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EXISTENCE OF THE σ-MESON BELOW 1 GeV AND THE $f_0(1500)$ GLUEBALL

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The isoscalar s-waves of the $\pi\pi$ scattering (from the threshold up to 1.9 GeV) and of the $\pi\pi \to K\overline{K}$ process (from the threshold to $\sim$ 1.46 GeV) are analyzed simultaneously in the model-independent approach. A confirmation of the σ-meson at $\sim$ 665 MeV and an indication for the glueball nature of the $f_0(1500)$ state are obtained. It is shown that the large $\pi\pi$-background, usually obtained, combines, in reality, the influence of the left-hand branch-point and the contribution of a wide resonance at $\sim$ 665 MeV. A minimum scenario of the simultaneous description of the processes $\pi\pi \to \pi\pi, K\overline{K}$ does wihtout the $f_0(1370)$ resonance. A version including this state is also considered. In this case the $f_0(1370)$ resonance has the dominant $s\bar{s}$ component (the ratio of its coupling constant with the $\pi\pi$ channel to the one with the $K\overline{K}$ channel is 0.12). The coupling constants of the observed states with the $\pi\pi$ and $K\overline{K}$ systems and the $\pi\pi$ and $K\overline{K}$ scattering lengths are obtained.

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1 Introduction

The problem of scalar mesons is the most troublesome and long-lived in light meson spectroscopy. Among difficulties in their understanding there is the one related to a strong model-dependence of information on multichannel states obtained in analyses based on the specific dynamic models or using an insufficiently-flexible representation of states (*e.g.*, the standard Breit – Wigner form). Earlier, we have shown [1] that an inadequate description of multichannel states gives not only their distorted parameters but also can cause the fictitious states when important (even energetic-closed) channels are neglected. In this paper we are going, conversely, to demostrate that the large background (*e.g.*, that happens in analyzing $\pi\pi$ scattering), can hide

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low-lying states, even such important for theory as a σ-meson [2]. The latter is required by most of the models (like the linear σ-models or the Nambu – Jona-Lasinio models [3]) for spontaneous breaking of chiral symmetry. Recent new analyses of old and new experimental data found a possible candidate for that state [4]- [8]. However, these analyses use either the Breit – Wigner form (even if modified) or specific forms of interactions; therefore, there one cannot talk about a model independence of results. Besides, in these analyses, a large $\pi\pi$-background is obtained.

A model-independent information on multichannel states can be obtained on the basis of the first principles (analyticity, unitarity) immediately applied to analyzing experimental data. The way of realization is a consistent allowance for the nearest singularities on all sheets of the Riemann surface of the S-matrix. Earlier, we have proposed this method for 2- and 3-channel resonances and developed the concept of standard clusters of poles on the Riemann surface as a qualitative characteristic of a state and a sufficient condition of its existence [1, 9]. The cluster kind is related to the state nature. At all events, we can, in a model-independent manner, distinguish between bound states of particles and the ones of quarks and gluons [1, 10], qualitatively predetermine the relative strength of coupling of a state with the considered channels, and obtain an indication on its gluonium nature.

The layout of the paper is as follows. In Section 2, we outline a model-independent version of two-coupled-channel formalism, determine the possible pole-clusters on the 4-sheeted Riemann surface as characteristics of multichannel states, and introduce a new uniformizing variable, allowing for the branch-points of the right-hand (unitary) and left-hand cuts of the $\pi\pi$-scattering amplitude. In Section 3, we analyze simultaneously experimental data on the processes $\pi\pi \to \pi\pi, K\overline{K}$ in the isoscalar s-wave in the presented approach. On the basis of obtained pole clusters for resonances, their coupling constants with the considered channels and the scattering lengths, compared with the results of various models, we make conclusion on the nature of the observed states and on the mechanism of chiral-symmetry breaking. In the Conclusion, the obtained results are discussed.

2 Two-Coupled-Channel Formalism

Here we restrict ourselves to a 2-channel simultaneous consideration of the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$. Therefore, we have the 2-channel S-matrix determined on the 4-sheeted Riemann surface. The matrix elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K})$, have the right-hand cuts along the real axis of the s-plane, starting at $4m_\pi^2$ and $4m_K^2$, and the left-hand cuts, beginning at $s = 0$ for S_{11} and at $4(m_K^2 - m_\pi^2)$ for S_{22} and S_{12}. We number the Riemann-surface sheets according to the signs of analytic continuations of the channel momenta $k_1 = (s/4 - m_\pi^2)^{1/2}$, $k_2 = (s/4 - m_K^2)^{1/2}$ as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --, +-$ correspond to the sheets I, II, III, IV.

To elucidate the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S_{\alpha\beta}^L$ $(L = II, III, IV)$ in terms of those on the physical sheet $S_{\alpha\beta}^I$. The latter have, except for the real axis, only zeros corresponding to resonances. Using the reality property of the analytic functions and the 2-channel unitarity, one can obtain

$$S_{11}^{II} = \frac{1}{S_{11}^I}, \qquad S_{11}^{III} = \frac{S_{22}^I}{\det S^I}, \qquad S_{11}^{IV} = \frac{\det S^I}{S_{22}^I},$$

$$S_{22}^{II} = \frac{\det S^I}{S_{11}^I}, \qquad S_{22}^{III} = \frac{S_{11}^I}{\det S^I}, \qquad S_{22}^{IV} = \frac{1}{S_{22}^I}, \tag{1}$$
$$S_{12}^{II} = \frac{iS_{12}^I}{S_{11}^I}, \qquad S_{12}^{III} = \frac{-S_{12}^I}{\det S^I}, \qquad S_{12}^{IV} = \frac{iS_{12}^I}{S_{22}^I},$$

Here $\det S^I = S_{11}^I S_{22}^I - (S_{12}^I)^2$; $(S_{12}^I)^2 = -s^{-1}\sqrt{(s-4m_\pi^2)(s-4m_K^2)}F(s)$; in the limited energy interval, $F(s)$ is proportional to the squared product of the coupling constants of the considered state with channels 1 and 2. Formulae (1) immediately give the resonance representation by poles and zeros on the 4-sheeted Riemann surface. One must distinguish between three types of 2-channel resonances described by a pair of conjugate zeros on sheet I: (**a**) in S_{11}, (**b**) in S_{22}, (**c**) in each of S_{11} and S_{22}. As seen from (1), to the zeroscorresponding to resonances of types (**a**) and (**b**) there correspond a pair of complex conjugate poles on sheet III, shifted relative to a pair of poles on sheet II and IV, respectively. To the states of type (**c**) one must consider corresponding two pairs of conjugate poles on sheet III. A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface) of size typical of strong interactions. The cluster kind is related to the state nature. The resonance, coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ one, is described by the cluster of type (**a**); in the opposite case it is represented by the cluster of type (**b**) (say, the state with the dominant $s\bar{s}$ component); the flavour singlet (*e.g.* glueball) must be represented by the cluster of type (**c**) as a necessary condition.

For the simultaneous analysis of experimental data of the coupled processes it is convenient to use the Le Couteur-Newton relations [11] expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, k_2)$, the real analytic function with the only square-root branch-points at $k_i = 0$. These branch points should be taken into account in the corresponding uniformizing variable. Earlier, this was done by us in the 2-channel consideration [9] with the uniformizing variable $z = (k_1 + k_2)/\sqrt{m_K^2 - m_\pi^2}$ which was proposed in Ref. [12] and maps the 4-sheeted Riemann surface with two unitary cuts, starting at $4m_\pi^2$ and $4m_K^2$, onto the plane. Note that other authors have also applied the parametrizations using the Jost functions at analyzing the s-wave $\pi\pi$ scattering in the one-channel approach [13] and in the 2-channel one [10]. In latter work, the uniformizing variable k_2 has been used, therefore, their approach cannot be applied near the $\pi\pi$ threshold.

When analyzing the processes $\pi\pi \to \pi\pi, K\overline{K}$ by the above methods in the 2-channel approach, two states ($f_0(975)$ and $f_0(1500)$) were found to be sufficient (χ^2/ndf ≈ 1.00). However, there has been obtained a large $\pi\pi$-background. The character of the representation of the background (the pole of 2nd order on the real axis on sheet II and the corresponding zero on sheet I) suggests that a wide light state is possibly hidden in the background. To check this, one must work out the background in some detail.

Now we take into account also the left-hand branch-point at $s = 0$ in the uniformizing variable

$$v = \frac{m_K\sqrt{s-4m_\pi^2} + m_\pi\sqrt{s-4m_K^2}}{\sqrt{s(m_K^2 - m_\pi^2)}}. \tag{2}$$

The variable v maps the 4-sheeted Riemann surface, having (in addition to two above-indicated

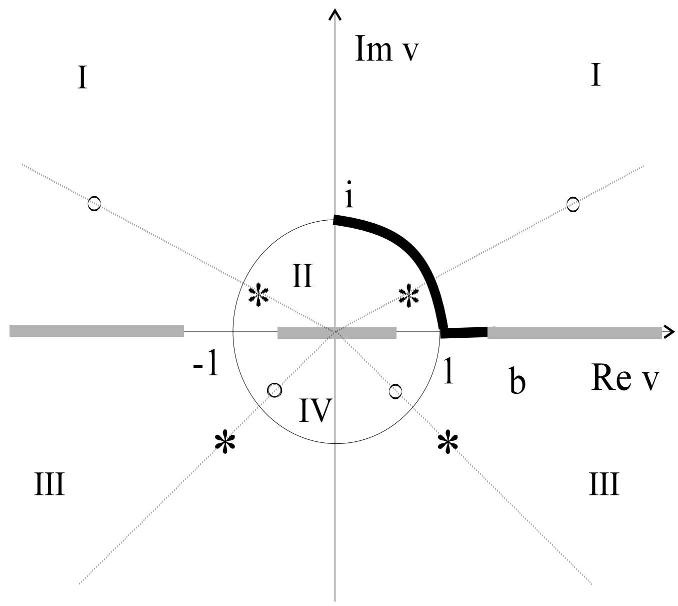

Fig. 1. Uniformization plane for the $\pi\pi$-scattering amplitude.

unitary cuts) also the left-hand cut starting at $s=0$, onto the v-plane [4]. In Fig. 1, the plane of the uniformizing variable v for the $\pi\pi$-scattering amplitude is depicted. The Roman numerals (I,..., IV) denote the images of the corresponding sheets; the thick line represents the physical region; the points i, 1 and $b=\sqrt{(m_K+m_\pi)/(m_K-m_\pi)}$ correspond to the $\pi\pi, K\overline{K}$ thresholds and $s=\infty$, respectively; the shaded intervals $(-\infty,-b],\ [-b^{-1},\ b^{-1}],\ [b,\infty)$ are the images of the corresponding edges of the left-hand cut. The depicted positions of poles ($*$) and of zeros ($\circ$) give the representation of the type (**a**) resonance in S_{11}.

On v-plane the Le Couteur-Newton relations are [9, 12]

$$S_{11}=\frac{d(-v^{-1})}{d(v)},\quad S_{22}=\frac{d(v^{-1})}{d(v)},\quad S_{11}S_{22}-S_{12}^2=\frac{d(-v)}{d(v)}. \tag{3}$$

The condition of the real analyticity implies $d(-v^*)=d^*(v)$ for all v, and the unitarity needs the following relations to hold true for the physical v-values: $|d(-v^{-1})|\le|d(v)|,\quad |d(v^{-1})|\le|d(v)|,\quad |d(-v)|=|d(v)|$.

The d-function that on the v-plane already does not possess branch-points is taken as $d=d_Bd_{res}$, where $d_B=B_\pi B_K$; B_π contains the possible remaining $\pi\pi$-background contribution, related to exchanges in crossing channels; B_K is that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude. The function $d_{res}(v)$ represents the contribution of resonances, described by one of three types of the pole-zero clusters, *i.e.*, except for the point

[4] The analogous uniformizing variable has been used, *e.g.*, in Ref. [14] at studying the forward elastic $p\bar{p}$ scattering amplitude.

$v = 0$, it consists of zeros of clusters:

$$d_{res} = v^{-M} \prod_{n=1}^{M} (1 - v_n^* v)(1 + v_n v), \tag{4}$$

where M is the number of pairs of the conjugate zeros.

3 Analysis of experimental data.

Here we analyze simultaneously the available experimental data on the $\pi\pi$-scattering [15] and the process $\pi\pi \to K\overline{K}$ [16] in the channel with $I^G J^{PC} = 0^+ 0^{++}$. As data, we use the results of phase analyses which are given for phase shifts of the amplitudes (δ_1 and δ_{12}) and for moduli of the S-matrix elements $\eta_1 = |S_{aa}|$ (a=1$\pi\pi$,2$K\overline{K}$) and $\xi = |S_{12}|$. The 2-channel unitarity condition gives $\eta_1 = \eta_2 = \eta, \ \xi = (1 - \eta^2)^{1/2}, \ \delta_{12} = \delta_1 + \delta_2$.

To obtain the satisfactory description of the $\pi\pi$ scattering from the threshold to 1.89 GeV, we have taken $B_\pi = 1$, and three states turned out to be sufficient: the two ones of the type (**a**) ($f_0(665)$ and $f_0(980)$) and $f_0(1500)$ of the type (**c**). The following zero positions on the v-plane, corresponding to these resonances, have been established:

for $f_0(665)$:	$v_1 = 1.36964 + 0.208632i,$	$v_2 = 0.921962 - 0.25348i,$
for $f_0(980)$:	$v_3 = 1.04834 + 0.0478652i,$	$v_4 = 0.858452 - 0.0925771i,$
for $f_0(1500)$:	$v_5 = 1.2587 + 0.0398893i,$	$v_6 = 1.2323 - 0.0323298i,$
	$v_7 = 0.809818 - 0.019354i,$	$v_8 = 0.793914 - 0.0266319i.$

Here for the $\pi\pi$ phase shift δ_1 and the elasticity parameter η, 113 and 50 experimental points [15], respectively, are used; when rejecting the points at 0.61, 0.65, and 0.73 GeV for δ_1 and at 0.99, 1.65, and 1.85 GeV for η, which give an anomalously large contribution to χ^2, we obtain for χ^2/ndf the values 2.7 and 0.72, respectively; the total χ^2/ndf in the case of the $\pi\pi$ scattering is 1.96. The corresponding curves (solid) demonstrating the quality of these fits are shown in Figs. 2 and 3.

With the presented picture, the satisfactory description for the modulus (ξ) of the $\pi\pi \to K\overline{K}$ matrix element is given from the threshold to $\sim$ 1.4 GeV (Fig. 4, the solid curve). Here 35 experimental points [16] are used; $\chi^2/\text{ndf} \approx 1.11$ when eliminating the points at energies 1.002, 1.265, and 1.287 GeV (with especially large contribution to χ^2). However, for the phase shift $\delta_{12}(s)$, slightly excessive curve is obtained. Therefore, keeping the *parameterless* description of the $\pi\pi$ background, one must take into account the part of the $K\overline{K}$ background that does not contribute to the $\pi\pi$-scattering amplitude.

Note that on the v-plane, S_{11} has no cuts; however, the amplitudes of the processes $K\overline{K} \to \pi\pi, K\overline{K}$ do have the cuts which arise from the left-hand cut on the s-plane, starting at $s = 4(m_K^2 - m_\pi^2)$. This left-hand cut will be neglected in the Riemann-surface structure, and the contribution on the cut will be taken into account in the $K\overline{K}$ background as a pole on the real s-axis on the physical sheet in the sub-$K\overline{K}$-threshold region. On the v-plane, this pole gives two poles on the unit circle in the upper half-plane, symmetric to each other with respect to the imaginary axis, and two zeros, symmetric to the poles with respect to the real axis, *i.e.*, one additional parameter is introduced (a position p of the pole on the unit circle). Therefore, for B_K

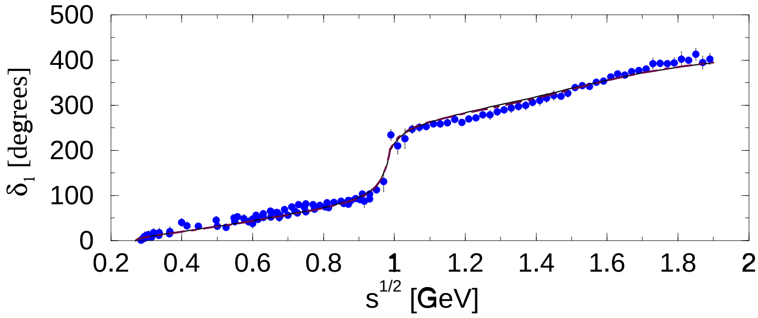

Fig. 2. The energy dependence of the phase shift δ_1 of the $\pi\pi$ scattering amplitude obtained on the basis of a simultaneous analysis of the experimental data on the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$ for both versions – without and with the $f_0(1370)$ state: the solid curve corresponds to version 1; the dot-dashed one, version 2 (for δ_1 the both curves practically coincide). The data on the $\pi\pi$ scattering are taken from Refs. [15].

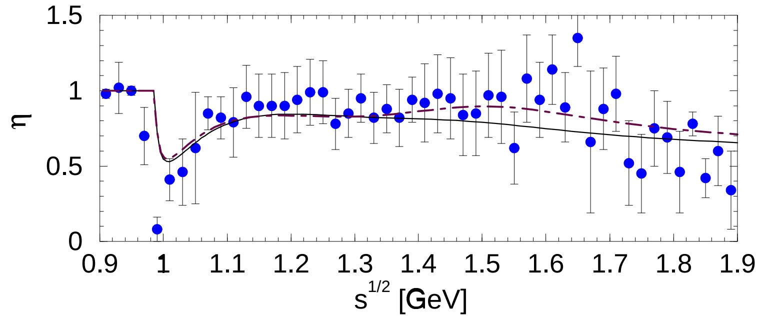

Fig. 3. The same as in Fig. 2 but for the elasticity parameter η.

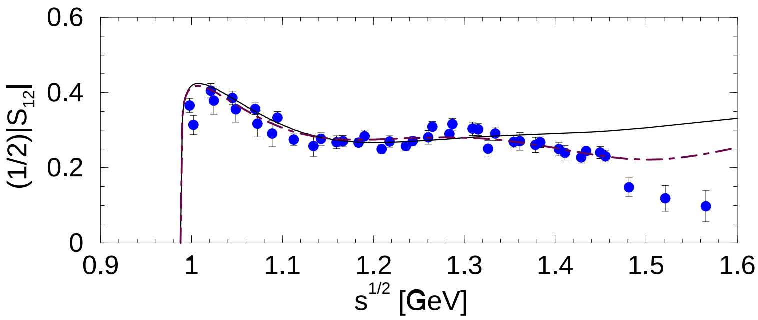

Fig. 4. The energy dependences of the ($|S_{12}|$) for both versions without and with the $f_0(1370)$ state: the solid curve corresponds to version 1; the dot-dashed one, version 2. The data on the process $\pi\pi \to K\overline{K}$ are taken from Refs. [16].

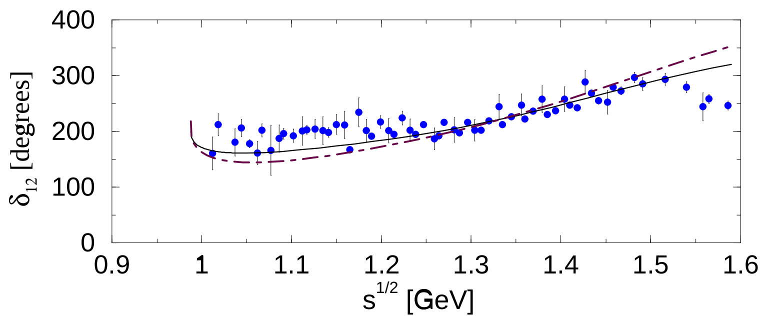

Fig. 5. The same as in Fig. 4 but for the phase shift (δ_{12}).

we take the form

$$B_K = v^{-4}(1 - pv)^4(1 + p^* v)^4. \tag{5}$$

The fourth power in (5) is stipulated by the following. First, a pole on the real s-axis on sheet I in S_{22} is accompanied by a pole on sheet II at the same s-value (as it is seen from eqs. (1)); on the v-plane, this implies the pole of second order. Second, for the s-channel process $\pi\pi \to K\overline{K}$, the crossing u- and t-channels are the $\pi - K$ and $\overline{\pi} - K$ scattering (exchanges in these channels give contributions on the left-hand cut). This results in the additional doubling of the multiplicity of the indicated pole on the v-plane. The expression (5) does not contribute to S_{11}, *i.e.* the parameterless description of the $\pi\pi$ background is kept intact.

A satisfactory description of the $\delta_{12}(\sqrt{s})$ (Fig. 5, the solid curve) is obtained to ~1.52 GeV with the parameter $p = 0.948201 + 0.31767i$ (this corresponds to the pole position on the s-plane at $s = 0.434 \mathrm{GeV}^2$). Here 59 experimental points [16] are considered; $\chi^2/\mathrm{ndf} \approx 3.05$ when eliminating the points at 1.117, 1.247, and 1.27 GeV (with especially large contribution to χ^2). The total χ^2/ndf for four analyzed quantities to describe the processes $\pi\pi \to \pi\pi, K\overline{K}$ is 2.12; the number of adjusted parameters is 17 i.e. the pole positions as in Table I and the pole corresponding to the parameter p.

Table I. Pole clusters for obtained resonances in version 1.

	$f_0(665)$		$f_0(980)$		$f_0(1500)$	
Sheet	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV
II	610±14	620±26	988±5	27±8	1530±25	390±30
III	720±15	55±9	984±16	210±22	1430±35	200±30
					1510±22	400±34
IV					1410±24	210±38

In Table I, the obtained poles on the corresponding sheets are shown on the complex energy plane ($\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r$). Since, for wide resonances, values of masses and widths are very model-dependent, it is reasonable to report characteristics of pole clusters which must be rather stable for various models.

Now we can calculate the coupling constants of obtained states with the $\pi\pi$ and $K\overline{K}$ systems through the residues of amplitudes at the pole on sheet II. Expressing the T-matrix via the S-matrix as $S_{ii} = 1 + 2i\rho_i T_{ii},\ S_{12} = 2i\sqrt{\rho_1\rho_2}T_{12}$, where $\rho_i = \sqrt{(s-4m_i^2)/s}$, taking the resonance part of the amplitude in the form: $T_{ij}^{res} = \sum_r g_{ir}g_{rj}D_r^{-1}(s)$, where $D_r(s)$ is an inverse propagator ($D_r(s) \propto s - s_r$), and denoting the coupling constants with the $\pi\pi$ and $K\overline{K}$ systems through g_1 and g_2, respectively, we obtain for $f_0(665)$: $g_1 = 0.7477 \pm 0.095$ GeV and $g_2 = 0.834 \pm 0.1$ GeV, for $f_0(980)$: $g_1 = 0.1615 \pm 0.03$ GeV and $g_2 = 0.438 \pm 0.028$ GeV, for $f_0(1500)$: $g_1 = 0.899 \pm 0.093$ GeV.

In this 2-channel approach, there is no point in calculating the coupling constant of the $f_0(1500)$ state with the $K\overline{K}$ system, because the 2-channel unitarity is valid only to 1.4 GeV, and, above this energy, there is a considerable disagreement between the calculated amplitude modulus S_{12} and experimental data.

We see that a minimum scenario of the simultaneous description of the processes $\pi\pi \to \pi\pi, K\overline{K}$ does not require the $f_0(1370)$ resonance. Therefore, if this meson exists, it must be relatively more weakly coupled to the $\pi\pi$ channel than to the $K\overline{K}$ one, *i.e.* be described by the pole cluster of type (**b**) (this would testify to the dominant $s\bar{s}$ component in this state). To confirm quantitatively this qualitative conclusion [17], which is distinctive from the one of other works [2], we consider the 2nd version including the $f_0(1370)$ of type (**b**)[5] in addition to three above observed resonances. In the figures this solution is represented by the dot-dashed curves. The description of the $\pi\pi$ scattering from the threshold up to 1.89 GeV is practically the same as without the $f_0(1370)$: χ^2/ndf for two quantities δ_1 and η is 2.01. The description of experimental data is improved a little for $|S_{12}|$ which is described now up to $\sim$ 1.46 GeV. For this quantity, we consider now 41 experimental points [16]; $\chi^2/\text{ndf} \approx 0.92$. However, on the whole, the description is even worse in comparison with the 1st version: the total $\chi^2/\text{ndf} \equiv 2.93$ for four analyzed quantities to describe the processes $\pi\pi \to \pi\pi, K\overline{K}$ (cf. 2.12 for the 1st case). The number of adjusted parameters is 21, where they all are positions of the poles describing resonances except a single one related to the $K\overline{K}$ background which is $p = 0.976745 + 0.214405i$ (this corresponds to the pole on the s-plane at $s = 0.622\text{GeV}^2$). Let us indicate the obtained zero positions on the v-plane, corresponding to considered resonances in version 2:

$$
\begin{array}{lll}
\text{for } f_0(665): & v_1 = 1.36783 + 0.212659i, & v_2 = 0.921962 - 0.25348i, \\
\text{for } f_0(980): & v_3 = 1.04462 + 0.0479703i, & v_4 = 0.858452 - 0.0925771i, \\
\text{for } f_0(1370): & v_5 = 1.22783 - 0.0483842i, & v_6 = 0.802595 - 0.0379537i, \\
\text{for } f_0(1500): & v_7 = 1.2587 + 0.0398893i, & v_8 = 1.24837 - 0.0358916i, \\
 & v_9 = 0.804333 - 0.0179899i, & v_{10} = 0.795579 - 0.0253985i.
\end{array}
$$

In Table II, the obtained poles on the corresponding sheets of the Riemann surface are shown on the complex energy plane ($\sqrt{s_r} = \text{E}_r - i\Gamma_r$).

When calculating the coupling constants in version 2, we should take, for the $f_0(1370)$, the residues of amplitudes at the pole on sheet IV. Then we obtain (in GeV units): for $f_0(665)$: $g_1 = 0.724 \pm 0.095$ and $g_2 = 0.582 \pm 0.1$, for $f_0(980)$: $g_1 = 0.153 \pm 0.03$ and $g_2 = 0.521 \pm 0.028$, for $f_0(1370)$: $g_1 = 0.11 \pm 0.03$ and $g_2 = 0.91 \pm 0.04$, for $f_0(1500)$: $g_1 = 0.866 \pm 0.09$.

So, the $f_0(980)$ and especially the $f_0(1370)$ are coupled essentially more strongly to the $K\overline{K}$ system than to the $\pi\pi$ one. This tells about the dominant $s\bar{s}$ component in the $f_0(980)$ state and

[5] The clusters of type (**a**) and (**c**) for the $f_0(1370)$ state are rejected as is shown by this analysis.

Table II. Pole clusters for obtained resonances in version 2.

Sheet	$f_0(665)$		$f_0(980)$		$f_0(1370)$		$f_0(1500)$	
	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV
II	610±14	610±26	986±5	25±8			1530±22	390±28
III	720±15	55±9	984±16	210±25	1340±21	380±25	1490±30	220±25
							1510±22	370±30
IV					1330±18	270±20	1490±20	300±35

especially in the $f_0(1370)$ one.

Let us also indicate the scattering lengths calculated for both versions. For the $K\overline{K}$ scattering:

$$a_0^0(K\overline{K}) = -1.188 \pm 0.13 + (0.648 \pm 0.09)i,\ [m_{\pi^+}^{-1}]; \quad (\text{version1}),$$
$$a_0^0(K\overline{K}) = -1.497 \pm 0.12 + (0.639 \pm 0.08)i,\ [m_{\pi^+}^{-1}]; \quad (\text{version2}).$$

The imaginary part in $a_0^0(K\overline{K})$ means that, already at the threshold of the $K\overline{K}$ scattering, other channels ($2\pi, 4\pi$ etc.) are opened. We see that the real part of the $K\overline{K}$ scattering length is very sensitive to the existence of the $f_0(1370)$ state.

In Table III, we compare our results for the $\pi\pi$ scattering length a_0^0, obtained for both versions, with results of some other works both theoretical and experimental. We see that our results

Table III. Comparison of results of various works for the $\pi\pi$ scattering length a_0^0.

$a_0^0,\ m_{\pi^+}^{-1}$	References	Remarks
0.27 ± 0.06 (1) 0.266 (2)	our paper	model-independent approach
0.26 ± 0.05	L. Rosselet et al. [15]	analysis of the decay $K \to \pi\pi e\nu$ using Roy's model
0.24 ± 0.09	A.A. Bel'kov et al. [15]	analysis of $\pi^- p \to \pi^+\pi^- n$ using the effective range formula
0.23	S. Ishida et al. [6]	modified analysis of $\pi\pi$ scattering using Breit-Wigner forms
0.16	S. Weinberg [18]	current algebra (non-linear σ-model)
0.20	J. Gasser, H. Leutwyler [19]	one-loop corrections, non-linear realization of chiral symmetry
0.217	J. Bijnens at al. [20]	two-loop corrections, non-linear realization of chiral symmetry
0.26	M.K. Volkov [21]	linear realization of chiral symmetry
0.28	A.N. Ivanov, N.I. Troitskaya [22]	a variant of chiral theory with linear realization of chiral symmetry

correspond best to the linear realization of chiral symmetry.

We have here presented model-independent results: the pole positions, coupling constants and scattering lengths. Masses and widths of these states that should be calculated from the obtained pole positions and coupling constants are highly model-dependent. Let us demonstrate this.

If we suppose, that the obtained state $f_0(665)$ is the σ-meson, then from the known relation of the σ-model between the coupling constant of the σ with the $\pi\pi$-system and masses $g_{\sigma\pi\pi} = (m_\sigma^2 - m_\pi^2)/\sqrt{2}f_{\pi^0}$ (here f_{π^0} is the constant of the weak decay of the π^0: $f_{\pi^0} = 93.1$ MeV), we obtain $m_\sigma \approx 342$ MeV.

If we take the resonance part of amplitude as $T^{res} = \sqrt{s}\Gamma/(m_\sigma^2 - s - i\sqrt{s}\Gamma)$, we obtain $m_\sigma \approx 850$ MeV and $\Gamma \approx 1240$ MeV.

4 Conclusions

In the present model-independent approach, a simultaneous description of the isoscalar s-wave channel of the processes $\pi\pi \to \pi\pi, K\overline{K}$ from the thresholds to the energy values, where the 2-channel unitarity is valid, is obtained with a parameterless description of the $\pi\pi$ background. Thus, a model-independent confirmation of the state, already discovered in other works [5]- [8] (or claiming this discovery) and denoted in the PDG issues by $f_0(400-1200)$ [2], is obtained.

A parameterless description of the $\pi\pi$ background is given by allowance for the left-hand branch-point in the proper uniformizing variable. Therefore, all the adjusted parameters in describing the $\pi\pi$ scattering are the positions of poles corresponding to resonances, and we conclude that our model-independent approach is a valuable tool for studying the realization schemes of chiral symmetry. The existence of the low-lying state $f_0(665)$ with the properties of the σ-meson and the obtained $\pi\pi$-scattering length ($a_0^0(\pi\pi) \approx 0.27$) suggest the linear realization of chiral symmetry.

The analysis of the used experimental data gives the evidence that, if the $f_0(1370)$ resonance exists (version 2), it has the dominant $s\bar{s}$ component, because the ratio of its coupling constant with the $\pi\pi$ channel to the one with the $K\overline{K}$ channel is 0.12 (as to that assignment of the $f_0(1370)$, we agree with the work [23]). However, this topic would need an additional investigation because in this kinematic region the influence of the $\eta\eta$ channel on the behaviour of the $\pi\pi \to K\overline{K}$ amplitude can be expected. On the other hand, the decay modes of the $f_0(1370)$ to $\pi\pi, 4\pi, \eta\eta, K\overline{K}$ are seen [2]. Note that a minimum scenario of the simultaneous description of processes $\pi\pi \to \pi\pi, K\overline{K}$ does without the $f_0(1370)$ resonance. The $K\overline{K}$ scattering length is very sensitive to whether this state exists or not.

The $f_0(1500)$ state is represented by the pole cluster which corresponds to a flavour singlet, *e.g.* the glueball.

We emphasize that the obtained results are model-independent, since they are based on the first principles and on the mathematical fact that a local behaviour of analytic functions, determined on the Riemann surface, is governed by the nearest singularities on all sheets.

We think that multichannel states are most adequately represented by clusters, *i.e.*, by the pole positions on all the corresponding sheets. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances.

Finally, note that in the model-independent approach, there are many adjusted parameters (although, *e.g.* for the $\pi\pi$ scattering, they all are positions of poles describing resonances). The number of these parameters can be diminished by some dynamic assumptions, but this is another approach and of other value.

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