- \exp(-R_- t)}{\Gamma_{g_-g_-} \bar{\rho}_{e_0 e_0}}, \quad (18)$$

where  $w_0(t)$  is the waiting time distribution for a two state atom. For large time ( $t > \Gamma_{g_-g_-}^{-1}$ )  $w(t)$  has a weak but finite slowly decaying tail. This means that after each photon emission on the strong transition, there is a small probability that the next photon takes a long time of the order  $R_-^{-1}$  to arrive. This corresponds to a dark period in the fluorescence. Naturally as  $\beta$  increases  $R_-$  increases, therefore the time period one has to wait to observe the next photon decreases. The above analysis indicates that a coherent control of the QJR is possible with a weak axial magnetic field.

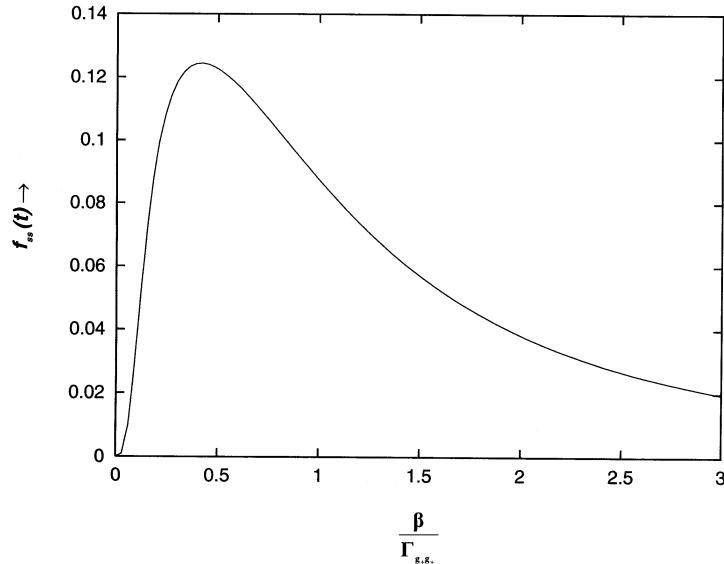


Fig. 4b. Steady state value of the correlation function as a function of  $\beta/\Gamma_{g+g_+}$  for  $\delta'/\Gamma_{g+g_+} = 0$ ,  $\Omega/\Gamma_{g+g_+} = 10$ ,  $\Gamma_{g-g_-}/\Gamma_{g+g_+} = 9$ ,  $\cos^2(kz) = 0.1$ , and  $\sin^2(kz) = 0.9$ .

## 5 Conclusions

In conclusion we have shown that certain points on the optical lattice induces quantum jumps in metastable Helium atom. At these special points if the atom is somehow localized, the statistics of the photons scattered by the atom can be controlled coherently by a weak magnetic field applied along the laser axis. We find that the evolution of the atom on a long time scale is governed by the weak magnetic field (which also controls the ground state coherence) by causing a slow transfer of probability from brightness to darkness and vice versa.

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