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THE PION-NUCLEON Σ-TERM IN A CHIRAL QUARK MODEL

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The pion-nucleon Σ-term is calculated in a linear σ-model based on the $U(3) \times U(3)$ quark effective Lagrangian. The importance of the pole diagram with the scalar meson $f_0(400 - 1200)$ is demonstrated. For the mass of this meson the value 400 MeV was chosen, which corresponds to the theoretical predictions taking into account singlet-octet mixing of scalar isoscalar mesons and glueball on the one hand and to recent experimental data on the other. The resulting value $\sigma = 75$ MeV is in agreement with the latest analysis of experimental data on the π-N scattering. It is shown that the hypothesis of the content of strange quarks in the valence structure of a nucleon is not necessary to reach agreement with experimental data.

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1 Introduction

In last ten years the hypothesis of the content of strange quarks in the valence structure of a nucleon [1, 2] has been many times discussed on the basis of the analysis of the pion-nucleon Σ-term. This was caused by the fact that the theoretical estimates of the Σ-term obtained in the cited papers without using this hypothesis were substantially smaller than experimental data. Note that so far there have been no reliable experimental data for this quantity. In [1, 2] small values of the Σ-term were obtained without using the above mentioned hypothesis. Even considering the content of strange quarks in a nucleon, they obtained the estimates $\sigma = 43$ MeV. Other authors give considerably larger values:

$$\Sigma_{\rm exp} = 64 \pm 8 \ {\rm MeV} \ \ [3], \tag{1}$$

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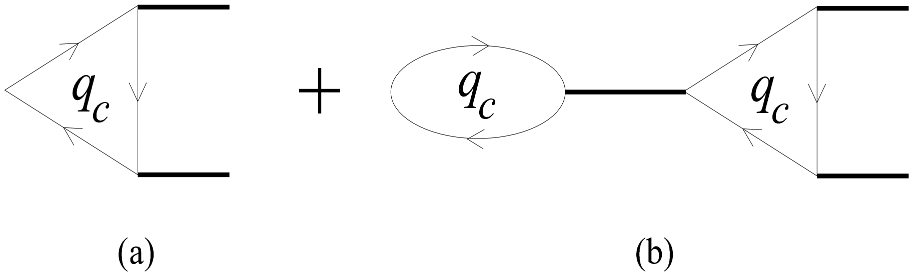

Fig. 1. The quark diagrams describing the matrix element $\langle \pi^+(p_1)|\bar{u}u + \bar{d}d|\pi^+(p_2)\rangle$.

$$\Sigma_{\rm exp} = 92 \pm 6 \text{ MeV} \;\; [4]. \tag{2}$$

The latter value has been obtained quite recently within a new method of analyzing experimental data.

In [5] we showed that in the framework of the linear σ model one can quite satisfactorily explain results [1, 2] without using strange quarks, considering both the diagram describing the scalar form factor of the pion and the diagram with the intermediate σ-meson (see Fig. 1).

Indeed, it was shown that the latter diagram (Fig. 1b) basically determines the value of the Σ-term, and completely cancels the contribution from the scalar form factor of the pion described by the quark triangle diagram (Fig. 1a). The remaining part allows us to describe, in a quite satisfactory way, the experimental data without using the hypothesis of the content of strange quarks in a nucleon.

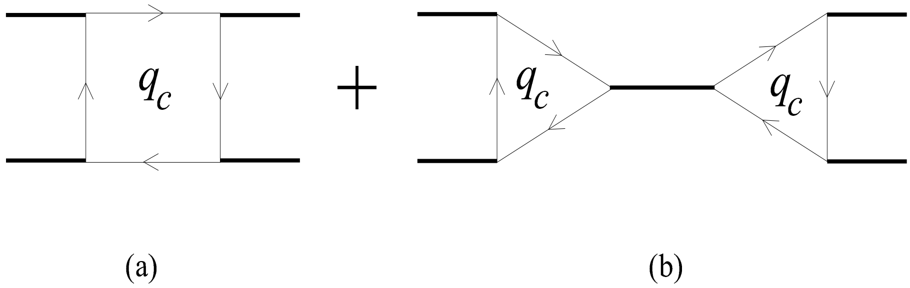

Fig. 2. The quark diagrams describing $\pi\pi$-scattering.

The similar situation takes place in the description of the $\pi\pi$ scattering in the linear σ model where the contribution of the contact π^4 term is completely cancelled by the diagram with the intermediate σ meson (see Fig. 2), and the remaining part of the pole diagram gives the Weinberg formula for the amplitude of the $\pi\pi$ scattering

$$A_{\pi\pi} \simeq \frac{s}{F_\pi^2}, \tag{3}$$

where $s = (p_1 + p_2)^2$, p_1 and p_2 are the momenta of the incoming pions and $F_\pi = 93 MeV$ is the pion decay constant [6].

To evaluate the Σ-term, in [5] the scalar form factor of the pion was calculated in the chiral symmetric limit when $M_\sigma = 2m$ and $p_1^2 = p_2^2 = 0$, where $m = 280$ MeV is the mass of the constituent u-quark, p_1 and p_2 are the pion momenta (Fig. 1), and we obtained for the Σ-term:

$$\sigma = 50\,\mathrm{MeV}. \tag{4}$$

This value is close to the result given in [1,2], where the hypothesis of strange quarks content in a nucleon was used[4].

In the present paper a more realistic value for the σ-meson mass than in [5] is used, namely,

$$M_\sigma = 400\,\mathrm{MeV}. \tag{5}$$

It was obtained in our recent works [8, 9]. In our case the σ-meson is identical to the experimental scalar state $f_0(400 - 1200)$. The experimental measurements of the mass of this meson and its width are very discrepant [7], but from the theoretical point of view, considering the singlet-octet mixing of scalar and isoscalar mesons with one other and with the glueball, a lower value is preferable. The singlet-octet mixing is defined by the 't Hooft interaction which appears in the $U(3) \times U(3)$ quark model because of the interaction of instantons [9–12]. Value (5) is in agreement with some of the experimental data (for example [13–15]).

Here we also take into account dependence of the scalar pion form factor (Fig. 1) on external pion momenta. This dependence arises from two sources. One is, taking account of the $\pi - a_1$ transitions at external pion legs, other is dependence of the quark triangle diagram (Fig. 1a) on external pion momenta. The dependence of the first kind was considered in [5] and the dependence of the second kind is considered here for the first time.

Let us very briefly define the pion-nucleon Σ-term. The chiral symmetry allows us to connect this term with the even (respective to isotopic transformations) amplitude of the πN scattering, $D^{(+)}(\nu, t)$ evaluated at the Cheng-Dashen point [16]: $\nu = (s - u)/4M_p = 0$ and $t = 2M_\pi^2$: [5]

$$\Sigma = F_\pi^2 \bar{D}^{(+)}(0, 2M_\pi^2) = \sigma + \Delta. \tag{6}$$

The first term on the RHS of (6) can be considered as the contribution from the amplitude of the scattering of the massless pion on the physical nucleon, whereas the second term, Δ, represents the correction arising from the nonvanishing pion mass. The pion-nucleon σ-term is defined by the following matrix element:

$$\sigma = \frac{m_{0u} + m_{0d}}{4M_p} \langle P(p)|\bar{u}u + \bar{d}d|P(p)\rangle, \tag{7}$$

where $|P(p)\rangle$ is the one-proton physical state, $M_p = 938$ MeV is the proton mass, and m_{0q} is the mass of the current q-quark $(q = u, d)$.

[4]In this approximation all $\pi\pi$ scattering lengths are equal to zero (see (3)). In order to obtain nontrivial values of the $\pi\pi$ scattering lengths, it is necessary to use the real value for the mass of the $f_0(400 - 1200)$ meson.

[5]Here s, u and t denote the kinematic invariants of the πN scattering: $s + u + t = 2M_\pi^2 + 2M_p^2$.

Recall that the value of the first term on the RHS of (6) obtained by estimating the matrix element $\langle \pi^+(p_1)|\bar{u}u + \bar{d}d|\pi^+(p_2)\rangle$ on the basis of the Gell-Mann–Oakes–Renner low-energy theorem [17] in the current algebra and the PCAC approach is

$$\langle \pi^+(p_1)|\bar{u}u + \bar{d}d|\pi^+(p_2)\rangle \simeq \langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle = -\frac{1}{F_\pi^2}\langle 0|\bar{u}u + \bar{d}d|0\rangle = \frac{4v}{F_\pi^2}, \quad (8)$$

where $2v = -\langle 0|\bar{u}u|0\rangle = -\langle 0|\bar{d}d|0\rangle$ is the quark condensate.

Now one can get this formula in a chiral quark model of the Nambu–Jona–Lasinio (NJL) type calculating the quark-loop diagrams depicted in Fig. 1. For this purpose it is necessary to use the lagrangian $L_{\rm int}$ of the quark-meson interaction obtained in our model [6]

$$L_{int} = \bar{q}(g_\sigma\sigma + i\gamma^5 g_\pi \vec{\pi}.\vec{\tau})q + \frac{i}{F_\pi}(1 - 1/Z)\bar{q}\frac{1}{2}\gamma^\mu\gamma^5\partial_\mu\vec{\pi}.\vec{\tau}q, \quad (9)$$

where $q = (u, d)$ denotes the field operators of the constituent u and d quarks, $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the isospin Pauli matrices. In this model we have the following relations between the meson-quark coupling constants

$$g_\sigma = \frac{g_\rho}{\sqrt{6}}, \quad g_\pi = \frac{m}{F_\pi}, \quad (10)$$

where $g_\rho = 6.14$ is the $\rho \to 2\pi$ decay constant and $m = m_u = m_d$ is the mass of a constituent quark. The first relation in (10) was obtained for the first time in [6, 18], and the second one is the Goldberger-Treiman relation. On the other hand, the constants g_σ and g_π are linked by the relation

$$g_\pi = \sqrt{Z} g_\sigma, \quad (11)$$

where

$$Z = \left(1 - \frac{6m^2}{M_{a_1}^2}\right)^{-1} \quad (12)$$

is the additional renormalization of pion fields appearing after taking into account the $\pi - a_1$ transitions, $M_{a_1} = 1.26 \pm 0.03$ GeV [7] is the mass of the axial a_1-meson. We can consider (11) as the equation defining the constituent quark mass m. This allows us to express the mass of the constituent u-quark through the a_1-meson mass M_{a_1}, the pion decay constant F_π, and the coupling constant g_ρ, namely

$$m^2 = \frac{M_{a_1}^2}{12}\left[1 - \sqrt{1 - \frac{4g_\rho^2 F_\pi^2}{M_{a_1}^2}}\right] \quad (13)$$

and we obtain $m = 280$ MeV [6].[6]

[6]The equality $Z = 1$ is formally possible when $M_{a_1} \to \infty$; it corresponds to the absence of a_1-mesons in the intermediate state.

The second part of L_{int} (9) appears after redefinition of axial-vector fields by taking into account the $\pi - a_1$ transitions [6, 19]. This part of the lagrangian is necessary to define the momentum dependence of the scalar pion form factor. To reproduce the GMOR result (8) [5], it is enough to use the first part of L_{int} (9), because in the low-energy approximation, where $p_i^2 \to 0$, the second part of L_{int} (9) vanishes.

Let us write the result of the calculation of the diagrams depicted in Fig. 1 [5]:

$$\langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle = 4mZ\{1 + [I_1(m) - 2m^2 I_2(m)]\frac{8g_\sigma^2}{M_\sigma^2}\}, \tag{14}$$

where the integrals $I_n(m)$ in the Euclidean space have the following form [6]

$$I_n(m_u) = \frac{3}{(2\pi)^4}\int \frac{d_E^4 k}{(m^2 + k^2)^n}\theta(\Lambda^2 - k^2), \tag{15}$$

where Λ is the cut-off parameter identified in our chiral quark model with the scale of spontaneous breaking of the chiral symmetry (SBCS). Using the expressions $g_\sigma^2 = (4I_2(m))^{-1} = g_\rho^2/6$ we obtain the value $\Lambda = 1.25$ GeV [6].

In the chiral limit, where $M_\sigma^2 = 4m^2$, the first and the third terms on the RHS of (14) cancel in pairs. Then, using the relation for the quark condensate $2v = 4mI_1(m)$ [6] we obtain the result that coincides with the GMOR result (8).

The latest analysis of the experimental data gives a relatively large value for the pion-nucleon σ-term [4]. We show here how one can obtain this result in our model using the more realistic value of the σ-meson ($f_0(400 - 1200)$) mass $M_\sigma = 400$ MeV. This value was obtained in [8, 9], where the singlet-octet mixing of scalar isoscalar mesons and the scalar glueball were taken into account.

To estimate the σ-term corresponding to $M_\sigma = 400$ MeV we can use (14), where the coefficient C_σ^2 describing the content of u- and d-quark components in the scalar state $f_0(400 - 1200)$ should be introduced into the second term on the RHS containing the σ-meson mass. In [8] the value $C_\sigma = 0.94$ was obtained. As a result, we have the following formula for the matrix element defining the σ-term (see also (7)) [5]

$$\begin{aligned}\langle P(p)|\bar{u}u + \bar{d}d|P(p)\rangle &= \frac{g_{\sigma pp}}{g_\sigma}\bar{u}(p)u(p)\langle 0|\bar{u}u + \bar{d}d|0\rangle = \\ &= \frac{g_{\sigma pp}}{g_\sigma}\left(1 + [I_1(m) - 2m^2 I_2(m)]\frac{8C_\sigma^2 g_\sigma^2}{M_\sigma^2}\right)\bar{u}(p)u(p),\end{aligned} \tag{16}$$

where $u(p)$ is the bispinor normalized by the condition $\bar{u}(p)u(p) = 2M_p$, $g_{\sigma pp} = M_p/F_\pi$ denotes the coupling constant of the σpp interaction on the mass shell of the proton $(p^2 = M_p^2)$. Using (16) and the GMOR relation

$$m_\pi^2 = \frac{2v}{F_\pi^2}(m_{0u} + m_{0d}) \tag{17}$$

we obtain in our chiral model[7]

$$\sigma = 75\,\mathrm{MeV}. \tag{18}$$

[7]We used the following value for the integral $I_1(m_u) = \frac{3}{(4\pi)^2}\left[\Lambda^2 - m_u^2 \ln\left(\frac{\Lambda^2}{m_u^2} + 1\right)\right]$.

Now let us calculate the dependence of the scalar pion form factor on the external momenta p_1 and p_2. The first source of this dependence appears after taking into account the possibility of $\pi - a_1$ transitions at the external pion legs. These transitions are described by the last term of the lagrangian L_{int} (9). The contributions were estimated in our work [5] and, as a result, the factor $A(M_\pi^2)$ appears on the RHS of (14) and (16)[8]:

$$A(M_\pi^2) = 1 + \left(1 - \frac{1}{Z}\right) \frac{M_\pi^2}{2m^2 Z} = 1.026, \tag{19}$$

where we used $p_1^2 = p_2^2 = M_\pi^2$. The second source of the momentum dependence of the scalar pion form factor is related to the momentum dependence of the triangle quark diagram (see Fig. 1). This dependence was neglected in our previous calculations. After taking it into account we obtain the expression

$$\begin{aligned} I^{(\Delta)} &= -i \frac{3}{(2\pi)^4} \int^\Lambda d^4k \, Tr \left\{ \frac{1}{m - \hat{k} - \hat{p}_1} \gamma_5 \frac{1}{m - \hat{k}} \gamma_5 \frac{1}{m - \hat{k} - \hat{p}_2} \right\} = \\ &= 4m[I_2(m) + p_1 p_2 I_3(m)] = 4m I_2(m) \left(1 + \frac{3M_\pi^2}{8\pi^2 F_\pi^2 Z}\right). \end{aligned} \tag{20}$$

Here we put $p_1 p_2 = M_\pi^2$, $4I_2(m) = g_\sigma^{-2}$, $m^2/g_\sigma^2 = F_\pi^2 Z$ and $I_3(m) = 3(32\pi^2 m^2)^{-1}$. Now we can extract the second additional factor on the RHS in (14) and (16) describing the p^2-dependence

$$B(M_\pi^2) = 1 + \frac{3M_\pi^2}{8\pi^2 F_\pi^2 Z} = 1.06 \tag{21}$$

Finally, for the pion-nucleon Σ-term we have

$$\Sigma = \sigma A(M_\pi^2) B(M_\pi^2) = \sigma + \Delta = 81 \text{ MeV} \tag{22}$$

As a consequence, taking into account the p^2-dependence leads to the value

$$\Delta = \Sigma(M_\pi^2) - \sigma|_{p^2=0} = 6 \text{ MeV} \tag{23}$$

Now let us briefly analyze the results. In this paper we have shown, as in [5], that after taking into account the diagrams with the intermediate σ-meson one can get agreement with the last experimental data (see [4]) without using the hypothesis of the content of strange quarks in a nucleon. Therefore, in the framework of the linear σ-model it is unnecessary to use this hypothesis. Analysis of the terms desribing the dependence on the external pion momenta given in the present paper shows that their contributions are relatively small and the numerical value of the Σ-term mainly depends on the mass M_σ of the scalar meson $f_0(400 - 1200)$. The difference between the results obtained here and in [5] is for the most part caused by different values of M_σ.

We already mentioned that there is a large discrepancy in the experimental data for the Σ-term and for the mass and width of the scalar σ-meson $f_0(400 - 1200)$. Because our model is

[8]We note that in our previous paper [5] there is a mistake in the coefficient of the p^2-term. Actually, this coefficient must be two times larger.

very sensitive to M_σ, more precise measurements of the scalar meson $f_0(400-1200)$ mass can yield important information on the Σ-term.

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