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PANCHARATNAM PHASE OF AN EFFECTIVE TWO-LEVEL ATOM IN A KERR-LIKE MEDIUM

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The influence of Stark shift on the Pancharatnam phase when an ideal cavity is filled with a Kerr-like medium and coupling is affected through a degenerate two-photon process is studied. The exact results are employed to perform a careful investigation of the temporal evolution of atomic inversion and Pancharatnam phase. We invoke the mathematical notion of maximum variation of a function to construct a measure for Pancharatnam phase fluctuations. It is shown that the Pancharatnam phase explicity contains information about the statistics of the field and atomic coherence. It is shown that the addition of the Kerr medium and the Stark shift has an important effect on the properties of the Pancharatnam phase. The results show that, the effect of the Kerr medium and the Stark shift changes the quasiperiod of the Pancharatnam phase evolution. The general conclusions reached are illustrated by numerical results.

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1 Introduction

In recent years much attention has been paid to the quantum phases such as the Pancharatnam phase [1-4] and geometric phase (Berry phase) [5-6]. Physical sciences contain many examples of objects whose behavior is specified up to a phase by certain parameters. The total phase acquired by the wavefunction of a quantum system in a cyclic or noncyclic evolution contains two parts, normally, the dynamical phase part and the geometric phase part. As it is well known, the dynamical phase of the state vector of a system is Hamiltonian dependent, but the geometric phase simply depends on the chosen path in the space spanned by all the likely quantum states for the system. The Pancharatnam phase is very important in the propagation of a light beam where its polarization state is changing periodically [1]. Most experimental demonstrations of Pancharatnam phase [7,8] involve splitting a laser beam, transporting the state of polarization of one or both

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of the split beams along variable paths on the Poincaré sphere, and detecting the resulting change in the phase difference of the beam by interferometric superposition. The quantum phase, including the total phase as well as its dynamical and geometric parts, of Pancharatnam type are derived for a general spin system in a time-dependent magnetic field based on the quantum invariant theory [3]. Another approach that provides a unified way to discuss geometric phases in both photon (massless) and other massive particle system was developed by [9].

One of the simplest non-trivial models of quantum optics is the Jaynes-Cummings model [10], which describes the interaction of two-level atom with single mode of the quantized field, is considered to be one of the most successful models in quantum optics. In addition to the standard Jaynes-Cummings model, some generalized models [11,12] have been constructed and extensively studied. One of these generalizations (multilevels, multiphotons) is to replace the mediated photon by a degenerate two-photon, i.e., photons of the same mode are either emitted or absorbed in pairs by the atom. To make the two-photon processes closer to the experimental realization, we include the effect of the dynamic Stark shift in the evolution of the Pancharatnam phase, which is necessary and interesting. Furthermore, we examine the effect of the dynamic Stark shift in the evolution of the Pancharatnam phase in the presence and absence of a Kerr-like medium. The model considered, consists of a single two-level atom undergoing a two-photon processes in a single-mode field surrounded by a nonlinear Kerr-like medium contained inside a very good quality cavity. The cavity mode is coupled to the Kerr medium as well as to the two-level atom. The Kerr medium can be modeled as an anharmonic oscillator with frequency ω. Physically this model may be realized as if the cavity contains two different species of Rydberg atoms, of which one behaves like a two-level atom undergoing two-photon transition and the other behaves like an anharmonic oscillator in the single-mode field of frequency $\omega_\circ$ [11,12].

Such a model is interesting by itself as another exactly solvable quantum model [13] that gives nontrivial results, but we can also think of its possible applications. This Hamiltonian is natural for local modes in molecular physics or for a nonlinear Jahn-Teller effect, although long-time behavior in either case might be obscured by omnipresent damping. There may also be optical applications, since this type of nonlinearity may be realized by letting the electromagnetic radiation pass through a nonlinear Kerr medium [14]. One can think of an experiment with a Rydberg atom in a nonlinear Kerr-like cavity. The field of quantum information and computing is based on manipulation of quantum coherent states [15]. Existing device of quantum optics have been proposed as experimental implementation and employed to realize quantum computers. A scheme depending on applications of the displacing operator and propagating a laser beam in a nonlinear Kerr medium has been proposed to perform quantum gates [16].

The material of this paper is arranged as follows: Section 2, we introduce the model and write the expressions for the final state vector at any time $t > 0$ and the Pancharatnam phase calculation including the Stark shift and the Kerr-like medium effects. By a numerical computation, we examine the influence of the Stark shift and the Kerr-like medium on the atomic inversion and the Pancharatnam phase for a coherent field input and a variety of initial atomic states in section 3. Take into consideration the total phase is calculated in this case. Different cases are studied numerically to demonstrate the effects due to both Stark and Kerr effects. Finally, conclusions are presented.

2 Basic equation

The total phase has both the dynamical and the geometric phase parts for an arbitrary quantum evolution of a system from a state at $t = 0$ to a final state at time t. Without invoking that any initial state vector $\psi(0)$ and the final state vector $\psi(t)$ correspond to different rays, we use the Pancharatnam phase approach [17] of defining the phase between them. On subtracting the dynamical phase ϕ_d from the Pancharatnam phase (or the total phase ϕ_t) we obtain the geometric phase ϕ_g. The Pancharatnam phase ϕ_t between the vectors $\psi(0)$ and $\psi(t)$ is given by [17]

$$\phi_t = \arg\langle\psi(0) \mid \psi(t)\rangle. \tag{1}$$

The dynamical phase for an arbitrary quantum evolution from time $t = 0$ to a time t is given by the time integral of the expectation value of the Hamiltonian over the time interval $[0, t]$,

$$\phi_d = -\frac{1}{\hbar}\int_0^t \langle\psi(t) \mid H \mid \psi(t)\rangle dt. \tag{2}$$

So, the geometric phase under Schrödinger evolution is given by $\phi_g = \phi_t - \phi_d$. Both the dynamical as well as the geometric phases studied separately for the quantum optical system under consideration are not producing significantly interesting results pertaining to photon statistics of the cavity field and the type of transition the atom is undergoing. On the other hand, the sum of the two phases reveals interesting results. Hence in this sense both ϕ_d and ϕ_g are very important in determining the behavior of ϕ_t. The phase can be determined experimentally in several ways. It can for instance be measured with an interferometer, which is a special device using the one-state two-Hamiltonian strategy, also can be measured by the superposition technique using the two-state one-Hamiltonian [18] strategy. This technique has been employed [19-20] for determining the phase acquired by a two-state quantum system in a cyclic evolution.

The model considered here consists of a single-mode interacting with an effective two-level atom when the dipole forbidden transition is replaced by a two-photon one. We consider the degenerate case, in which pairs of photons with the same frequency are created or absorbed and the quantized radiation field in the rotating wave approximation in an ideal cavity ($Q = \infty$) filled with a nonlinear Kerr-like medium. We also assume that the cavity mode interacts with both the atom and the Kerr-like medium. However, a real cavity cannot be ideal. But in Ref. [21] the influence of a cavity with finite bandwidth at nonzero temperature T was studied and it was shown that for new available experimental values of $Q = 2 \times 10^{10}$ and $T = 0.5K$ the effect of the bandwidth and the temperature are negligible until the time $t \sim 10^{-3}(\lambda t = 30)$ from the start of the interaction. The excited and ground states of the atom will be designated by $\mid e\rangle$ and $\mid g\rangle$, respectively. We assume that these states have identical parity, whereas the intermediate states, labeled $\mid j\rangle(j = 3, 4, ...)$, are coupled to $\mid e\rangle$ and $\mid g\rangle$ by a direct dipole transition and so located as to give rise to a significant Stark shift. The intensity dependent Stark effect can be employed in quantum nondemolition measurements [22-24]. Kerr effects can be observed by surrounding the atom by a non-linear medium inside a high Q-cavity [25]. The effective Hamiltonian of the model under consideration in this paper in the rotating-wave approximation can be written as [11,26,27] ($\hbar = c = 1$),

$$\hat{H}_{eff} = \omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\omega_a \hat{\sigma}_z + \hat{a}^\dagger \hat{a}(\beta_2 \mid e\rangle\langle e \mid + \beta_1 \mid g\rangle\langle g \mid) \\ + \chi \hat{a}^{\dagger 2}\hat{a}^2 + \lambda(\hat{a}^{\dagger 2}\hat{\sigma}_- + \hat{a}^2 \hat{\sigma}_+), \tag{3}$$

where ω_c is the field frequency and ω_a is the transition frequency between the excited and ground states of the atom, $\hat{a}$ and $\hat{a}^\dagger$, are the annihilation and the creation operators of the cavity field respectively, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transitions to the intermediate relay level, λ is the effective coupling constant between the atomic and field mode system, σ_z and $\sigma_\pm$ are the atomic pseudo-spin operators. We denote by χ the dispersive part of the third-order nonlinearity of the Kerr-like medium [27]. The initial state of the total atom-field system can be written as

$$\mid \psi(0) \geq (a \mid g\rangle + b \mid e\rangle) \otimes \sum_{n=0}^{\infty} C_n \mid n\rangle, \tag{4}$$

where $a = e^{-i\nu}\sin(\theta)$, $b = \cos(\theta)$. Equation (4) means that the atom is initially in the superposition of its two states $\mid g\rangle$ and $\mid e\rangle$ and the field is assumed to be initially in a coherent state where $C_n = e^{(-\bar{n}/2)}\frac{\alpha^n}{\sqrt{n!}}$, and $\bar{n} = \mid \alpha \mid^2$ is the mean photon number of the coherent field. However, at any time $t > 0$ the atom-field coupling is described by the entangled state (in the interaction picture),

$$\mid \psi(t) \geq \sum_{n=0}^{\infty}\Big(a_n(t) \mid n, g\rangle + b_n(t) \mid n, e\rangle\Big), \tag{5}$$

where the coefficients a_n and b_n are given by the formulae

$$a_n(t) = e^{-i\lambda t \eta_n}\Big(aC_n F_n^*(t) - ibC_{n-2}R_n(t)\Big), \tag{6}$$

$$b_n(t) = e^{-i\lambda t \eta_{n+2}}\Big(bC_n F_{n+2}(t) - iaC_{n+2}R_{n+2}(t)\Big), \tag{7}$$

$$\eta_n = \frac{\chi}{\lambda}(n^2 - 3n + 3) + \frac{n}{2}\beta_1 + \frac{n-2}{2}\beta_2,$$

$$F_n(t) = \cos \lambda t \mu_n - i\xi_n \frac{\sin \lambda t \mu_n}{\mu_n},$$

$$R_n(t) = \sqrt{n(n-1)}\frac{\sin \lambda t \mu_n}{\mu_n},$$

$$\mu_n^2 = \xi_n^2 + n(n-1),$$

$$\xi_n = \frac{\delta}{2} - \frac{1}{2\lambda}(2\chi(2n-3) + n\beta_1 - (n-2)\beta_2),$$

$\delta = \Delta/\lambda$. With the wave function $\mid \psi(t)\rangle$ calculated, any statistical property related to the atom or the field can be calculated. With the help of equations (1), (2) and (5) it is straightforward to find the expressions for ϕ_d and ϕ_t in the initial coherent field in the following form,

$$\phi_d = -\lambda t\Bigg(\sum_{n=0}^{\infty} P(n)\Big[\cos^2(\theta)(\eta_{n+2} + \xi_{n+2})$$
$$+\sin^2(\theta)(\eta_n - \xi_n)\Big] + \mid \alpha \mid^2 \sin(2\theta)\cos(\nu)\Bigg), \tag{8}$$

$$\phi_t = arg\sum_{n=0}^{\infty}(a^* C_n^* a_n(t) + b^* C_n^* b_n(t))$$
$$= -\sin^{-1}\left(\frac{Y(t)}{\sqrt{X^2(t)+Y^2(t)}}\right), \tag{9}$$

$$Y(t) = \sum_{n=0}^{\infty}\Bigg(\cos^2(\theta)[\cos\lambda t\mu_{n+2}\sin\lambda t\eta_{n+2} + \xi_{n+2}\frac{\sin\lambda t\mu_{n+2}}{\mu_n+2}\cos\lambda t\eta_{n+2}]$$
$$+\sin^2(\theta)[\cos\lambda t\mu_n \sin\lambda t\eta_n - \xi_n\frac{\sin\lambda t\mu_n}{\mu_n}\cos\lambda t\mu_n]$$
$$+\mid \alpha \mid^2 \frac{\sin\lambda t\mu_{n+2}}{\mu_n+2}\sin(2\theta)\cos(\nu)\cos\lambda t\eta_{n+2}\Bigg)P(n), \tag{10}$$

$$X(t) = \sum_{n=0}^{\infty}\Bigg(\cos^2(\theta)[\cos\lambda t\mu_{n+2}\sin\lambda t\eta_{n+2} - \xi_{n+2}\frac{\sin\lambda t\mu_{n+2}}{\mu_n+2}\sin\lambda t\eta_{n+2}]$$
$$+\sin^2(\theta)[\cos\lambda t\mu_n \cos\lambda t\eta_n - \xi_n\frac{\sin\lambda t\mu_n}{\mu_n}\sin\lambda t\mu_n]$$
$$-\mid \alpha \mid^2 \frac{\sin\lambda t\mu_{n+2}}{\mu_n+2}\sin(2\theta)\cos(\nu)\sin\lambda t\eta_{n+2}\Bigg)P(n). \tag{11}$$

With $P(n) =\mid C_n \mid^2$. In what follows we shall consider the effect of both Kerr and Stark shift on dynamical behavior of the atomic inversion and the Pancharatnam phase of the system for two-photon process.

3 Results of calculations

On the basis of the analytical solution presented in the previous section, we shall examine the evolution in time of the atomic inversion and the scaled Pancharatnam phase ϕ_t. It should be emphasized that in computing all infinite series for the atomic wave function $\psi(t)$, we have invoked mathematically sound truncation criteria. To ensure an excellent accuracy the behavior of each function from $\langle\sigma_z\rangle$ and ϕ_t has been determined with great precision. For regions exhibiting strong fluctuation a resolution of 10^3 point per unit of time has been employed. For all our plots the initial condition has been chosen, with coherence parameter α real. Its square is equal to the mean photon number. We recall that time t has been scaled; one unit of time is given by the inverse of the coupling constant λ.

The dynamical phase ϕ_d for the initial coherent field with the absence of both Stark and Kerr-like medium is given by $\phi_d = -(a^*b\alpha^* + \alpha ab^*)\lambda t$. In this case, both of ϕ_t and ϕ_d vanish whenever $a = 0$ or $b = 0$. From equations (8) and (9), it follows that for the cavity field in a Fock state $\mid n\rangle$, the phases are zero. Thus a more general radiation state, i.e., other than Fock state, and atomic coherence are necessary for development of the Pancharatnam and dynamical phases. The dynamical phase being linear in t has periodic saw-tooth kind variation over the time λt bounded between the limits 0 and 2π. Note also that the dynamical phase ϕ_d does not contain any information about the quantal nature of the cavity field. Similarly, the geometric phase part also does not give any information about the field distribution. On the other hand, the evolution of the total phase ϕ_t is quite revealing. In Fig. 1 we have plotted the atomic inversion and Pancharatnam phase ϕ_t as a function of the scaled time λt, in the absence of the Stark shift and the Kerr-like medium, for the mean photon number $\bar{n} = 25$. The atomic inversion Fig. 1a which is clearly exhibiting the periodic collapse and revival phenomenon of Rabi oscillation with a period of π. It is important to note that the Pancharatnam phase ϕ_t for the two-photon transition is also similar to the periodic collapse-revival phenomenon of Rabi oscillation but with a period of 2π (see Fig.1b). Thus the behavior of the Pancharatnam phase ϕ_t is in contrast with the behavior of the dynamical phase ϕ_d Eq. (8) as it shows the phenomenon of collapses revivals. In the figures that follow we investigate the effects of Stark shift and Kerr-like medium. In Fig. 2, we show the case in which the two levels have unequal Stark shifts ($r < 1$, in Fig. 2, $r = 0.5$). We see that the Stark shift leads to changes in the quasiperiods of the atomic inversion (see Fig. 2a) and the Pancharatnam phase (see Fig. 2b), we basically see that the Stark shift produces a large effective detuning, which causes a weak interaction between the field and the atom which produces the asymmetrical splitting in the Pancharatnam phase. A quick look at the Rabi frequency for the collapses and revivals of the inversion, in this case it can be shown that the quasi periods occur at distances $\pi/\sqrt{1+\frac{1-r^2}{4r^2}}$ instead of π for the two-photon JCM. Also it is noted that the atomic inversion oscillates around values < 0 in this case; which means that the atomic system loses some of its mean energy to the field. While the effect in the phase is rather remarkable especially around $\lambda t \sim 3\pi$ where it is almost chaotic for a short period, then after that quasi periodic behavior occurs. In case $r = 1(\beta_1 = \beta_2)$, which corresponds to the case in which the two levels of the atom are equally strongly coupled with the intermediate relay level, we see that the evolutions of the atomic inversion and the Pancharatnam phase are almost similar for the case in which $r = 0$. This may be interpreted physically as follows:- This result corresponds to the fact that the Stark shift creates an effective intensity dependent

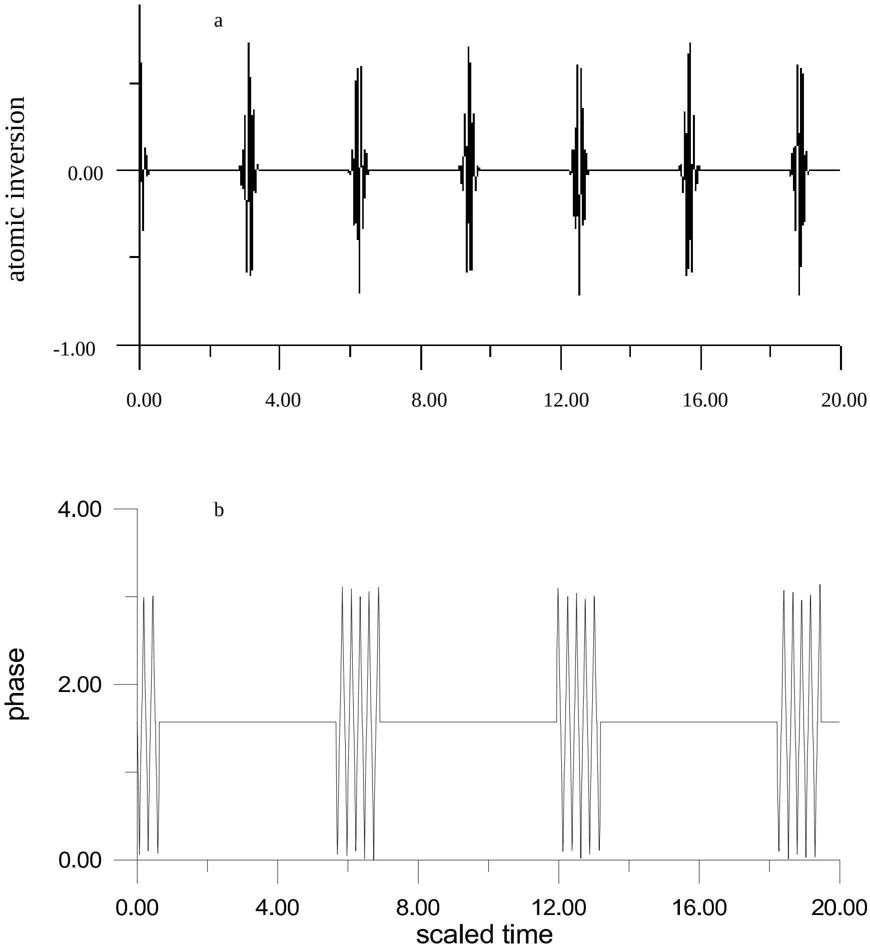

Fig. 1 (a) Population inversion for a two-level atom undergoing a two-photon transition as a function of scaled time λt with the cavity field initially in the coherent state having $\bar{n} = 25$ and the atom is in the symmetric superposition of its two states ($a = b = 1/\sqrt{2}$); $\chi/\lambda = 0$ and $r = \sqrt{\beta_1/\beta_2} = 0$. (b) Pancharatnam phase ($\phi_t + \pi/2$) as a function of scaled time λt for the same system under similar conditions of parameters.

detuning $\Delta_N = \beta_2 - \beta_1$. When $r = 1, \Delta_N = 0$, in this case, the Stark shift does not affect the time evolution of the atomic inversion or the Pancharatnam phase.

To visualize the influence of the Kerr-like medium in the atomic inversion and the Pancharatnam phase ϕ_t we set different values of χ/λ, while all the other parameters are the same as in Fig. 1. The outcome is presented in Figures 3 and 4. One can distinguish between two stages of evolution, each of which has been pictured separately. It is to be remarked that the atomic inversion has similar behavior but oscillates around

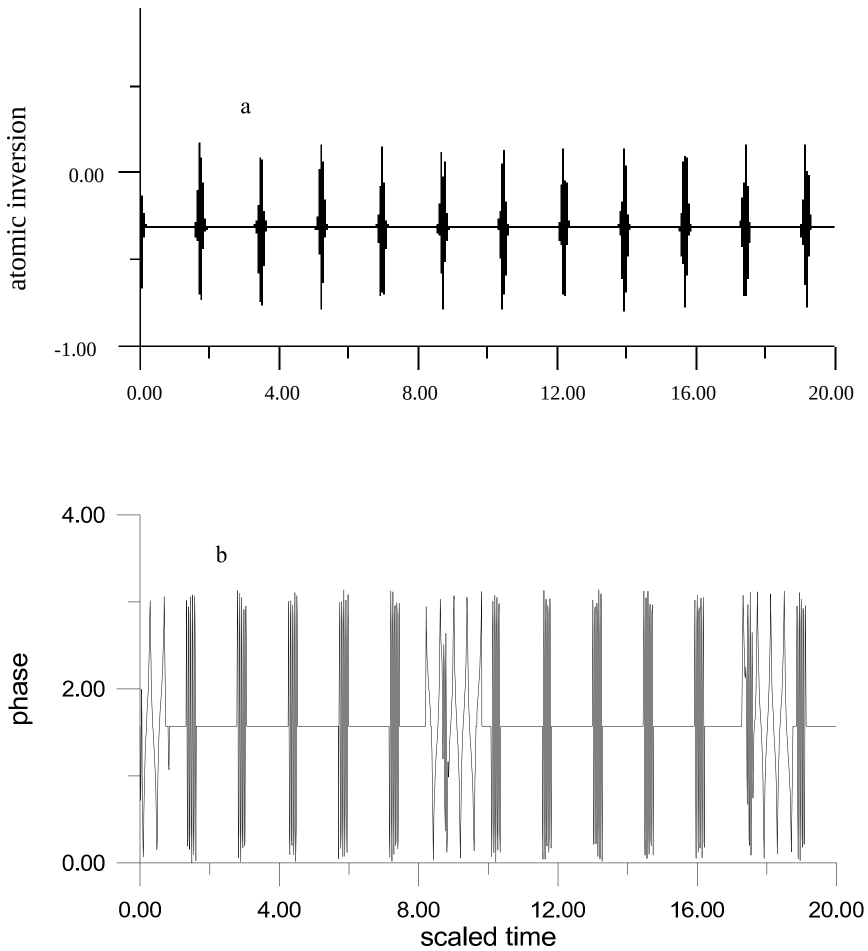

Fig. 2 (a) Population inversion for a two-level atom undergoing a two-photon transition as a function of scaled time λt with the cavity field initially in the coherent state having $\bar{n} = 25$ and the atom is in the symmetric superposition of its two states ($a = b = 1/\sqrt{2}$); $\chi/\lambda = 0$ and $r = \sqrt{\beta_1/\beta_2} = 0.5$. (b) Pancharatnam phase ($\phi_t + \pi/2$) as a function of scaled time λt for the same system under similar conditions of parameters.

positive value Fig. 3a, which means that due to the Kerr nonlinearity more energy is stored in the atomic system leading to energy inhibition, in contrast to the Stark shift effect. In other words, by increasing the nonlinearity of the Kerr-like medium there is a growing tendency of the atom to trap the excitation energy. Physically it is due to the change in energy-level structure of the system under consideration. The symmetry shown in the standard two-photon transition Fig. 1b for Pancharatnam phase ϕ_t is no longer present once Kerr like medium is added (see Fig. 3b). It is noted that after two

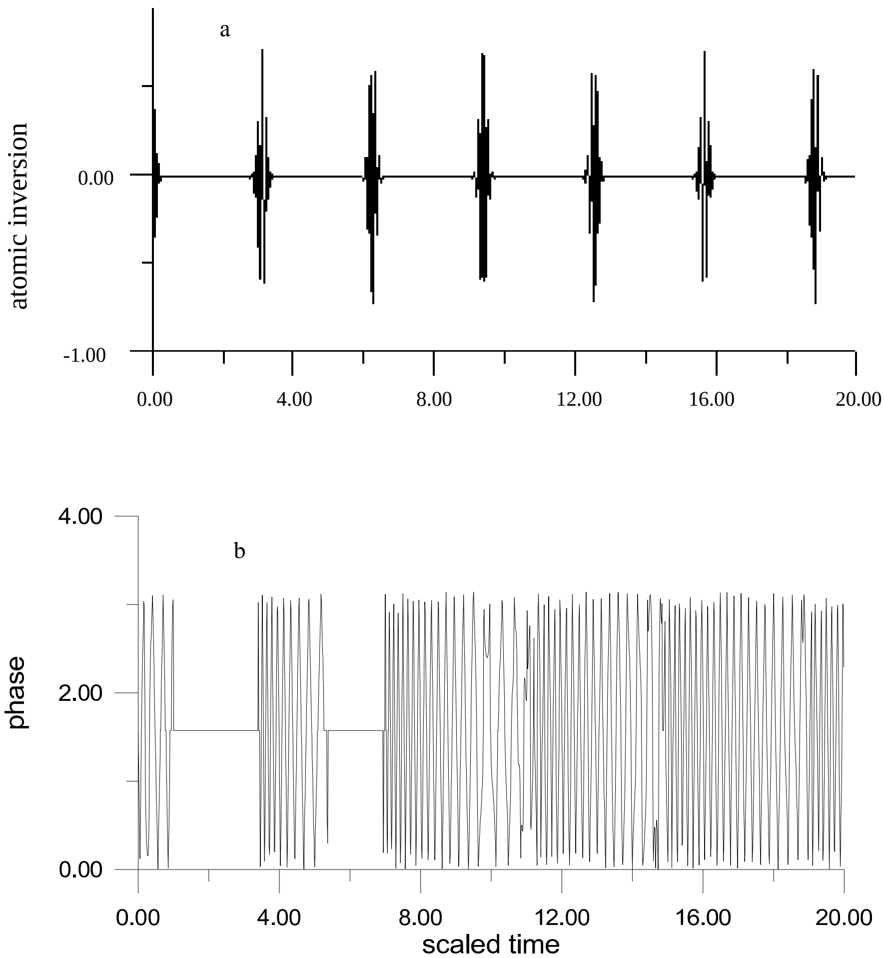

Fig. 3 (a) Population inversion for a two-level atom undergoing a two-photon transition as a function of scaled time λt with the cavity field initially in the coherent state having $\bar{n} = 25$ and the atom is in the symmetric superposition of its two states ($a = b = 1/\sqrt{2}$); $\chi/\lambda = 0.01$ and $r = \sqrt{\beta_1/\beta_2} = 0$. (b) Pancharatnam phase ($\phi_t + \pi/2$) as a function of scaled time λt for the same system under similar conditions of parameters.

quasi periods, interference becomes more effective, and one can not distinguish quasi periods. For larger values of the Kerr medium χ/λ, the effect in the atomic inversion and in the Pancharatnam phase ϕ_t become more pronounced, not only the amplitude of Rabi oscillation which depend on the inverse of χ/λ, but also the time average of the atomic inversion and the Pancharatnam phase ϕ_t is affected remarkably, only one period is shown and chaotic behavior starts (see Fig. 4). The influence of Stark shift in the presence of a Kerr-like medium is plotted in Fig. 5. It is to be noted that

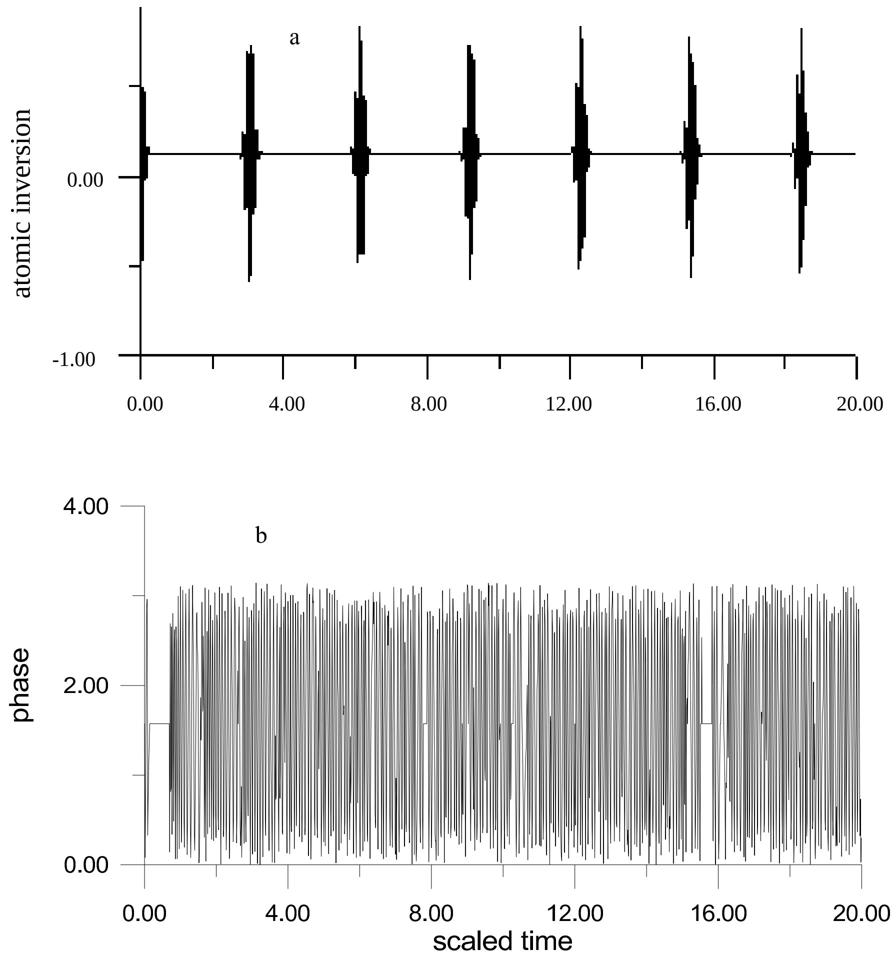

Fig. 4 (a) Population inversion for a two-level atom undergoing a two-photon transition as a function of scaled time λt with the cavity field initially in the coherent state having $\bar{n} = 25$ and the atom is in the symmetric superposition of its two states ($a = b = 1/\sqrt{2}$); $\chi/\lambda = 0.1$ and $r = \sqrt{\beta_1/\beta_2} = 0$. (b) Pancharatnam phase ($\phi_t + \pi/2$) as a function of scaled time λt for the same system under similar conditions of parameters.

Stark interaction behaves like the limiting case of the Kerr interaction. This may be understood in the following way: the Kerr interaction produces two separate effects, (a) a Kerr one, which splits the field in phase space, producing a Schrödinger cat [28], and (b) a Stark interaction with the field in a cat state. The atom-field interaction when the field is initially in a cat state has been shown to be less pure than for the field in a coherent state. It has been shown [29] that taking into account Stark shifts in the atom-field interaction aggress with experimental results of micromasers [30]. Such as

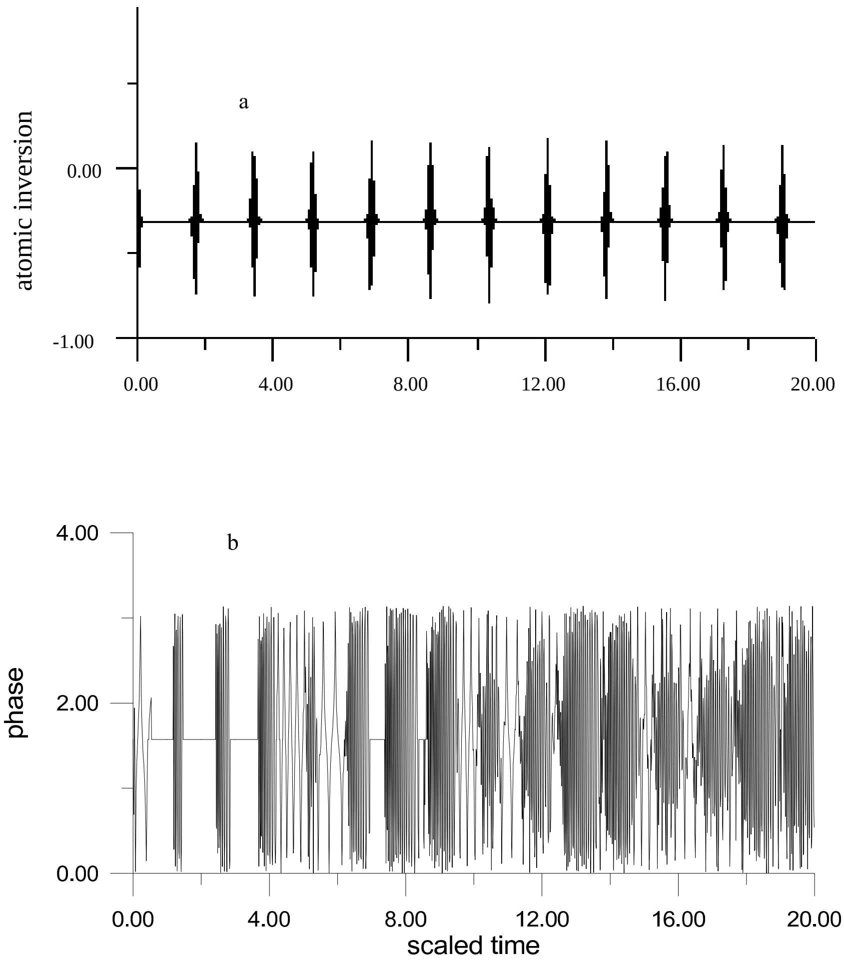

Fig. 5 the same as in Fig. 4 but with $\chi/\lambda = 0.01$ and $r = \sqrt{\beta_1/\beta_2} = 0.5$.

shifted transition lineshapes and those asymmetrically distorted. The phases discussed here are physically observable interferometrically through a "structured" approach [31].

In conclusion, we have studied the Pancharatnam phase of a coherent field interacting with a two-level atom when the dipole forbidden transition is replaced by a two-photon transition. We considered the degenerate case, in which pairs of photons with the same frequency are created or absorbed taking into account the presence of the Stark shift and the Kerr-like medium. Such systems are potentially interesting for their ability to process information in a novel way and might find application in models of quantum logic gates. It is observed that the periodicity shown in the standard two-photon processes for the Pancharatnam phase ϕ_t is no longer present once Kerr-like medium or Stark shift is added. Both effects have been studied for the Pegg-Barnett definition for the phase [26].

In the Pegg-Barnett phase the Stark shift tends to localize the phase while the Kerr effect on the other hand tend to damp and diffuse the phase distribution. The Pancharatnam phase explicitly derived here is quite sensitive to the initial conditions of its constituent sub-system. If the initial conditions are such that they produce some kind of entanglement, then only one observes the Pancharatnam phase ϕ_t as well as the dynamical phase. In such cases the phase ϕ_t can distinguish between different statistics of radiation in the cavity as well as the type of photon transition involved. Pancharatnam's ideas in optics have also led to better understanding of Berry's phase in quantum mechanics [32].

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